聚类——认识WKFCM算法

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参考文献：Shen H, Yang J, Wang S, et al. Attribute weighted mercer kernel based fuzzy clustering algorithm for general non-spherical datasets[J]. Soft Computing, 2006, 10(11):1061-1073.

一、WKFCM概述

WKFCM是在KFCM的基础上对数据的属性（或维度、特征空间）进行加权，加权矩阵W是一个C*L的矩阵，与聚类中心的大小一致。其中，C是聚类数，L是数据的维度。

目标函数：

$$J_{\mathrm{WFKCA}}=\sum_{i=1}^{C}\sum_{j=1}^{N}\sum_{k=1}^{L}\mu_{ij}^{m}w_{ik}^{\beta}\|\phi(x_{jk})-\phi(v_{ik})\|^{2}$$

约束条件：

$$\mu_{ij}\in[0,1]\ \mathrm{And}\ \sum_{i=1}^{C}\mu_{ij}=1\quad 1\le j\le N$$

$$w_{ik}\in[0,1]\ \mathrm{And}\ \sum_{k=1}^{L}w_{ik}=1\quad 1\le i\le C$$

更新公式：

$$\begin{cases} v_{ik} = 0 & if \ w_{ik} = 0 \\ v_{ik} = \frac{\sum_{j=1}^{N} \mu_{ij}^{m} \cdot K(x_{jk}, v_{ik}) \cdot x_{jk}}{\sum_{j=1}^{N} \mu_{ij}^{m} \cdot K(x_{jk}, v_{ik})} & if \ w_{ik} \neq 0 \end{cases}. \tag{9}$$

$$\mu_{ij} = \frac{1}{\left(\sum_{r=1}^{C} \frac{\sum_{k=1}^{L} w_{ik}^{\beta} (1 - K(x_{jk}, v_{ik}))}{\sum_{k=1}^{L} w_{rk}^{\beta} (1 - K(x_{jk}, v_{rk}))} \right)^{\frac{1}{m-1}}}. \tag{14}$$

$$w_{ik} = \frac{1}{\left(\sum_{t=1}^{L} \frac{\sum_{j=1}^{N} \mu_{ij}^{m} (1 - K(x_{jk}, v_{ik}))}{\sum_{j=1}^{N} \mu_{ij}^{m} (1 - K(x_{jt}, v_{it}))} \right)^{\frac{1}{\beta-1}}}. \tag{16}$$

注意:

Remark 1 When $\mathbf{x}_j = \mathbf{v}_i$, that is to say, the data point is the same as one of the cluster centers, we have $\sum_{k=1}^{L} w_{ik}^{\beta} \left(1 - K(x_{jk}, v_{ik})\right) = 0$. In such a case, we deviate from (14) and assign $\mathbf{x}_j$ with membership degree 1 to the cluster with $\sum_{k=1}^{L} w_{ik}^{\beta} \left(1 - K(x_{jk}, v_{ik})\right) = 0$ and choose $\mu_{ij} = 0$ for the other clusters i.

Remark 2 The parameter β plays an important role in the clustering process. When β is large enough, then the weight value of each attribute is almost equal to $1/L$, in other words, each attribute is of the same influence to every cluster; and if $\beta \to 1$, then the influence of the weight will reach the maximum; Especially, when $\beta = 1$, $w_{ik} = \{0, 1\}$ indicates whether the kth attribute is absolutely relevant to the ith cluster ($w_{ik} = 1$) or not ($w_{ik} = 0$).

二、算法步骤

Weighted Fuzzy Kernel Clustering Algorithm (WFKCA)

1. Let $t = 1$, initialize $\mathbf{v}_i^{(t-1)} = (v_{i1}, v_{i2}, \ldots, v_{iL})$, and $w_{ik}^{(t-1)} = \frac{1}{L}$ $(1 \le i \le C, 1 \le k \le L)$; Set $J_{\text{WFKCA}}^{(t-1)} = \xi$, where ξ is a large constant;
2. Compute $\mu_{ij}^{(t)}$ according to (14);
3. Calculate $\mathbf{v}_i^{(t)} = (v_{i1}, v_{i2}, \ldots, v_{iL})$ based on (9);
4. Update $w_{ik}^{(t)} (1 \le i \le C, 1 \le k \le L)$ according to (16);
5. Obtain $J_{\text{WFKCA}}^{(t)}$ with (1);
6. If $|J_{\text{WFKCA}}^{(t)} - J_{\text{WFKCA}}^{(t-1)}| < \varepsilon$ //ε is a small constant predefined
   Stop. WFKCA terminates;
   Else
   $t \leftarrow t + 1$, goto step 2;
   End If