MATLAB实例：非线性方程数值解法(迭代解)

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很久之前写过一篇关于“MATLAB用二分法、不动点迭代法及Newton迭代（切线）法求非线性方程的根”，本博文相当于之前这一篇的延续与拓展，介绍四种求解一元非线性方程的数值解法(迭代解)，包括：牛顿迭代法，Halley迭代法，Householder迭代法以及预测校正牛顿-哈雷迭代法(Predictor-Corrector Newton-Halley，PCNH)，具体参考文献[1]，来源于这篇文章：THREE-STEP ITERATIVE METHOD WITH EIGHTEENTH ORDER CONVERGENCE FOR SOLVING NONLINEAR EQUATIONS。

1. 迭代更新公式

- 牛顿迭代法

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)},n=0,1,\cdots$$

- Halley迭代法

$$x_{n+1}=x_n-\frac{2f(x_n)f'(x_n)}{2f'^2(x_n)-f(x_n)f''(x_n)},n=0,1,\cdots$$

- Householder迭代法

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}-\frac{f^2(x_n)f''(x_n)}{2f'^3(x_n)},n=0,1,\cdots$$

- 预测校正牛顿-哈雷迭代法(Predictor-Corrector Newton-Halley, PCNH)

预测
$$w_n=x_n-\frac{f(x_n)}{f'(x_n)}$$

校正
$$\begin{cases} y_n=w_n-\dfrac{2f(w_n)f'(w_n)}{2f'^2(w_n)-f(w_n)f''(w_n)} \\ x_{n+1}=y_n-\dfrac{f(y_n)}{f'(y_n)}-\dfrac{f^2(y_n)f''(y_n)}{2f'^3(y_n)},n=0,1,\cdots \end{cases}$$

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2. MATLAB程序

newton.m

```
function [x1, k]=newton(t1,esp,m)
syms x;
fun=x^3+4*(x^2)-10;
for k=1:m
   if abs(subs(diff(fun,'x'),x,t1))<esp
      x1=t1;
      break;
   else
      if subs(diff(fun,'x',2),x,t1)==0
         break;
         disp('解题失败！')
      else
         t0=t1;
         t1=t0-subs(fun,x,t0)/subs(diff(fun,'x'),x,t0);
         if abs(t1-t0)<esp
            x1=t1;
            break;
         end
      end
   end
end
% x1=vpa(x1,15);
```

halley.m

```
function [x1, k]=halley(t1,esp,m)
syms x;
fun=x^3+4*(x^2)-10;
for k=1:m
   if abs(subs(diff(fun,'x'),x,t1))<esp
      x1=t1;
      break;
   else
      if subs(diff(fun,'x',2),x,t1)==0
         break;
         disp('解题失败！')
      else
         t0=t1;
```

```
            t1=t0-(2*subs(fun,x,t0)*subs(diff(fun,'x'), x, t0))/(2*(subs(diff(fun,'x'), x, t0))^2-subs(fun, x, t0)*subs(diff(fun,'x',2),x,t0));
            if abs(t1-t0)<esp
                x1=t1;
                break;
            end
        end
    end
end
% x1=vpa(x1,15);
```

householder.m

```
function [x1, k]=householder(t1,esp,m)
syms x;
fun=x^3+4*(x^2)-10;
for k=1:m
    if abs(subs(diff(fun,'x'),x,t1))<esp
        x1=t1;
        break;
    else
        if subs(diff(fun,'x',2),x,t1)==0
            break;
            disp('解题失败！')
        else
            t0=t1;
            t1=t0-(subs(fun, x, t0))/(subs(diff(fun,'x'),x,t0))-(((subs(fun, x, t0))^2)*subs(diff(fun,'x',2),x,t0))/(2*(subs(diff(fun,'x',2),x,t0))^3);
            if abs(t1-t0)<esp
                x1=t1;
                break;
            end
        end
    end
end
% x1=vpa(x1,15);
```

PCNH.m

```
function [x1, k]=PCNH(t1,esp,m)
syms x;
fun=x^3+4*(x^2)-10;
for k=1:m
    if abs(subs(diff(fun,'x'),x,t1))<esp
        x1=t1;
        break;
    else
        if subs(diff(fun,'x',2),x,t1)==0
            break;
            disp('解题失败！')
        else
            t0=t1;
            w=t0-subs(fun,x,t0)/subs(diff(fun,'x'),x,t0);
            y=w-(2*subs(fun,x,w)*subs(diff(fun,'x'), x, w))/(2*(subs(diff(fun,'x'), x, w))^2-subs(fun, x, w)*subs(diff(fun,'x',2),x,w));
            t1=y-(subs(fun, x, y))/(subs(diff(fun,'x'),x,y))-(((subs(fun, x, y))^2)*subs(diff(fun,'x',2),x,y))/(2*(subs(diff(fun,'x',2),x,y))^3);
            if abs(t1-t0)<esp
                x1=t1;
                break;
            end
        end
    end
end
% x1=vpa(x1,15);
```

demo.m

```
clear
clc
% Input: 初始值，迭代终止条件，最大迭代次数
[x1, k1]=newton(1,1e-4,20); % 牛顿迭代法
[x2, k2]=halley(1,1e-4,20); % Halley迭代法
[x3, k3]=householder(1,1e-4,20); % Householder迭代法
[x4, k4]=PCNH(1,1e-4,20); % 预测校正牛顿-哈雷迭代法(PCNH)
fprintf('牛顿迭代法求解得到的方程的根为：%.15f, 实际迭代次数为：%d次\n', x1, k1);
fprintf('Halley迭代法求解得到的方程的根为：%.15f, 实际迭代次数为：%d次\n', x2, k2);
fprintf('Householder迭代法求解得到的方程的根为：%.15f, 实际迭代次数为：%d次\n', x3, k3);
fprintf('预测校正牛顿-哈雷迭代法(PCNH)求解得到的方程的根为：%.15f, 实际迭代次数为：%d次\n', x4, k4);
```

