英文论文中Remark所放位置及其作用

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Remark是备注、评论的意思，是对上述所提内容做进一步补充解释说明。有以下几种情况：

(1) 放在定理(/引理/命题/推论/假设…)证明完毕之后

- 对上述定理作进一步解释说明。（例如：针对上述定理，我们有另一种证明方式xxx）
- 讨论定理中出现的特殊情况。（例如：当f(x, y)=0时，这个模型就变成xxx，在这种情况下，我们的方法可以扩展到xxx中去）
- 分情况讨论。（当f(x, y)>0，xxx，当f(x, y)<0，xxx）
- 解释我们的定理与别人的不同之处，我们的定理的好处。
- 此定理与证明可能晦涩难懂，这里给出一个具体例子来解释该定理的用法。
- 承上启下。上述定理只适用于xxx情况，针对xxx情况，引出下述定理。
- 通过上述定理，我们能得到以下几点结论(结果)。

Remark 3.4.

(i) In the above proof, we only use the dissipativity arising from $F(u)$ not using the one from $-\partial_x^2 u$. Obviously, the dissipativity $F''(u) \geq C_2 > 0$ in ODEs' context (in Theorem 2.7) can be replaced by the positivity of the second order Fréchet derivative of the potential energy: $\frac{1}{2}\|\partial_x^2 u\|_{L^2}^2 + \int F(u)dx$ in PDEs. Indeed, for example, for the problem with $F(u) = -u^4/4$ under the Dirichlet boundary condition, we need to use a contribution of $-\partial_x^2 u$ to the dissipativity.

(ii) As we have seen above, the procedure is independent of a space discretization. Thus, we may be able to apply the result to the other schemes such as the one defined by the finite element method with respect to the space variable.

Remark 3. When $A = A(n; \mathbf{b}, \mathbf{a}, \mathbf{c})$, with $\mathbf{a}$ nonzero, the reduction provided in Theorem 2 does not yield a tridiagonal structure anymore. In fact, it can be verified that

$$P_n^T A P_n = P_n^T A' P_n + A(n; \mathbf{0}, \mathbf{a}', \mathbf{0}),$$

where $A' = A(n; \mathbf{b}, \mathbf{0}, \mathbf{c})$ and

$$\mathbf{a}' = \begin{cases} (a_1, a_{n-1}, a_3, a_{n-3}, \ldots, \ldots, a_4, a_{n-2}, a_2, a_n) & \text{if } n \text{ is even} \\ (a_1, a_{n-1}, a_3, a_{n-3}, \ldots, a_{\frac{n+1}{2}}, \ldots, a_4, a_{n-2}, a_2, a_n) & \text{if } n \text{ is odd} \end{cases}.$$

The form of $P_n^T A' P_n$ is given by Theorem 2.

Remark 10 The Nussbaum-type functions $N_i(\eta) = \Omega\sqrt{\alpha^2 + \beta_i^2} e^{\alpha|\eta|} \sin \beta_i \eta$ are designed in this paper. The proposed Nussbaum-type functions have better properties:

(*i*) By integrating them, new Nussbaum-type functions can be obtained as $\bar{G}_i(\eta) = b_i \int_{sign(\eta)\frac{\theta_i}{\beta_i}}^{\eta} N(s)ds = \Omega b_i e^{\alpha|\eta|} sin(\beta_i|\eta| - \theta_i)$.

(*ii*) There exist intervals $(\underline{c}_{n_m}, \bar{c}_{n_m})$, $n = 1, 2, \ldots$, such that $\bar{G}_i(\eta) < 0, \forall \eta \in (\underline{c}_{n_m}, \bar{c}_{n_m}), i = 1, 2, \ldots, m$, by selecting appropriate parametersα and β_i, $i = 1, 2, \ldots, m$.

