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Experiment 8: Analyzing Spin-Orbit Splitting in Sodium Doublet

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Abstract

My partner and I analyzed the spectra of sodium doublet to (1) observe spin-orbit splitting of sodium 3p and (2) calculate the magnitude of the B-field produced by the transitioning electron. Using a spectrometer and a sodium gas lamp, the peak values for the sodium doublet were determined to be: $D_1 = 588.995$ $[nm]$ and $D_2 = 589.655$ $[nm]$. Using this result, the magnitude of the magnetic field of an electron is calculated to be 20.35 ± 0.38 $[T]$—which is within 1σ of the given theoretical value.

1 Introduction and Background

When an atom is excited, an electron can be excited to a lower to higher orbital–assuming enough energy was added. In an sodium atom, an electron can transition from the 3s (1/2) to 3p (1/2 & 3/2) state where spin-orbit splitting can be observed. When this electron transitions back down to its 3s(1/2) orbital, photons are released, fig.1, producing the famous yellow sodium doublet lines.

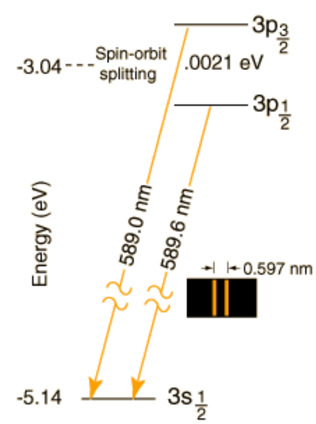

Figure 1: Sodium doublet orbital transition (*HyperPhysics* 2012).

By analyzing the wavelength of this light, one can calculate the corresponding photon energy using the Planck-Einstein relation, where h is the Planck's constant (6.626×10^{-34} $[m^2kgs^{-1}]$),

$$E = hf = \frac{hc}{\lambda} \tag{1.1}$$

Recall the following:

$$U = -\vec{\mu} \cdot \vec{B} \tag{1.2}$$

Where U is the potential energy, $\vec{\mu}$ is the magnetic moment, and $\vec{B}$ is the magnetic field vector. Also, recall that $\mu_B = \frac{e\hbar}{2m_e}$, where $e \equiv$ charge of the electron, $\hbar \equiv$ the reduced Planck's constant, $m_e \equiv$ mass of the electron. By substituting μ_B into equation (1.2), we get the following:

$$U = -\left(\frac{e\hbar}{2m_e}|B|cos(\theta)\right) \Rightarrow |B| = \frac{2m_e}{e\hbar}U \tag{1.3}$$

Recall that $U = \Delta E_{spin} \Rightarrow \Delta E_{spin} = \frac{1}{2}\Delta E$; substituting this into equation (1.3):

$$|B| = \frac{m_e}{e\hbar}\Delta E \tag{1.4}$$

By combining equations (1.1) and (1.4), the magnitude of the B-field due to an electron can be calculated using the following expression:

$$|B| = \frac{m_e}{e\hbar}\left(\frac{hc}{\lambda_{D1}} - \frac{hc}{\lambda_{D2}}\right) \tag{1.5}$$

2 Procedure

1. Assemble the following apparatus: sodium lamp, spectrometer with entrance slit aligned with the lamp, and a computer with *SpectraArray-SL* software.

2. Turn on the sodium lamp and the computer. Let the lamp warm up for at least 5 minutes (although the longer the better).

3. Record the entrance slit width of the spectrometer and the number of lines per millimeter on the diffraction grating.

4. Using *SpectraArray-SL* software, take background data by placing an opaque cover over the entrance slit which accounts for instrumental errors/biases. Optional: Another background measurement can be taken by blocking the sodium lamp, which accounts for light pollution.

5. Turn the software into real-time mode and align spectrometer with sodium lamp such that the two sodium doublet peaks are distinct. Record the corresponding wavelengths for these peaks, insure that D1=589.995 nm, if not, adjust the spectrometer. Using these values, calculate the magnitude of the B-field using equation (1.5), and propagate errors with equation (5.2).