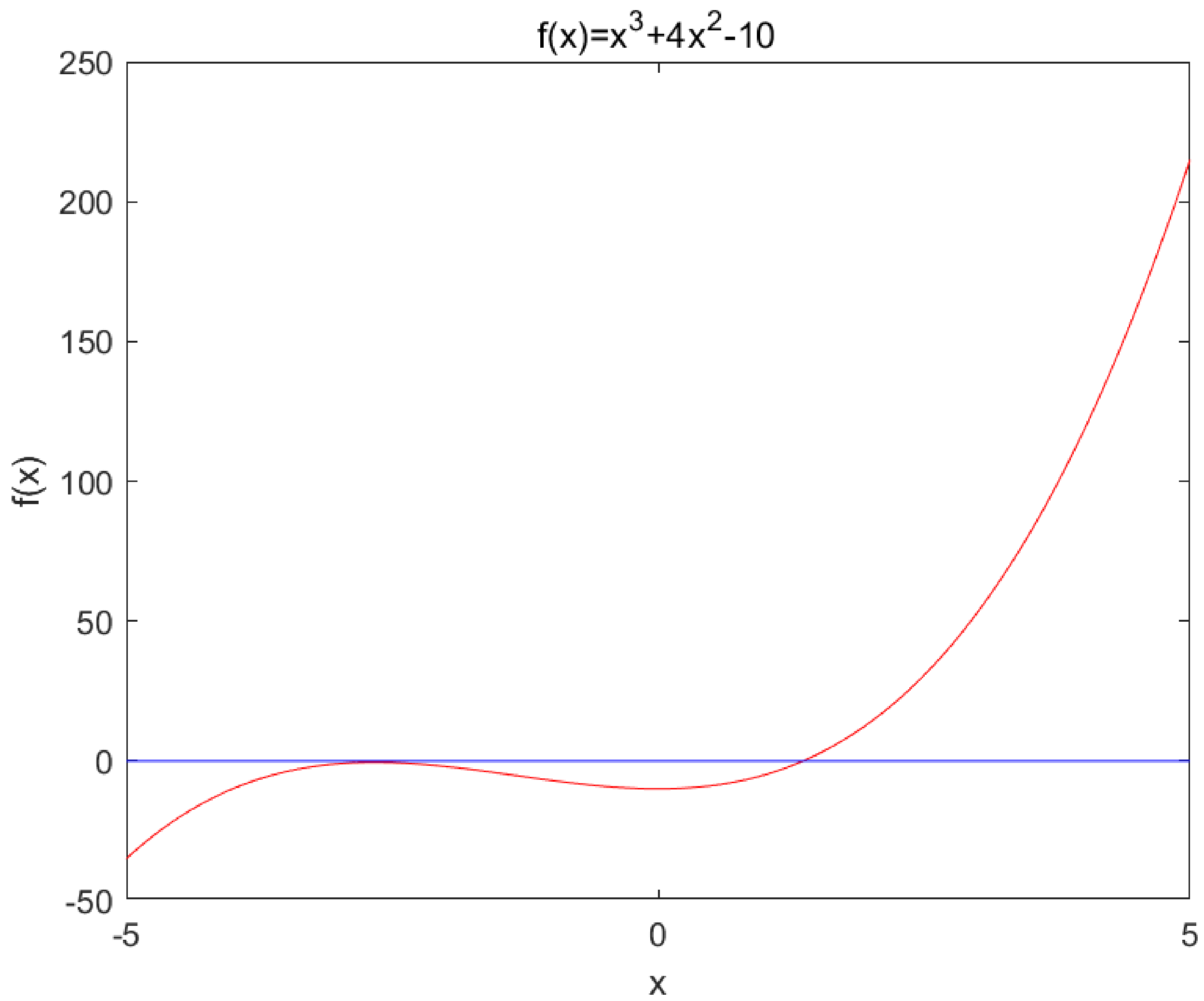
```
%% 函数图像
x=-5:0.01:5;
y=x.^3+4.*(x.^2)-10;
y_0=zeros(length(x));
plot(x, y, 'r-', x, y_0, 'b-');
xlabel('x');
ylabel('f(x)');
title('f(x)=x^3+4{x^2}-10');
saveas(gcf,sprintf('函数图像.jpg'),'bmp'); %保存图片
```

3. 数值结果

求解$f(x)=x^3+4{x^2}-10=0$方程在$x_0=1$附近的根。

牛顿迭代法求解得到的方程的根为：1.365230013435367, 实际迭代次数为：4次
Halley迭代法求解得到的方程的根为：1.365230013414097, 实际迭代次数为：3次
Householder迭代法求解得到的方程的根为：1.365230013391664, 实际迭代次数为：3次
预测校正牛顿-哈雷迭代法(PCNH)求解得到的方程的根为：1.365230013414097, 实际迭代次数为：2次

函数图像：



4．参考文献

[1] Bahgat, Mohamed & Hafiz, Mohammad. (2014). THREE-STEP ITERATIVE METHOD WITH EIGHTEENTH ORDER CONVERGENCE FOR SOLVING NONLINEAR EQUATIONS. International Journal of Pure and Applied Mathematics. 93.