(*iii*) The boundedness of the $\sum_{i=1}^{m} \bar{G}_i(\eta_i(t))$ is given, which plays a key role in establishing the inequalities $V(t) \leq \sum_{i=1}^{m} \int_{t_0}^{t} (b_i N_i(\eta_i) - 1)\dot{\eta}_i(\tau)d\tau + w = \sum_{i=1}^{m} \bar{G}_i(\eta_i(t)) - \sum_{i=1}^{m} \eta_i(t) + \bar{w}$ and $V_n(t) + \int_{t_0}^{t} [\sum_{j=1}^{n} k_j z_j^2]d\tau \leq \int_{t_0}^{t} [\sum_{j=1}^{n} (b_j N_j(\eta_j) - 1)\dot{\eta}_j]d\tau + \epsilon$.

(*iv*) Compared with Nussbaum-type functions of literatures in [35] and [36], picking proper constant Ω of proposed Nussbaum-type functions, the performance of the control system can be improved.

Remark 4 For example, choose $m = 2, b_1 > 0, b_2 > 0, M = 5, g' = 3,$ $\bar{b} = 3$, $\beta_0 = 1, \beta_1 = 1/5, \beta_2 = 1$. Let $n_1 = n$, $n = 0, 1, 2, \ldots$, then $n_2 = 5n + 3$, $(\underline{c}_{n_1}, \bar{c}_{n_1}) = (a_{n_1}, \bar{a}_{n_1})$ $= (5(2n\pi + \pi + \theta_1), 5(2n\pi + 2\pi + \theta_1))$ and $(\underline{c}_{n_2}, \bar{c}_{n_2}) = (\underline{a}_{n_2}, a_{n_2}))$ $= (2(5n + 3)\pi + \theta_2, 2(5n + 3)\pi + \pi + \theta_2)$. Choose $\mu = \frac{1}{2}$, $L_0 = \frac{\pi}{3}$, $\mathcal{L}_1 = \frac{\pi}{3}$, then $\frac{\bar{b}}{\mathcal{L}_1} ln(\frac{1}{\mu}(m-1)) = \frac{9}{\pi} ln2 \approx 1.9858$, $\beta_2 \cot\frac{3}{M} = cot\frac{3}{5} \approx 1.4617$, we can let $\alpha = 2$.

However, in [36], the length of the interval $[\underline{a}_\infty, \bar{a}_\infty]$: $\mathcal{L} = \frac{2.094}{5^2} \approx 0.0838$. Considering $[\underline{a}_\infty + \delta, \bar{a}_\infty] \subset [\underline{a}_\infty, \bar{a}_\infty]$, it is easily shown that a positive constant $\delta \leq \mathcal{L}$, choose $\bar{b} = 2$, then $\alpha > \frac{\bar{b}}{\delta} ln(2m) > \frac{\bar{b}}{L_2} ln(2m) = \frac{2}{0.0838} ln(4) \approx 33.0858$, or $\alpha > \frac{2}{0.0838} ln(2) \approx 16.5429$. If we choose $\alpha = 16.5$, $e^{\alpha|\eta|} \approx e^{16.5|\eta|}$ may be very large when $\eta \nrightarrow 0$, which might make the systems unstable.

Remark 6. We give an alternative proof of the above proposition, which is constructive. Let the notations be as above. In addition, let G be a generator matrix of C. By Lemma 2, C^* is LCD. Thus, C^* is not self-orthogonal. Hence, there is a nonzero vector x of C^* with $\mathrm{wt}(x) \not\equiv 0 \pmod{p}$. Note that $x \notin C$ and $\langle x, x \rangle \neq 0$. Consider the following matrix:

$$G' = \begin{pmatrix} x \\ G \end{pmatrix}.$$

Then we have

$$G'G'^* = \begin{pmatrix} \langle x, x \rangle & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & GG^* & \\ 0 & & & \end{pmatrix}.$$

Remark 6 When the compact set of the definition of the T–S model is not $\mathbb{R}^n$, only the local stability comes at hand. Hence, a very crucial point is to find the largest so-called region of attraction. In this case, the following lemma is useful.

（2）放在算法步骤(伪代码)后面

- 进一步解释算法流程/算法迭代过程（例如：Case 1: xxx, Case 2: xxx）。

Remark 2: For the proposed TGHRR algorithm, if the hidden mapping is known, when the number of the training data is larger than the number of dimensionality of the hidden-mapping features, i.e., $(N >> d_\rho)$, obtaining the solution with (41) is more efficient than that with (45) considering the computational complexity of matrix inverse; otherwise, (45) is more efficient.