3 Data

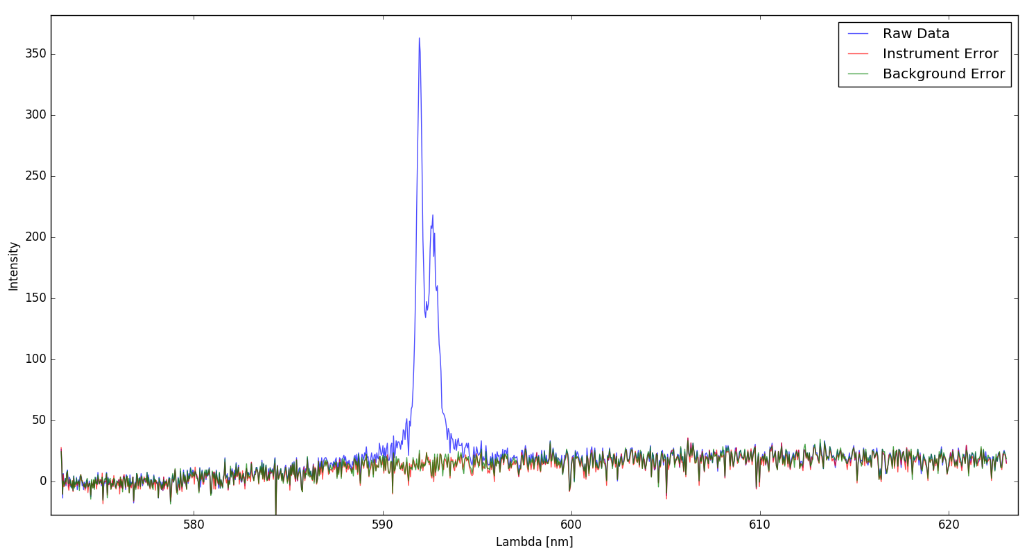

Figure 2: Raw Data from the spectrometer.

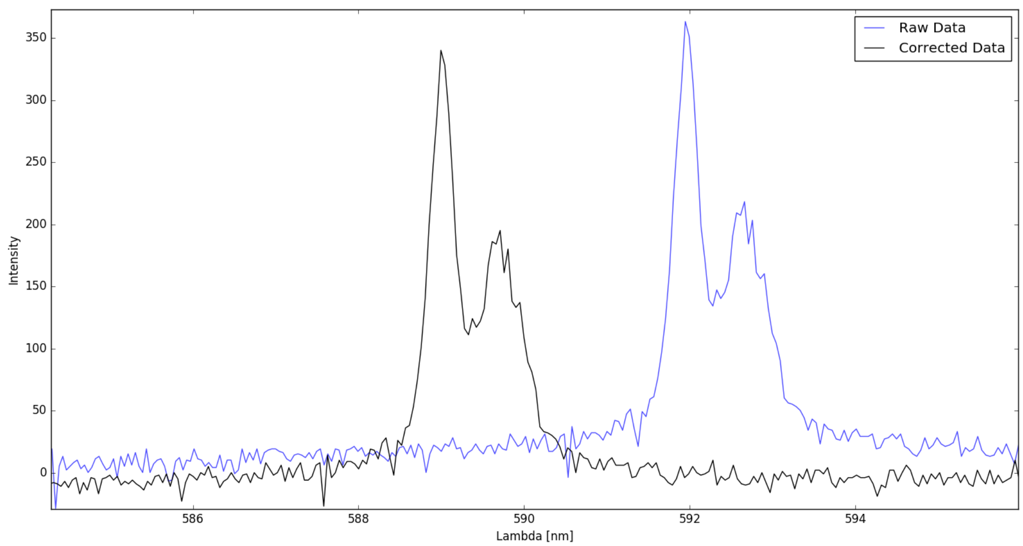

Figure 3: Corrected Data: Accounting for λ shift of 2.95 [nm], and intensity error.

