MATLAB实例：PCA降维

作者：凯鲁嘎吉 - 博客园 http://www.cnblogs.com/kailugaji/

1. iris数据

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```

2. MATLAB程序

```
function [COEFF,SCORE,latent,tsquared,explained,mu,data_PCA]=pca_demo()
x=load('iris.data');
[~,d]=size(x);
k=d-1; %前k个主成分
x=zscore(x(:,1:d-1));  %归一化数据
[COEFF,SCORE,latent,tsquared,explained,mu]=pca(x);
% 1）获取样本数据 X ，样本为行，特征为列。
% 2）对样本数据中心化，得S（S = X的各列减去各列的均值）。
% 3）求 S 的协方差矩阵 C = cov(S)
% 4) 对协方差矩阵 C 进行特征分解 [P,Lambda] = eig(C);
% 5）结束。
% 1、输入参数 X 是一个 n 行 p 列的矩阵。每行代表一个样本观察数据，每列则代表一个属性，或特征。
% 2、COEFF 就是所需要的特征向量组成的矩阵，是一个 p 行 p 列的矩阵，没列表示一个出成分向量，经常也称为（协方差矩阵的）特征向量。并且是按照对应特征值降序排列的。所以，如果只需要前 k 个主成分向量，可通过：COEFF(:,1:k) 来获得。
% 3、SCORE 表示原数据在各主成分向量上的投影。但注意：是原数据经过中心化后在主成分向量上的投影。即通过：SCORE = x0*COEFF 求得。其中 x0 是中心平移后的 X（注意：是对维度进行中心平移，而非样本。），因此在重建时，就需要加上这个马
% 4、latent 是一个列向量，表示特征值，并且按降序排列。
% 5、tsquared Hotelling的每个观测值X的T平方统计量
% 6、explained 由每个主成分解释的总方差的百分比
% 7、mu 每个变量X的估计平均值
% x= bsxfun(@minus,x,mean(x,1));
data_PCA=x*COEFF(:,1:k);
latent1=100*latent/sum(latent);%将latent总和统一为100，便于观察贡献率
pareto(latent1);%调用matla画图 pareto仅绘制累积分布的前95%，因此y中的部分元素并未显示
xlabel('Principal Component');
ylabel('Variance Explained (%)');
% 图中的线表示的累积变量解释程度
print(gcf,'-dpng','Iris PCA.png');
iris_pac=data_PCA(:,1:2) ;
save iris_pca iris_pac
```

3. 结果

iris_pca：前两个主成分

```
-2.25698063306803       0.504015404227653
-2.07945911889541       -0.653216393612590
-2.36004408158421       -0.317413944570283
-2.29650366000389       -0.573446612971233
-2.38080158645275       0.672514410791076
-2.06362347633724       1.51347826673567
-2.43754533573242       0.0743137171331950
-2.22638326740708       0.246787171742162
-2.33413809644009       -1.09148977019584
-2.18136796941948       -0.447131117450110
-2.15626287481026       1.06702095645556
-2.31960685513084       0.158057945820095
```

