1. $T(n) = \sum_{i=0}^{k} 2^i (k-i)$ where $n = 2^k$

$\Rightarrow \quad T(n) = 2^0(k-0) + 2^1(k-1) + \cdots + 2^{k-1}(k-(k-1)) + 2^k(k-k)$

$\quad\; 2T(n) = 2^1(k-0) + 2^2(k-1) + \cdots + 2^k(k-(k-1)) + 2^{k+1}(k-k)$

$\Rightarrow \quad 2T(n) - T(n) = 2^0(k-0) + 2^1 + \cdots + 2^k + 2^{k+1} \cdot 0$

$= k + \frac{2^k - 2}{2-1} + 0$

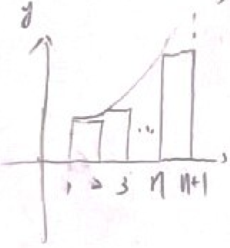
$= 2^k + k - 2$

$= T(n)$

$\Rightarrow \quad T(n) = \Theta(2^k) = \Theta(n)$ ※

2. $T(n) = \sum_{i=0}^{n} i^2 \lg(i)$

$\Rightarrow \quad T(n) = 1^2 \lg(1) + 2^2 \lg(2) + \cdots + n^2 \lg(n)$



$T(n) \le \int_1^{n+1} n^2 \lg(n)\, dn$

$\Rightarrow \quad \int_1^{n+1} n^2 \lg(n)\, dn = \left[\frac{1}{3} n^3 \lg(n)\right]\Big|_1^{n+1} - \int_1^{n+1} \frac{1}{3} n^3 \cdot \frac{1}{n}$

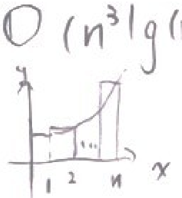
$\left(\int u'v = uv - \int uv'\right)$

$\Rightarrow \quad \left[\frac{1}{3} n^3 \lg(n)\right]\Big|_1^{n+1} - \frac{1}{3}\left(\frac{1}{3} n^3\right)\Big|_1^{n+1}$

$= \left(\frac{1}{3}(n+1)^3 \lg(n+1) - 0\right) - \left(\frac{1}{3}\left(\frac{1}{3}(n+1)^3\right) - \frac{1}{3} \cdot \frac{1}{3}\right)$

$\Rightarrow \quad T(n) = O(n^3 \lg(n))$

同理可證：



$\Rightarrow \quad T(n) \ge \int_1^{n} n^2 \lg(n)\, dn$

$\Rightarrow \quad T(n) = \Omega(n^3 \lg(n))$

$\Rightarrow \quad T(n) = \Theta(n^3 \lg(n))$ ※

3.

polynomially - bounded

$\Rightarrow T(n) = O(n^a)$ ($a \in$ positive number)

$\Rightarrow \log(T(n)) = O(\log n)$

$\Rightarrow \log((\log\log(n))^{\log(n)}) = \log(n)\log\log\log(n)$

$\because \log(n) \times \log^3(n) > \log(n)$

$\therefore \log((\log\log(n))^{\log(n)}) > \log(n)$

$\Rightarrow \log((\log\log(n))^{\log(n)}) \neq O(\log(n))$

$\Rightarrow$ It's not polynomially - bounded