

MATLAB中“fitgmdist”的用法及其GMM聚类算法

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高斯混合模型的基本原理：[聚类——GMM](#)，MATLAB官方文档中有关于fitgmdist的介绍：[fitgmdist](#)。我之前写过有关GMM聚类的算法：[GMM算法的matlab程序](#)。这篇文章主要应用MATLAB自带的函数来进行聚类。

1. fitgmdist函数介绍

fitgmdist的使用形式：`gmm = fitgmdist(X,k,Name,Value)`

输入

`'RegularizationValue', 0`。（取值：0, 0.1, 0.01,..., 正则化系数，防止协方差奇异）

`'CovarianceType', 'full'`。（取值：'full'，协方差矩阵是非对角阵，'diagonal'，协方差矩阵为对角阵）

`'Start', 'plus'`。（取值：'randSample'，随机初始化，'plus'，k-means++初始化，'S'，自定义初始化），其中S = `struct('mu',init_Mu,'Sigma',init_Sigma,'ComponentProportion',init_Components);`

`'Options', statset('Display', 'final', 'MaxIter', MaxIter, 'TolFun', TolFun)`。（'Display'有三个取值：'final' 显示最终的输出结果、'iter' 显示每次迭代的结果、'off' 不显示优化参数信息；'MaxIter'：默认100，最大迭代次数；'TolFun'：默认1e-6，目标函数的终止误差）

输出

`gmm.mu`：更新完后的聚类中心（均值）

`gmm.Sigma`：更新完后的协方差矩阵

`gmm.ComponentProportion`：更新完后的混合比例

`gmm.NegativeLogLikelihood`：更新完后的负对数似然函数

`gmm.NumIterations`：实际迭代次数

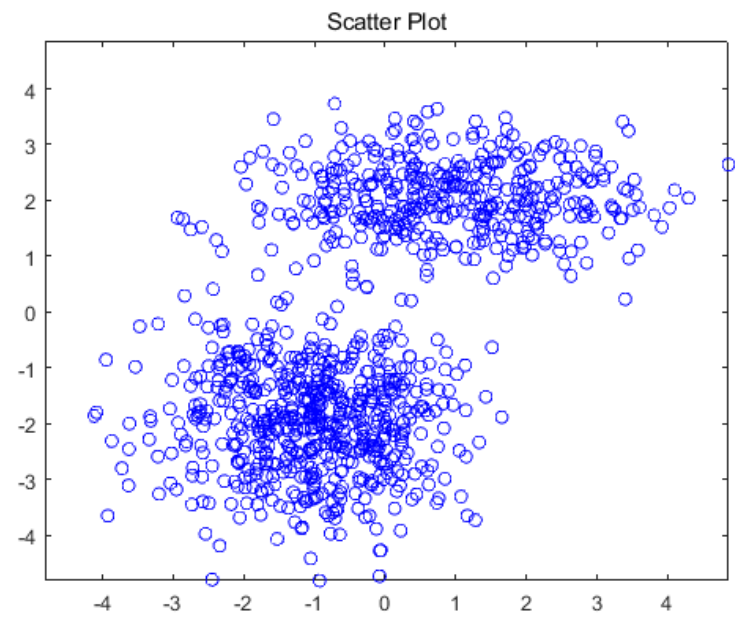
`gmm.BIC`：贝叶斯信息准则，用于模型选择

更多参数，请在命令行输入`properties(gmm)`

2. 高斯混合模型聚类实例

generate.m

```
function data=generate()  
%生成数据  
mu1 = [1 2];  
Sigma1 = [2 0; 0 0.5];  
mu2 = [-1 -2];  
Sigma2 = [1 0;0 1];  
data = [mvnrnd(mu1,Sigma1,400), ones(400,1);mvnrnd(mu2,Sigma2,600), 2*ones(600,1)];  
X=[data(:, 1), data(:, 2)];  
figure(1)  
plot(X(:,1), X(:,2),'bo')  
title('Scatter Plot')  
xlim([min(X(:)) max(X(:))]) % Make axes have the same scale  
ylim([min(X(:)) max(X(:))])
```



具体数据

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-0.798625575507672	2.14835001132099	1
2.82002920206994	1.97084621196340	1
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-1.83227505417862	-1.40561210105766	2
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-2.10309516122273	-2.36908619866192	2
-1.37657318496037	-1.33643223891869	2

