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Experiment 1: Verifying Theoretical Results in SS and DS Diffraction

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Abstract

Using a 632λ $[nm]$ He-Ne laser and an oscilloscope, my partner and I verified the theoretical location of a first order minimum in single slit diffraction. Similarly, we sought to verify the slit separation of a double slit grating. In the end, my partner and I were able to verify the location of the first minimum with a 0.2 $[mm]$ slit to be 3.63 ± 0.12—within 1σ of the theoretical. Similarly, with a 0.4 $[mm]$ slit, the location was determined to be 2.12 ± 0.12—within 2σ. For the two slit separation, d is measured to be 5.89 ± 0.03, which verifies the manufactured separation with 1σ accuracy.

1 Introduction

Through the use of a He-Ne laser, my partner and I seek to verify various theoretical results attained via geometric approximations. Specifically, the location of the first order minimum in single slit diffraction with both 0.2 and 0.4 [mm] slit widths. Similarly, we seek to verify the reported slit separation, d, of the manufactured double slit diffraction grating—which is reported to be 0.6 [mm].

Thomas Young conducted an experiment that exhibited the interference of light from two sources—referred to as Young's double-slit interferometer. In his experiment, monochromatic light was used as the source, in this experiment a He-Ne is the monochromatic light source. The brightest band, or the band with the highest intensity, is at $\theta = 0$ [rad], where the integral number, m, is at $m = 0$. My partner and I use the approximation that the distance from the slits to the screen, R, is much larger than the separation distance between the two slits, d.

Henceforth, I will be referring to Single Slit as SS, and Double Slit as DS.

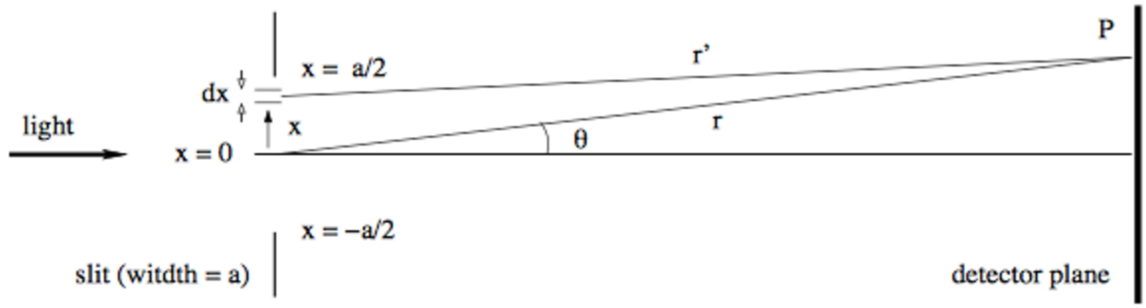

Figure 1: Single slit geometry [1]

2 Theory

Geometric analysis will be used to verify the theoretical values for the both experiments. See figure 1 and figure 2 for the geometry of SS and DS, respectively.

2.1 Single Slit

In the single-slit experiment, the position of the first minimum ($m = 1$) can be solved for by using the following relationships.

$$y = R \cdot tan(\theta) \tag{1}$$

Using the small angle approximation, we know $tan(\theta) \approx \theta$ and $sin(\theta) \approx \theta$. So it can be seen that:

$$y \cong R \cdot tan(\theta) \cong R \cdot sin(\theta) \tag{2}$$

To find the angles at which minima regions form on the screen, we use the following relationship:

$$m\lambda = asin\theta \tag{3}$$

Combining equation (3) with equation (2), the value position for $m = 1$ can be determined:

$$y_{theo} = \frac{R(m)\lambda}{a} = \frac{R\lambda}{a} \tag{4}$$

The error in y_{theo} can be quantified using the following, where δa is the error in the slit width, and δR is the error in distance:

$$\delta y_{theo} = \sqrt{\left(\frac{\lambda \cdot \delta R}{a}\right)^2 + \left(\frac{\lambda \cdot R \cdot \delta a}{a^2}\right)^2} \tag{5}$$

To determine y in the experiment,

$$2y = v_s \cdot \Delta t \tag{6}$$

where v_s is the speed at which the photo-diode moves, and Δt is length on the oscilloscope between $\pm m$.