Remark 3: When the hidden mapping is known, only the knowledge $\mathbf{w}_s$ is used for transfer learning and the data in the source domain is not required. This means that the proposed algorithm has good privacy protection ability for the data in the source domain. However, if the hidden feature mapping is unknown, the data in the source is also required, as shown in (50) and (51), to effectively implement transfer learning. In this case, the proposed algorithm can no longer protect the privacy of the data in the source domain.

（3）承上启下（上述是本文新提的方法，下面是本文新提的另一种方法）

- 尽管上述提出的方法已经很好地解决了xxx问题，但针对xxx问题，不能得到很好地解决，因此，在下一小节中，我们引出了另一种方法。

Remark: Although the S-TL-SSL-TSK algorithm introduces the transductive TL and SSL mechanism for fuzzy system training, the learning ability of this algorithm can still be further enhanced. Since the label membership parameter $\hat{\mu}_{ij}$ in S-TL-SSL-TSK is a fixed parameter, i.e. directly inheriting from the source domain in step 4 of Algorithm 2, the algorithm is weak in adapting to $\hat{\mu}_{ij}$. In the next subsection, a more adaptive algorithm will be proposed.

Remark 6 It should be emphasised that in some related works, for example, in [36], similar control method is considered. However, in [36, Theorem 1], it is assumed that (1) there is an interval $[\underline{a}_{\infty}, \bar{a}_{\infty}]$ such that $sign(b_i)sin(\frac{\chi}{\beta^i} - \epsilon_i) < -0.5$ and (2) χ_m has entered this interval satisfying $\chi_m \in [\underline{a}_{\infty} + \delta, \bar{a}_{\infty}]$. In fact, in consideration of the continuity of function χ_m in $t \in (-\infty, +\infty)$, an interval (a, b), $t \in (a, b)$ exists, which satisfies $\chi_m \in [\underline{a}_{\infty} + \delta, \bar{a}_{\infty}]$, if the function χ_m is unbounded. In addition, the proof of the Case 1 in [36, Theorem 1] was omitted. In this paper, we will solve this problem in the following section.

(4) 放在本文新提出方法这一节的最末

- 为了xxx目的，本文提出了xxx方法。
- 与之前(前人)所提的方法相比，我们的方法更具有xxx优势，能有效推广到实际应用中。

Remark 1: *To measure the distance between two fuzzy vectors is a key to define the fuzzy relation between them. Thus, we first propose a new measurement represented in Definition 5.* $\mathcal{D}\left(\overline{\mathrm{A}}_i, \overline{\mathrm{A}}_j\right)$ *is the longest distance among 1) distances between v and* $\overline{\mathrm{A}}_i$ *and 2) distances between u and* $\overline{\mathrm{A}}_j$.

Remark 1: Regarding the effectiveness of the proposed epileptic EEG recognition algorithms, as the physiological EEG signals of healthy people differ from those of patients with epilepsy, detecting epilepsy by classifying the EEG signals is in principle a feasible approach. However, EEG signals are complicated in nature and the recording can be influenced by many factors, which present a great challenge to ensuring the accuracy of the classification algorithms. In this regard, transfer learning is an important technique that can be used to construct robust intelligent models for classifying EEG signals. The algorithms proposed in the study are for the above purpose. They are expected to be a promising tool for classifying EEG signals due to their transfer learning abilities and the advantages inherited from fuzzy modeling.

Remark 2: Like other transductive learning-based methods, e.g. LMPROJ, the two transductive learning based methods proposed in this study not only use the labeled training data in the source domain to train the TSK FLS, but also the unlabeled test data in the target domain. This approach is different from the learning procedure of conventional classifiers. It means that for a new test dataset, the transductive transfer learning based model must be trained again. While this characteristic could lead to a computational burden, the proposed transductive learning based methods are more adaptive than other methods and thus practical for many applications.

欢迎补充。

参考文献：

[1] 英文论文中remark，lemma，theorem，assumption这些怎么用比较合适？--知乎

[2] 论文中的定理(Theorem)、引理(Lemma)、推论(Corollary) --CSDN