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-1.48450148214270	-3.47471076726943	2
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-0.831643709133855	-2.56984989677121	2
-0.928267641983258	-4.81476418501057	2
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-2.16720737476316	-1.19244168426693	2
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-0.574981863784512	-2.29540002373887	2
-0.742720166810230	-3.40044065166058	2
-1.55818044803369	-1.89982065599376	2
-2.08530289864125	-2.69614408848973	2
-2.27668246827953	-3.03166385728041	2
-0.928842867753549	-1.94813604188420	2
-3.02265903316559	-2.53321405989764	2
0.152454341677487	-1.21043788373565	2
-2.32574943116236	-0.949640745392916	2
-2.09064044178726	-3.12828760205670	2
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0.134886977667473	-3.52136629978657	2
-1.89474855779492	-2.02192726086667	2
-0.318034104228869	-2.35306642253285	2
-1.61800574924745	-0.825524936471070	2
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-0.0936742645656620	-1.31823674467175	2
-1.79926641137595	-2.05147102529154	2
-0.777846784073033	-2.19294306164927	2
-0.747185845555017	-0.865097342773531	2
-0.0769862371374327	-1.75091909063040	2
-0.388773373440707	-1.67536345759349	2
-1.09941472513134	-2.71704690192925	2
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-0.755676079038423	-1.36263377313167	2
-1.84601940010296	-1.42132892959564	2
-2.05171794126506	-0.805237134419860	2
-2.68044484862042	-1.91632039544155	2
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-0.996608692945777	-2.07791925611247	2
-1.44213421076704	-1.80431573962877	2
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-0.677318908345893	-2.04659672596894	2
-2.59014848084703	-2.95994822892238	2
-0.343753351611487	-1.40446767547124	2
0.0136515736996878	-0.546580984222088	2
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-1.91665282758399	-1.23681086631105	2

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-1.34295133105490	-1.89929569418714	2
-2.36159542799308	-1.22859416624455	2
-3.62495313921221	-2.45809254186177	2
-0.740548874058931	-1.29966302603733	2
-2.67921920336005	-1.18542623160203	2
-0.847522322899486	-1.15560224742330	2
0.741840918731699	-0.496338051365575	2
-2.54221691809446	-3.97571910611358	2
-0.640870911917068	-3.98946416736222	2
-1.27601757941708	-2.19574158817790	2
-1.99518769488596	-2.50955319147105	2
-0.0971121023003621	-1.94763245438242	2
0.758929387865149	-3.34051699723595	2
-1.46945090437368	0.131656964194621	2
-0.774828435874664	-1.98917132945057	2
-1.44524731731601	-2.87227670173351	2
0.206600225332415	-1.61159479256529	2
-0.890612193740134	-3.25470953464458	2
-0.783526850981737	-1.46513244158462	2
0.229291351306705	-2.30369791576758	2
-0.0766703376419500	-2.93769333289539	2
-1.64705658546377	-2.19363222244332	2
-2.02564827857175	-2.23661197841419	2
0.580602656916380	-1.38403756935533	2
-1.27308082491469	-2.20514718975902	2
-2.92790668922674	-1.98439888443983	2
-1.62181912776895	-0.851506362212275	2

GMM_main.m

```
function [accuracy,NumIterations]=GMM_main(data, K)
%主函数
[~, data_dim]=size(data);
X=data(:, 1:data_dim-1); %数据
real_label=data(:, data_dim);
[label,~, NumIterations]=Matlab_gmm_2(X, K);
accuracy=succeed(real_label,K, label);
```

Matlab_gmm.m

```
function [label, NegativeLogLikelihood, NumIterations]=Matlab_gmm(X, K)
%协方差矩阵为对角阵，数据独立同分布
[X_num,X_dim]=size(X);
para_sigma_inv=zeros(X_dim, X_dim, K);
N_pdf=zeros(X_num, K); %单高斯分布的概率密度函数
RegularizationValue=0.001; %正则化系数，协方差矩阵求逆
MaxIter=100; %最大迭代次数
TolFun=1e-8; %终止条件
% 自己设置初始化参数
% init_Mu = [1 1; 2 2];
% init_Sigma(:, :, 1) = [1 1; 1 2];
% init_Sigma(:, :, 2) = 2*[1 1; 1 2];
% init_Components = [1/2, 1/2];
% S = struct('mu', init_Mu, 'Sigma', init_Sigma, 'ComponentProportion', init_Components);
% gmm=fitgmdist(X, K, 'RegularizationValue', RegularizationValue, 'CovarianceType', 'diagonal', 'Start', 'S', 'Options', statset('Display', 'final', 'MaxIter', MaxIter, 'TolFun', TolFun));
gmm=fitgmdist(X, K, 'RegularizationValue', RegularizationValue, 'CovarianceType', 'diagonal', 'Start', 'plus', 'Options', statset('Display', 'final', 'MaxIter', MaxIter, 'TolFun', TolFun));
```

```

NegativeLogLikelihood=gmm.NegativeLogLikelihood;
NumIterations=gmm.NumIterations; %迭代次数
mu=gmm.mu; %均值
Sigma=gmm.Sigma; %协方差矩阵
ComponentProportion=gmm.ComponentProportion; %混合比例
for k=1:K
    sigma_inv=1./Sigma(:, :,k); %sigma的逆矩阵, (X_dim, X_dim)的矩阵
    para_sigma_inv(:, :, k)=diag(sigma_inv); %sigma^(-1)
end
for k=1:K
    coefficient=(2*pi)^(-X_dim/2)*sqrt(det(para_sigma_inv(:, :, k))); %高斯分布的概率密度函数e左边的系数
    X_miu=X-repmat(mu(k,:), X_num, 1); %X-miu: (X_num, X_dim)的矩阵
    exp_up=sum((X_miu*para_sigma_inv(:, :, k)).*X_miu,2); %指数的幂, (X-miu)'*sigma^(-1)*(X-miu)
    N_pdf(:,k)=coefficient*exp(-0.5*exp_up);
end
responsivity=N_pdf.*repmat(ComponentProportion,X_num,1); %响应度responsivity的分子, (X_num,K)的矩阵
responsivity=responsivity./repmat(sum(responsivity,2),1,K); %responsivity:在当前模型下第n个观测数据来自第k个分模型的概率, 即分模型k对观测数据Xn的响应度
%聚类
[~,label]=max(responsivity,[],2);
figure(2)
scatter(X(:,1),X(:,2),10,'.') % Scatter plot with points of size 10
hold on
gmPDF = @(x,y)reshape(pdf(gmm,[x(:) y(:)]),size(x));
fcontour(gmPDF,[-6 6])