The error in y_{exp} can be quantified using the following, where δt is the error in the time measurement and δv is the error in the velocity.

$$\delta y_{exp} = \sqrt{\left(\frac{v_s \cdot \delta t}{2}\right)^2 + \left(\frac{\Delta t \cdot \delta v}{2}\right)^2} \tag{7}$$

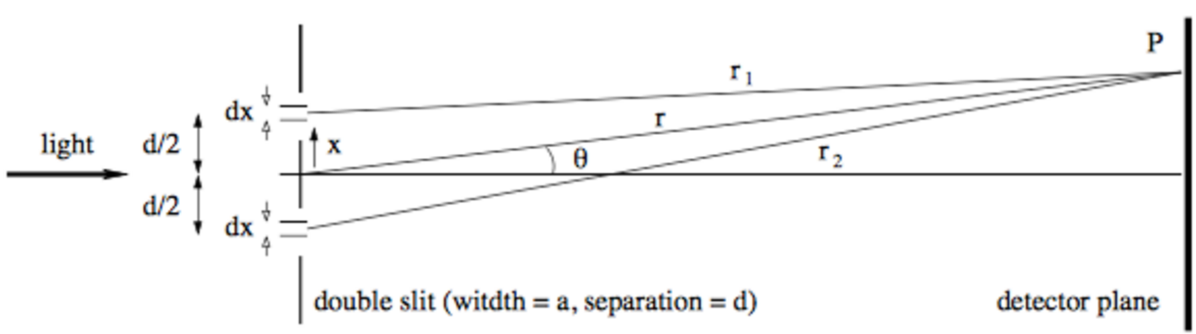

Figure 2: Double slit geometry [1]

2.2 Double Slit

To determine the slit separation, d, the following relationships can be manipulated:

$$m\lambda = dsin\theta \tag{8}$$

Using equation (2) and (6), it can be seen that

$$y = \frac{R(m)\lambda}{d} \Rightarrow d = \frac{R(m)\lambda}{y} \tag{9}$$

x

By producing a plot of y against m, the slope of the plot will be used to calculate d with equation (8).

$$y = \frac{\lambda R}{d}m + 0 \Rightarrow \frac{\lambda R}{d} = slope$$

$$\Rightarrow d = \frac{\lambda R}{slope} \tag{10}$$

The error in this relationship is quantified by,

$$\delta d = \sqrt{\left(\frac{\lambda \cdot \delta R}{(slope)}\right)^2 + \left(\frac{-\lambda \cdot R \cdot \delta(slope)}{(slope)^2}\right)^2} \tag{11}$$

3 Procedure

3.1 Single Slit 0.2 [mm] and 0.4 [mm]

1. Change the width of the slit to 0.2 [mm], report an error. Attach it to the slit apparatus (ensuring the slit is clamped down). Also check to see that the laser is properly collimated.
2. Record the distance between the slit and the screen, L, and an error.
3. Calculate a theoretical value and error of the position of the first minimum ($m = \pm 1$) using equation 4 and 5.
4. Start the screen on one side of the laser and start recording data with the oscilloscope. Move the screen across the laser using the motorized tracks and obtain a trace from the oscilloscope (ensure ample maxima/minima are captured.)
5. Record the Δt between $m = -1$ and $m = 1$
6. Using equation 6, calculate the position of the first minimum ($m = \pm 1$) and its associated error with equation 7. Determine if the theoretical and experimental values agree.
7. Repeat aforementioned steps using a 0.4 [mm] slit width.

3.2 Double Slit

1. Using the same apparatus, replace the single slit with a double slit.
2. Record the distance between the slit and the laser, and its associated error.
3. Similar to section 3.1, attain a trace from the oscilloscope (ensure ample maxima/minima are captured—preferably from $m = -6 \rightarrow 6$).

4. Record the positions of as many maxima/minima that can be distinguished. Find the Δt between $m = -0.5$ and $m = 0.5$ and divide it by 2, which is your t_m for $|m| = 0.5$, multiply t_m by the screen velocity ($v_s = 3.2$ $[cm/s]$) to find determine the position y_m $[cm]$; repeat for all maxima/minima.

