Week 13

Kai Lukowiak

May 1, 2018

1 Question

Use integration by substitution to solve the integral below:

$$\int 4e^{-7x} dx$$

Solved using substitution

$$u = -7x \tag{1}$$
$$\frac{\partial u}{\partial x} = 7 \tag{2}$$
$$\partial u = (7)\partial x \tag{3}$$
$$\frac{-4}{7}\int e^{u}\partial u \tag{4}$$
$$\frac{-4}{7}e^{-7x} + C \tag{5}$$

2 Question

Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of $\frac{dN}{dt} = \frac{-3150}{t^4} - 220$ bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a function $N(t)$ to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimetre.

$$\frac{dN}{dt} = \frac{-3150}{t^4} - 220 \tag{6}$$
$$= \tag{7}$$
$$\frac{dN}{dt} = -3150t^{-4} - 220 \tag{8}$$

Taking the integral of this we get:

$$\int -3150t^{-4} - 220\partial t = \tag{0}$$

$$\frac{3150}{3}t^{-3} - 220t + C \tag{1}$$

$$6350 = \frac{3150}{3}1^{-3} - 220 \cdot 1 + C \tag{2}$$

$$C = 6530 + 1270 = 7800 \tag{3}$$

Therefore the level of contamination is given by:

$$6350 = \frac{3150}{3}1^{-3} - 220 \cdot 1 + 7800 \tag{4}$$

3 Question

Find the total area of the red rectangles in the figure below, where the equation of the line is $f(x) = 2x - 9$.

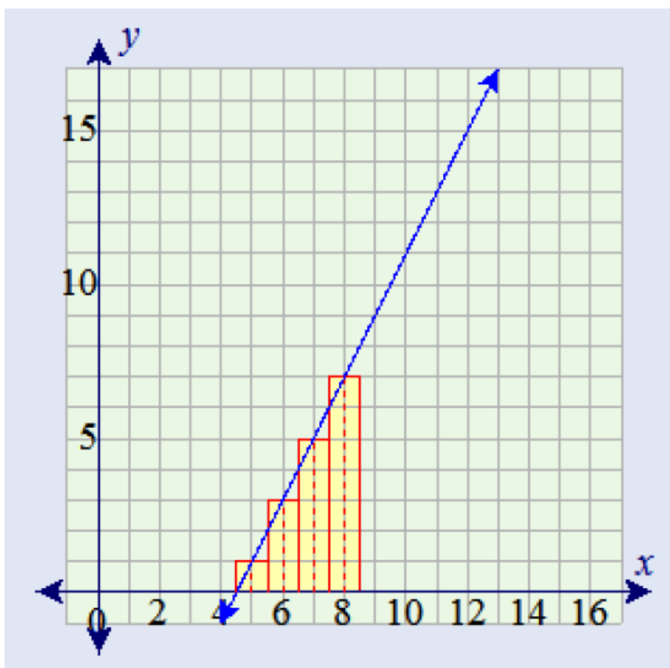

Figure 1: The Graph

The equation for the rectangles is:

$$f(x) = 2x - 9 \tag{0}$$

$$\int_{4.5}^{8.5} 2x - 9dx \tag{1}$$

$$= x^2 - 9x + C \tag{2}$$

We can ignore C

$$[x^2 - 9x]_{4.5}^{8.5} \tag{3}$$
$$8.5^2 - 9 \cdot 8.5 - (4.5^2 - 9 \cdot 4.5) \tag{4}$$
$$= 16 \tag{5}$$

4 Question

Find the area of the region bounded by the graphs of the given equations. $y = x^2 - 2x - 2, y = x + 2$

Solving the system of equations gives:

$$\begin{aligned} x^2 - 2x - 2 &= x + 2 \\ x^2 - 3x - 4 &= 0 \\ &= -1, \quad 4 \end{aligned}$$

Taking the difference between the equations:

$$\begin{aligned} x^2 - 3x - 4 \\ \int_{-1}^{4} -x^2 - 3x - 4 \\ [\frac{x^3}{3} - \frac{3x^2}{2} - 4x]_{-1}^{4} \\ \left[\frac{64}{3} - \frac{48}{2} - 16\right] - \left[\frac{-1}{3} - \frac{3}{2} + 4\right] \\ = 20.83 \end{aligned}$$

5 Question

A beauty supply store expects to sell 110 flat irons during the next year. It costs \$3.75 to store one flat iron for one year. There is a fixed cost of \$8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

The let α but the number of irons in an order and ρ be the number of orders per year. $\alpha \cdot \rho = 110$ which leads to $\alpha = 110/\rho$.d

We assume that the average number of irons at any given time is half the order size, α.

$$c = 8.25 \cdot \rho + \frac{3.75\alpha}{2} \tag{1}$$

$$c = 8.25 \cdot \rho + \frac{3.75 \cdot 110/\rho}{2} \tag{2}$$

$$c = 8.25 \cdot \rho + \frac{412.5}{2\rho} \tag{3}$$

$$c = 8.25 \cdot \rho + \frac{206.25}{\rho} \tag{4}$$

$$\frac{\partial c}{\partial \rho} = 8.25 - \frac{206.25}{\rho^2} \tag{5}$$

To minimize cost we must find the point where $\frac{\partial c}{\partial \rho} = 0$

$$0 = 8.25 - \frac{206.25}{\rho^2} \tag{6}$$

$$8.25 = \frac{206.25}{\rho^2} \tag{7}$$

$$\rho^2 = \frac{206.25}{8.25} \tag{8}$$

$$\rho = \sqrt{25} \tag{9}$$

$$= 5 \tag{10}$$

Therefore the company should make five orders of $110/5 = 22$ irons each.

6 Question

Use integration by parts to solve the integral below:

$$\int ln(9x)x^6 dx$$

$$\begin{align} U &= ln9x \tag{1} \\ dU &= \frac{1}{x}dx \tag{2} \\ dV &= x^6 dx \tag{3} \\ V &= \frac{x^7}{7} \tag{4} \\ \int UdV &= UV - \int VdU \tag{5} \\ ln9x \cdot \frac{x^7}{7} - \frac{1}{7}\int x^7 \cdot \frac{1}{x}dx \tag{6} \\ = ln9x \cdot \frac{x^7}{7} - \frac{1}{7} * \frac{x^7}{7} + c \tag{7} \\ = ln9x \cdot \frac{7x^7}{49} - \frac{x^7}{49} + c \tag{8} \\ = \frac{x^7}{49}(7 \cdot ln9x - 1) + c \tag{9} \end{align}$$

7 Question

Determine whether $f(x)$ is a probability density function on the interval $[1, \quad e^6]$. If not, determine the value of the definite integral:

A probability density function must sum to 1.

$$f(x) = \frac{1}{6x}$$

$$\begin{align} g(x) = \int_1^{6e} \frac{1}{6x}dx \tag{1} \\ \frac{1}{6}\int_1^{6e} \frac{1}{x}dx \tag{2} \\ \left[\frac{1}{6}lnx\right]_1^{6e} \tag{3} \\ \left[\frac{1}{6}lne^6\right] - \left[\frac{1}{6}ln1\right] \tag{4} \\ = \frac{1}{6}(6-0) = 1 \tag{5} \end{align}$$

So yes, the function is a probability density function.