```

Matlab_gmm_2.m

```

function [label, NegativeLogLikelihood, NumIterations]=Matlab_gmm_2(X, K)
%协方差矩阵为非对角阵, 数据不独立
[X_num,X_dim]=size(X);
N_pdf=zeros(X_num, K); %单高斯分布的概率密度函数
RegularizationValue=0.001; %正则化系数, 协方差矩阵求逆
MaxIter=100; %最大迭代次数
TolFun=1e-8; %终止条件
% 自己设置初始化参数
% init_Mu = [1 1; 2 2];
% init_Sigma(:, :,1) = [1 1; 1 2];
% init_Sigma(:, :,2) = 2*[1 1; 1 2];
% init_Components = [1/2,1/2];
% S = struct('mu',init_Mu,'Sigma',init_Sigma,'ComponentProportion',init_Components);
% gmm=fitgmdist(X, K, 'RegularizationValue', RegularizationValue, 'CovarianceType', 'diagonal', 'Start', 'S', 'Options', statset('Display', 'final', 'MaxIter', MaxIter, 'TolFun', TolFun));
gmm=fitgmdist(X, K, 'RegularizationValue', RegularizationValue, 'CovarianceType', 'full', 'Start', 'plus', 'Options', statset('Display', 'final', 'MaxIter', MaxIter, 'TolFun', TolFun));
NegativeLogLikelihood=gmm.NegativeLogLikelihood;
NumIterations=gmm.NumIterations; %迭代次数
mu=gmm.mu; %均值
Sigma=gmm.Sigma; %协方差矩阵
ComponentProportion=gmm.ComponentProportion; %混合比例
for k=1:K
    X_miu=X-repmat(mu(k,:), X_num, 1); %X-miu: (X_num, X_dim)的矩阵
    sigma_inv=inv(Sigma(:, :,k)); %sigma的逆矩阵, (X_dim, X_dim)的矩阵
    exp_up=sum((X_miu*sigma_inv).*X_miu,2); %指数的幂, (X-miu)'*sigma^(-1)*(X-miu)
    coefficient=(2*pi)^(-X_dim/2)*sqrt(det(sigma_inv)); %高斯分布的概率密度函数e左边的系数
    N_pdf(:,k)=coefficient*exp(-0.5*exp_up);
end
responsivity=N_pdf.*repmat(ComponentProportion,X_num,1); %响应度responsivity的分子, (X_num,K)的矩阵
responsivity=responsivity./repmat(sum(responsivity,2),1,K); %responsivity:在当前模型下第n个观测数据来自第k个分模型的概率, 即分模型k对观测数据Xn的响应度
%聚类
[~,label]=max(responsivity,[],2);
figure(2)

```

```
scatter(X(:,1),X(:,2),10,'.') % Scatter plot with points of size 10
hold on
gmPDF = @(x,y)reshape(pdf(gmm,[x(:) y(:)]),size(x));
fcontour(gmPDF,[-6 6])
```

succeed.m

```
function accuracy=succeed(real_label,K,id)
%输入K: 聚的类，id: 训练后的聚类结果，N*1的矩阵
N=size(id,1); %样本个数
p=perms(1:K); %全排列矩阵
p_col=size(p,1); %全排列的行数
new_label=zeros(N,p_col); %聚类结果的所有可能取值，N*p_col
num=zeros(1,p_col); %与真实聚类结果一样的个数
%将训练结果全排列为N*p_col的矩阵，每一列为一种可能性
for i=1:N
    for j=1:p_col
        for k=1:K
            if id(i)==k
                new_label(i,j)=p(j,k); %iris数据库，1 2 3
            end
        end
    end
end
end
%与真实结果比对，计算精确度
for j=1:p_col
    for i=1:N
        if new_label(i,j)==real_label(i)
            num(j)=num(j)+1;
        end
    end
end
end
accuracy=max(num)/N;
```

结果

以第二种情况为例，数据不独立，协方差矩阵不是只在对角线上有元素。

```
>> [accuracy,NumIterations]=GMM_main(data, 2)
32 iterations, log-likelihood = -3449.42
```

```
accuracy =

    0.9950000000000000
```

```
NumIterations =

    32
```