5. Plot y_m against $|m|$ and determine the slope and its associated error. Use this value and equation 10, calculate d and its error with equation 11.

6. Determine if the result agrees with the manufacturers reported d (0.6 [mm]).

4 Data

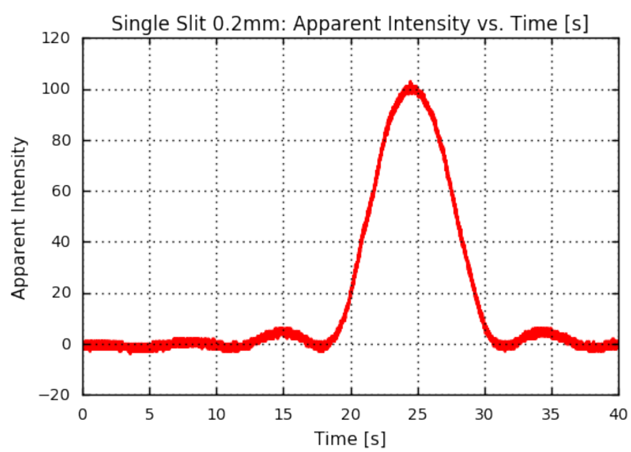

Figure 3: Oscilloscope trace for 0.2 [mm] slit.

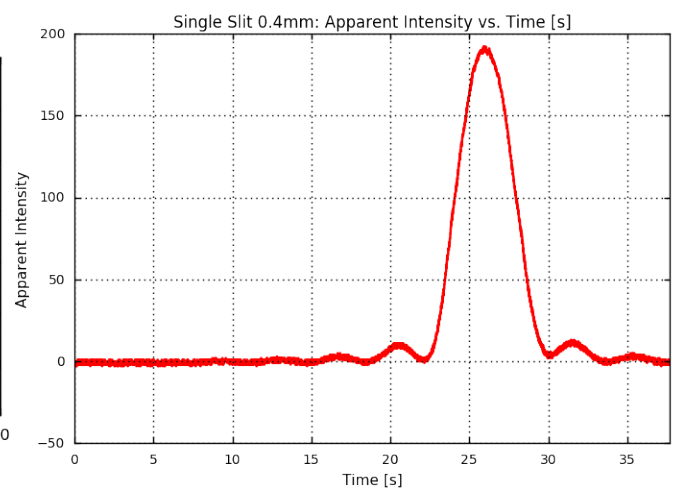

Figure 4: Oscilloscope trace for 0.4 [mm] slit.

Table 1: Double Slit $|m|$ measurements

$\lvert m\rvert$ $[-]$	0.5	1.0	1.5	2.0	2.5	3.5	6.0	6.5	7.0
t $[s]$	1.21	2.36	3.61	4.69	6.06	8.52	14.24	15.37	16.64
dt $[s]$	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05

