Computer Graphics - CS402

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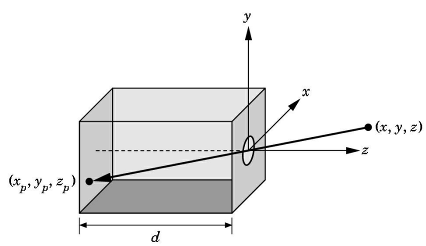
Homework # 1

1. For a pinhole camera (Figure 1(a)), the focal length is the distance from the pinhole to the film plane. The dimensions of a frame of 35-mm film are about 24mm (width along x-direction) $\times$ 36mm (height along y direction). The **field of view** or **angle of view** of the pinhole camera is the angle made by the largest object that the camera can image on its film plane. A real **field of view** should be a volume angle which is hard to deal with. People use surface angles of view such as the one in the xz-plane, the yz-plane (Figure 1(b)), or in the planes passing through one of the diagonals of the film plane. Assuming that the human visual system has an angle of view of 90^o, what focal length should we use with 35-mm film to achieve a natural view in

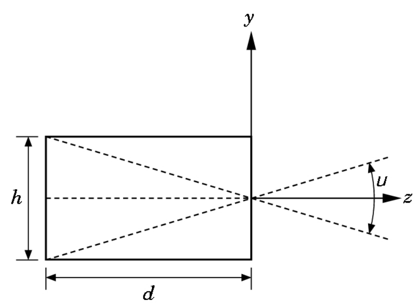
   (a) the xz-plane,

   (b) the yz-plane, and

   (c) one of any of the planes passing through one of the diagonals of the film plane?



(a)



(b)

Figure 1: (a) Pinhole Camera and Perspective Projection (b) Angle of View in yz-plane.

- As per (Figure 1(b)), we can write the following formula to find the focal length d:

$$\tan\frac{u}{2} = \frac{\frac{h}{2}}{d} \Rightarrow d = \frac{h}{2\tan\frac{u}{2}}.$$

For an angle of view of $u = 90^0$, the focal length d is equal to

$$d = \frac{h}{2\tan\frac{u}{2}} = \frac{h}{2}.$$

But, what do we use as h?

(a) We can take h as the dimension of the film plane along the x direction for which case we get

$$d = \frac{h}{2} = \frac{24}{2} = 12 \text{ mm}.$$

(b) We can take h as the dimension of the film plane along the y direction for which case we get

$$d = \frac{h}{2} = \frac{36}{2} = 18 \text{ mm}.$$

(c) We can take h as the diagonal of the film plane, i.e. $h = \sqrt{24^2 + 36^2} = 12\sqrt{13}$ mm, for which case the focal length d is equal to

$$d = \frac{h}{2\tan\frac{u}{2}} = \frac{h}{2} = 6\sqrt{13} \approx 21.6 \text{ mm}$$

2. Consider a pinhole camera (Figure 1). What is the shape of the image of circular disk? Assume perspective projection and allow the disk to lie in a plane parallel to the image plane within the field of view of the camera.

- Without loss of generality, we can assume the disk lying in the plane $z = z_0$, centered at $(0, 0)$ in the coordinates (x, y), and of radius R. It follows that its equation is $x^2 + y^2 = R^2$. The coordinates of the projected points on the film plane are

$$X = -\frac{xd}{z_0}, \qquad Y = -\frac{yd}{z_0}$$

Solving for x, y and plugging in their values in the previous equation, we get

$$\left(-\frac{Xz_0}{d}\right)^2 + \left(-\frac{Yz_0}{d}\right)^2 = R^2 \Longrightarrow X^2 + Y^2 = \left(\frac{Rd}{z_0}\right)^2$$

3. Consider a wire-frame object defined by the following four 3D points (X, Y, Z) lying in the field of view of pinhole camera with focal length d:

- $p_1 : (0.5, 0.5, 1)$
- $p_2 : (1, 0, 1)$
- $p_3 : (0, 0, 1)$
- $p_4 : (1, 1, 2)$

These vertices are connected with single lines between p_1 and p_2, p_2 and p_3, p_3 and p_4, but not between p_4 and p_1. Remembering that, for perspective projection, the coordinates of projection onto an image plane are given by:

$$y = -\frac{Yd}{Z}, \qquad x = -\frac{Xd}{Z}$$

The 3D coordinates of the object are given by uppercase letters (X, Y, Z) and the 2D coordinates of the projected image are given by lowercase letters (x, y).

(a) Compute the x and y coordinates for each point projected onto and $x - y$ image plane located at a distance of 1 from the center of projection ($d = 1$).

(b) Sketch the projected image of this object onto the $x - y$ projection plane, and label each of the projected points with their (x, y) coordinates.

- (a) Given the equations for perspective projection, with $d = 1$, the four points project as follow (recall that $x = Xd/Z; y = Yd/Z$):

$$p_1 : (0.5, 0.5, 1.0) \Longrightarrow x_1 = 0.5/1, y_1 = 0.5/1 \Longrightarrow P_1 = (0.5, 0.5)$$

$$p_2 : (1.0, 0.0, 1.0) \Longrightarrow P_2 = (1.0, 0.0$$

$$p_3 : (0.0, 0.0, 1.0) \Longrightarrow P_3 = (0.0, 0.0)$$

$$p_4 : (1.0, 1.0, 2.0) \Longrightarrow P_4 = (0.5, 0.5)$$

Note that p_1 and p_4 project to the same point on the image plane.

(b) The projection is drawn in the figure below

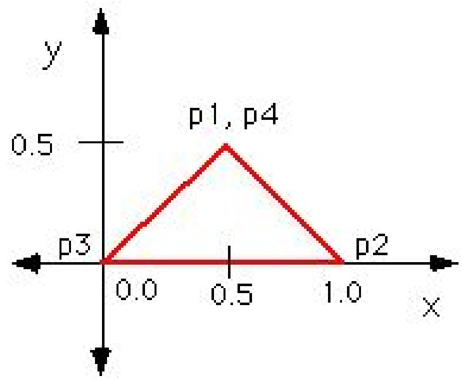


Figure 2: Projected figure

4. If we use direct coding of RGB values with 10 bits per primary color, how many possible colors do we have for each pixel?

- We will have $2^{10} \times 2^{10} \times 2^{10} = 1073741824$ possible colors.

5. If we use 12-bit pixel values in a lookup table representation, how many entries does the lookup table have?

- The lookup table will have $2^{12} = 4096$ entries.