-2.21665671559727	-0.706750478104682
-2.63090249246321	-0.935149145374822
-2.18497164997156	1.88366804891533
-2.24394778052703	2.71328133141014
-2.19539570001472	1.50869601039751
-2.18286635818774	0.512587093716441
-1.88775015418968	1.42633236069007
-2.33213619695782	1.15416686250116
-1.90816386828207	0.429027879924458
-2.19728429051438	0.949277150423224
-2.76490709741649	0.487882574439700
-1.81433337754274	0.106394361814184
-2.22077768737273	0.161644638073716
-1.95048968523510	-0.605862870440206
-2.04521166172712	0.265126114804279
-2.16095425532709	0.550173363315497
-2.13315967968331	0.335516397664229
-2.26121491382610	-0.313827252316662
-2.13739396044139	-0.482326258880086
-1.82582143036022	0.443780130732953
-2.59949431958629	1.82237008322707
-2.42981076672382	2.17809479520796
-2.18136796941948	-0.447131117450110
-2.20373717203888	-0.183722323644913
-2.03759040170113	0.682669420156327
-2.18136796941948	-0.447131117450110
-2.42781878392261	-0.879223932713649
-2.16329994558551	0.291749566745466
-2.27889273592867	0.466429134628597
-1.86545776627869	-2.31991965918865
-2.54929404704891	-0.452301129580194
-1.95772074352968	0.495730895348582
-2.12624969840005	1.16752080832811
-2.06842816583668	-0.689607099127106
-2.37330741591874	1.14679073709691
-2.39018434748641	-0.361180775489047
-2.21934619663183	1.02205856145225
-2.19858869176329	0.0321302060908945
1.10030752013391	0.860230593245533
0.730035752246062	0.596636784545418
1.23796221659453	0.612769614333371
0.395980710562889	-1.75229858398514
1.06901265623960	-0.211050862633647
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-0.496201068006129	-1.84269556949638
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-0.455486948803962	-1.53432438068788
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1.99505169024551	-0.721448439846371
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1.32475563655861	-0.485135293486995
1.69855040646181	1.01076227706927
1.95119099025002	0.999984474306318
1.16799162725452	-0.317831851008113
1.01637609822602	0.0653241212065782
1.78004554289349	-0.192627479858818
1.85855159177699	0.553527164026207
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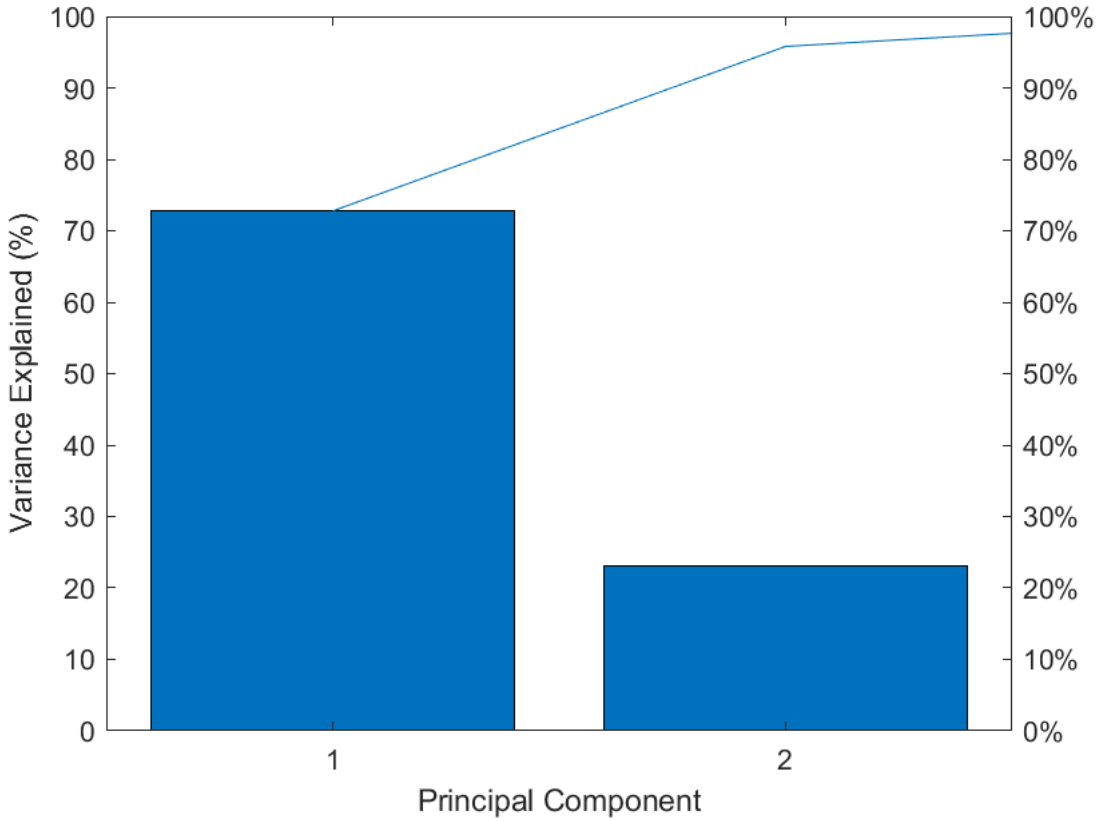
累计贡献率



可见：前两个主成分已经占了95%的贡献程度。这两个主成分可以近似表示整个数据。

4. pca_data.m

其中normlization.m见MATLAB实例：聚类初始化方法与数据归一化方法

```
function data=pca_data(data, choose)
% PCA降维，保留90%的特征信息
data = normlization(data, choose); %归一化
score = 0.90; %保留90%的特征信息
[num,dim] = size(data);
xbar = mean(data,1);
means = bsxfun(@minus, data, xbar);
cov = means'*means/num;
[V,D] = eig(cov);
eigval = diag(D);
[~,idx] = sort(eigval,'descend');
eigval = eigval(idx);
V = V(idx,:);
p = 0;
for i=1:dim
   perc = sum(eigval(1:i))/sum(eigval);
   if perc > score
       p = i;
```

```
        break;
    end
end
E = V(1:p,:);
data= means*E';
```

参考：

Junhao Hua. Distributed Variational Bayesian Algorithms. Github, 2017.

MATLAB实例：PCA（主成成分分析）详解