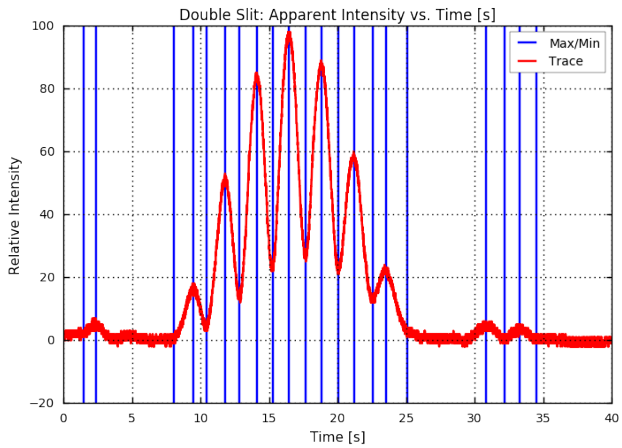

Figure 5: DS oscilloscope trace with identified maxima/minima

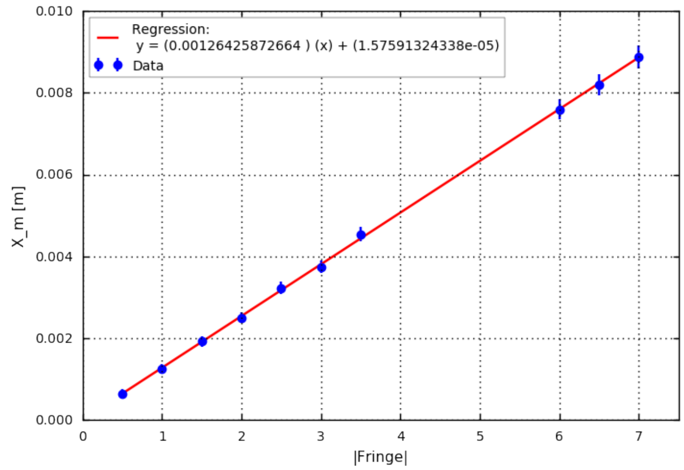

Figure 6: Distance from the origin, y, vs. $|magnitude|$ generated with Table 1

5 Results

All calculations were conducted in Python 2.7 using the relationships defined in Section 2: Theory and Section 3: Procedure, see Table 2 for results.

Table 2: Results for SS and DS experiments

	SS 0.2 [mm]	**SS 0.4 [mm]**		**DS**
$y_{theoretical}$ **[mm]**	3.68 ± 0.16	1.91 ± 0.05	$d_{theoretical}$ **[mm]**	0.6
Δt [s]	13606	7.97		
$y_{experimental}$ **[mm]**	3.63 ± 0.12	2.1 ± 0.12	$d_{experimental}$ **[mm]**	0.59 ± 0.04
z-score $[\sigma]$	0.42	1.67	**z-score** $[\sigma]$	0.25

6 Discussion

In the case of the 0.2 $[mm]$ slit, the theoretical result (3.68 ± 0.16 $[mm]$) for the location of the first magnitude does agree with the experimental value (3.63 ± 0.12 $[mm]$)—within 1σ. However, in the case of the 0.4 $[mm]$ slit, the theoretical result (1.91 ± 0.05 $[mm]$) does not *fully* agree with the experimental value of (2.12 ± 0.12 $[mm]$)—approximately 1.8σ, which is within an acceptable 2σ range, but not as accurate as the 0.2 [mm] experiment.

6.1 Discussion of Errors

In the single slit experiments, there is a systematic error due to the slit width. When the slit width is reduced to 0 [mm], light can still be observed traveling between the slit—meaning it is not exactly 0 [mm]. Recall that a smaller slit width yields a larger dispersion of maxima/minima. So, with this in mind, we can conclude that the displacement of the first order minima is slightly smaller than it would be with a perfectly 0.2/0.4 [mm] slit width.

In the double slit experiment, the maxima and minima for $m = 4 \rightarrow 5$ could not be distinguished accurately in the plot. So Figure 6 is missing data points for those magnitudes. This problem is most likely due to improper collimation of the laser apparatus. The traces (Figure 3, 4, 5) are observed to be slightly asymmetric, especially so in the DS trace. The lower intensity maxima/minima in all of traces are indistinguishable from noise—see Figure 3 and 4, where the peaks rapidly become indistinguishable from noise.

7 Conclusion

From the SS and DS measurements that were made using the He-Ne laser, we conclude that our calculated values a and d agreed with the predicted values attained via geometric approximation—specifically small angle approximation. In the case of the SS 0.2 [mm] experiment, the position of the first minimum was determined to be 3.68 ± 0.16 $[mm]$, while the position was determined, theoretically, to be 3.63 ± 0.12. Similarly, for the SS 0.4 $[mm]$ experiment, the position of the first minimum was determined to be 2.12 ± 0.12 $[mm]$, while the position was determined, theoretically, to be 1.91 ± 0.05. Also, my partner and I were able to experimentally verify the manufactures report of the slit separation, d (0.6 $[mm]$), to be 0.59 ± 0.04.

If this experiment were to be repeated, we would ensure that the laser is properly collimated—as discussed in Section 6.1. In the both experiments, a better aligned apparatus would yield better results in terms of distinguishing less visible minima/maxima. We would also ensure a better slit was used—also discussed in Section 6.1.

References

[1] Hugh Young and Roger A. Freedman University Physics with Modern Physics, 13th ed., Pearson Education, Inc., 2012.