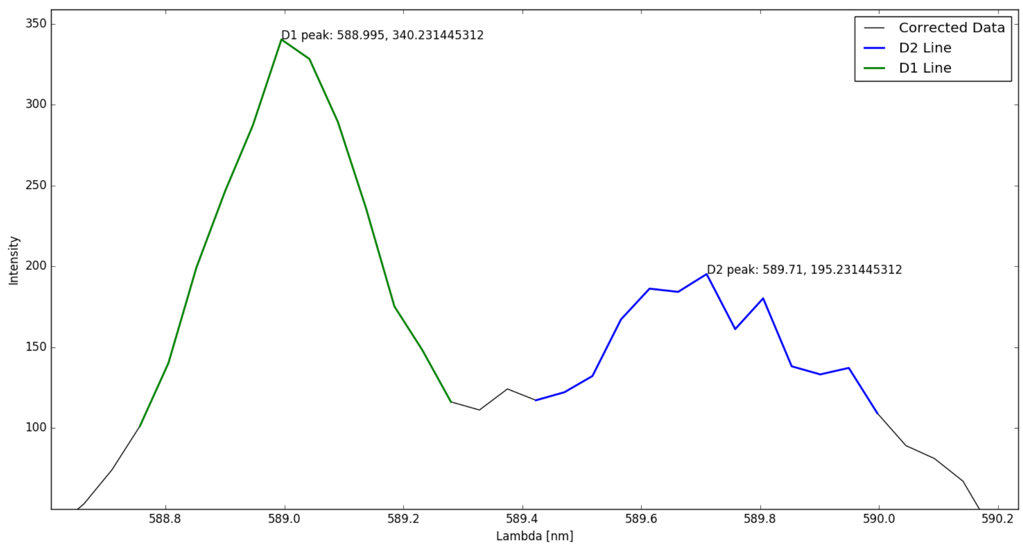

Figure 4: Close inspection of sodium doublet, with identified peaks (local maxima).

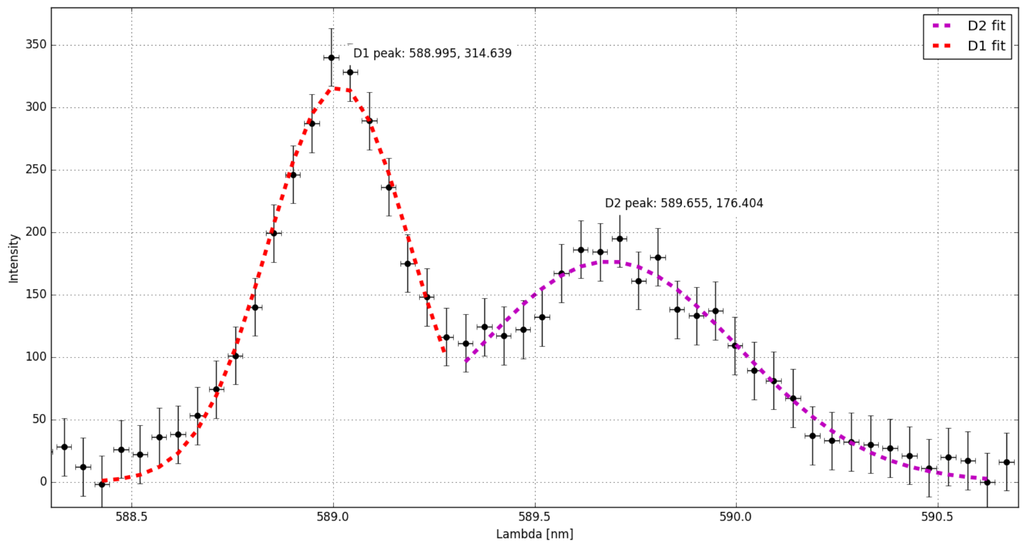

Figure 5: Gaussian fit to sodium doublet spectral lines, with identified peaks of Gaussian.

4 Results

I present two results for the magnitude of the B-field: (1) using the raw peak values from the spectrometer, fig. 4 and (2) using the peak values from a Gaussian fit, fig. 5. All calculations were done in python, utilizing the relationships described in **Section 2: Procedure**.

Raw output from the script:

(1) $\lambda_{D2} = 589.71\ [nm] \Rightarrow E_{D2} = 3.369e(-19)\ [J]$

$\lambda_{D1} = 588.995\ [nm] \Rightarrow E_{D1} = 3.373e(-19)\ [J]$

$\Delta E = (4.09 \pm 0.29)e(-22)\ [J]$

$|B| = 22.05 \pm 1.61\ [T]$

(2) $\lambda_{D2} = 589.655\ [nm] \Rightarrow E_{D2} = 3.369e(-19)\ [J]$

$\lambda_{D1} = 588.995\ [nm] \Rightarrow E_{D1} = 3.3726e(-19)\ [J]$

$\Delta E = (3.775 \pm 0.071)e(-22)\ [J]$

$|B| = 20.35 \pm 0.41\ [T]$

5 Discussion

The first result, (1) is approximately 1.3σ from the accepted theoretical value of 20 $[T]$; (2) is approximately 0.85σ from the accepted theoretical value.

To account for errors in the spectrometer and light pollution, two background measurements were taken. The first was collected by covering the spectrometer slit with the opaque cover, the second was collected by blocking only the sodium lamp. A moving average was calculated for each point and used to calibrate the data, see fig. 2 & 3.

