In Exercises 25 – 30, use the Taylor series given in Key Idea 32 to create the Taylor series of the given functions.

29) $f(x) = e^x sin \quad x$ (only find the first 4 terms)

We know that the Taylor Series Expansion for $sin(x)$ is:

$$sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

We also know, from the text book, that:

$$f(x) = e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

We now have to solve this by distributing the equations against each other (polynomial multiplication). We only do the first 4 terms.

$$\sum_{n=0}^{3} \frac{x^n}{n!} \cdot \sum_{n=0}^{3} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

Expanded they equal:

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3}$$
$$sin(x) = x - \frac{x^3}{3!} - \frac{x^5!}{5} - \frac{x^7}{7!} - \frac{x^9}{9!}$$

$$sin(x) = x - \frac{x^3}{6} - \frac{x^5!}{120} - \frac{x^7}{5040} - \frac{x^9}{362880}$$

$$sin(x) = x - \frac{60480x^3}{362880} - \frac{3024x^5!}{362880} - \frac{72x^7}{362880} - \frac{x^9}{362880}$$

$$sin(x) = \frac{362880x - 60480x^3 - 33024x^5 - 72x^7 - x^9}{362880}$$

This gives:

$$1(\frac{362880x - 60480x^3 - 33024x^5 - 72x^7 - x^9}{362880})+$$
$$x(\frac{362880x - 60480x^3 - 33024x^5 - 72x^7 - x^9}{362880})+$$

$$\frac{x^2}{2}\left(\frac{362880x - 60480x^3 - 33024x^5 - 72x^7 - x^9}{362880}\right)+$$
$$\frac{x^3}{3}\left(\frac{362880x - 60480x^3 - 33024x^5 - 72x^7 - x^9}{362880}\right)$$

This is going to be a very larger equation...

$$\frac{362880x - 60480x^3 - 33024x^5 - 72x^7 - x^9}{362880}+$$
$$\frac{362880x^2 - 60480x^4 - 33024x^6 - 72x^8 - x^{10}}{362880}+$$
$$\frac{362880x^3 - 60480x^5 - 33024x^7 - 72x^8 - x^{11}}{725760}+$$
$$\frac{362880x^4 - 60480x^6 - 33024x^8 - 72x^{10} - x^{12}}{1088640}$$

Taylor series was the bane of my existence in my B.A. in economics. This text did a lot better of a job explaining it.

Still, this is quite the amount of work.