The error in the energy difference and $|B|$ where determined using Equation (5.1) and (5.2),

$$\delta(\Delta E) = \sqrt{\left(\frac{-\delta\lambda_1}{\lambda_1^2}\right)^2 + \left(\frac{\delta\lambda_2}{\lambda_2^2}\right)^2} \tag{5.1}$$

$$\delta(|B|) = \frac{m_e}{c_e \times \hbar} \times (\delta E) \tag{5.2}$$

For (1), the $\delta\lambda_1$ and $\delta\lambda_2$ were determined using the spectral resolution of the diffraction grating (approximated to 0.037 $[nm]$) based on spacing of measurements in the raw data file. For (2), the error in λ was determined using the covariance matrix produced by the SciPy *curve_fit* python function ($\delta\lambda_1 \approx 0.006$ $[nm]$ and $\delta\lambda_2 \approx 0.012$ $[nm]$).

6 Conclusion

In conclusion, the magnitude of the magnetic field is calculated to be 20.35 ± 0.41 $[T]$, which is within 1σ of the given theoretical value of 20 $[T]$. If the experiment was conducted again, a higher diffraction grating would be utilized to ensure the peak values are sampled, as opposed to inferred. Another improvement would be to put the spectrometer on a rail perpendicular to the lamp to ensure an objectively better sampling, as opposed to guessing the alignment.

Appendix A: Spectrum Data ± 1.5nm of the Peaks

```
In [ ]: Lambda_D1 , I_D1
Out[ ]:
(array([ 587.534,  587.581,  587.627,  587.674,  587.721,  587.768,
         587.815,  587.862,  587.909,  587.956,  588.002,  588.05 ,
         588.097,  588.144]),
 array([  8.23144531, -26.76855469,  14.23144531,  -3.76855469,
          0.23144531,  10.23144531,   4.23144531,   9.23144531,
          9.23144531,   7.23144531,   3.23144531,  10.23144531,
          7.23144531,  19.23144531]))
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In [ ]: Lambda_D2, I_D2
Out[ ]:
(array([ 588.191,  588.238,  588.285,  588.332,  588.379,  588.426,
         588.474,  588.521,  588.568,  588.615,  588.663,  588.71 ,
         588.757,  588.805,  588.852,  588.9  ,  588.947,  588.995,
         589.042,  589.09 ,  589.137,  589.185,  589.232,  589.28 ,
         589.328,  589.375,  589.423,  589.471,  589.518,  589.566,
         589.614,  589.662,  589.71 ,  589.758,  589.805,  589.853,
         589.901,  589.949,  589.997,  590.045,  590.093,  590.141,
         590.189,  590.237,  590.285,  590.334,  590.382,  590.43 ,
         590.478,  590.526,  590.575,  590.623,  590.671,  590.72 ,
         590.768,  590.816,  590.865,  590.913,  590.962,  591.01 ,
         591.059,  591.107,  591.156]),
 array([  1.82314453e+01,   1.12314453e+01,   2.42314453e+01,
          2.82314453e+01,   1.22314453e+01,  -1.76855469e+00,
          2.62314453e+01,   2.22314453e+01,   3.62314453e+01,
          3.82314453e+01,   5.32314453e+01,   7.42314453e+01,
          1.01231445e+02,   1.40231445e+02,   1.99231445e+02,
          2.46231445e+02,   2.87231445e+02,   3.40231445e+02,
          3.28231445e+02,   2.89231445e+02,   2.36231445e+02,
          1.75231445e+02,   1.48231445e+02,   1.16231445e+02,
          1.11231445e+02,   1.24231445e+02,   1.17231445e+02,
          1.22231445e+02,   1.32231445e+02,   1.67231445e+02,
          1.86231445e+02,   1.84231445e+02,   1.95231445e+02,
          1.61231445e+02,   1.80231445e+02,   1.38231445e+02,
          1.33231445e+02,   1.37231445e+02,   1.09231445e+02,
          8.92314453e+01,   8.12314453e+01,   6.72314453e+01,
          3.72314453e+01,   3.32314453e+01,   3.22314453e+01,
          3.02314453e+01,   2.72314453e+01,   2.12314453e+01,
          1.12314453e+01,   2.02314453e+01,   1.72314453e+01,
          2.31445312e-01,   1.62314453e+01,   1.22314453e+01,
          1.12314453e+01,   4.23144531e+00,   3.23144531e+00,
          1.12314453e+01,   2.23144531e+00,   9.23144531e+00,
          1.22314453e+01,   6.23144531e+00,   6.23144531e+00]))
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