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**Experiment 5: Using Electron Diffraction to Caculate Atomic Plane Spacing of a Graphite Sample**

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# Abstract

Using an electron diffraction tube, my partner and I sought to reproduce the electron diffraction experiment conducted in 1927 by Davission and Germer. By combining de Broglie's hypothesis with Bragg's law, we obtain an expression relating the diameter of diffraction rings as function of accelerating potential. With this relationship, the atomic plane spacing, $d$, of graphite is measured. We find these values to be $d_1 = 227.0 \pm 17.0$ [pm ]and $d_2 = 125.0 \pm 6.0$ [pm], which are both in high agreement with theoretical values obtained via known geometry of the crystal lattice structure of graphite.


# 1 Introduction

In 1924 Louis de Broglie proposed that electrons have both wave-like and particle-like properties—much like light. De Broglie's hypothesis states matter consists of matter waves with a wavelength, $\lambda$, which depends on the momentum of the matter, $p$, by the following relationship,

$$\lambda = \frac{h}{p} \tag{1}$$

where $h$ is the Planck's constant. Essentially, de Broglie hypothesized that all "ordinary" matter exhibits the same particle-wave duality as light. This theory was verified by Davission and Germer in 1927 after showing electrons incident on a nickel target produced a wave-like diffraction pattern. This lead to de Broglie receiving the Nobel Prize in 1929. Davission and Germer later received the prize in 1937 for the discovery of electron diffraction. In this experiment, we are recreating this electron diffraction experiment.

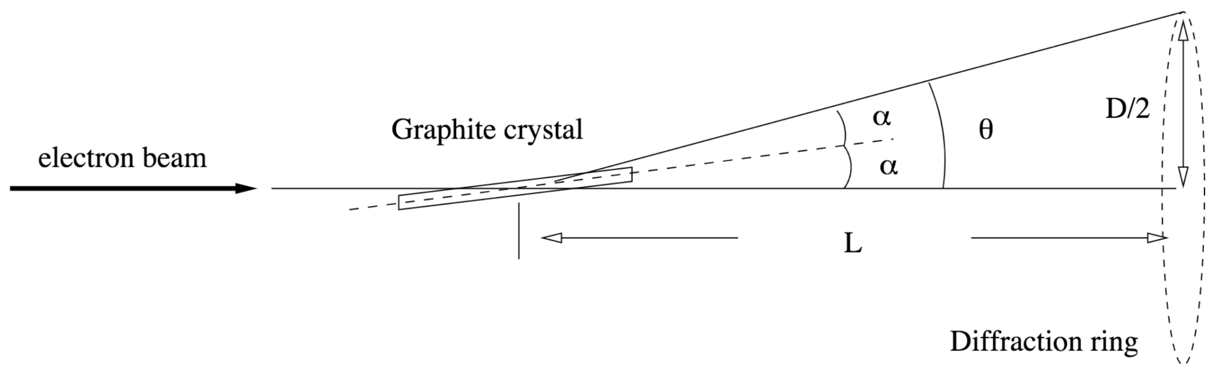


Figure 1: Geometry of the apparatus used in this experiment

## 2 Theory

In the context of this experiment, consider an electron with a—non-relativistic—Kinetic Energy, $K$, given by,

$$K = 0.5mv^2 = \frac{p^2}{2m} = eV \tag{2}$$

where $e$ is the electric charge and $V$ is the potential difference. Solving Eq. 1 for $p$ and substituting into Eq. 2,

$$\lambda = \frac{h}{\sqrt{2meV}} \tag{3}$$

In the cathode ray tube, an electron beam is shot at a polycrystalline sample of graphite which creates a diffraction pattern of concentric circles. Recall that Bragg's law states,

$$2dsin(\alpha) = n\lambda \tag{4}$$

where $d$ is the atomic plane spacing, $\alpha$ is the angle of incidence of the electron beam, and $n$

is the $n^{th}$ ring from the center of the diffraction pattern. Using Bragg's Law, $d$ can be related to $V$. In this particular experiment, two diffraction patterns are observed that correspond to $n = 1$. By inspecting Fig. 1, $\alpha = \theta/2$, so it can be seen that,

$$d = \frac{\lambda}{2sin(\alpha)} = \frac{\lambda}{2sin(\theta/2)} \approx \frac{\lambda}{\theta} \tag{5}$$

where the small angle approximation, $sin(\theta) \approx \theta$, is invoked. Using geometry, Fig. 1,

$$tan(\theta) \approx \theta = \frac{D}{2L} \tag{6}$$

where the small angle approximation, $tan(\theta) \approx \theta$, is invoked. Combining Eq. 5 and Eq. 6,

$$d = \left(\frac{2L\lambda}{D}\right) \tag{7}$$

With this in mind, it can be seen that,

$$D = \frac{2hL}{d\sqrt{2me}} \frac{1}{\sqrt{V}} \tag{8}$$

which is of the form $y = Mx + B$, therefore by plotting $D$ against $V^{-1/2}$, the slope, $M$, of a linear regression is given by,

$$M = \frac{h2L}{d\sqrt{2me}} \tag{9}$$

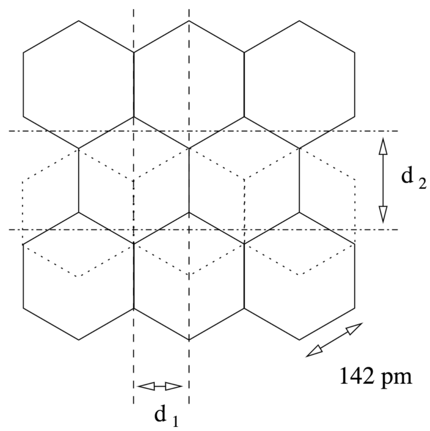


Figure 2: Overlapping crystal planes in graphite. Using the length of the side of the hexagon, the values of $d_1$ and $d_2$ can be calculated.

Solving for $d$,

$$d = \frac{2hL}{M\sqrt{2me}} \tag{10}$$

From our measurements, we will obtain 2 values of d, one for the inner diffraction ring ($d_1$) and one for the outer diffraction ring ($d_2$). These values are then compared with theoretical values predicted from the geometry of the crystal lattice structure of graphite, Fig. 2.

## 3 Procedure

In this experiment an electron diffraction tube is utilized which consists of an electron beam. The electron beam is diffracted off of a graphite sample which produces a diffraction pattern on the surface of the apparatus. Using calipers, measure the diameters of the resultant rings as a function of accelerating potential. Take 8 measurements between 2.40 [V] and 4.80 [V] in 0.3 [V] increments. To account for parallax, take at least 3 measurements for the diameter at each potential to obtain an accurate measurement. Plot diameters as a function of $V^{-1/2}$, and fit a linear regression. Using the slope of said regression, calculate the atomic plane spacing for each diffraction ring using Eq. 10.

# 4 Data

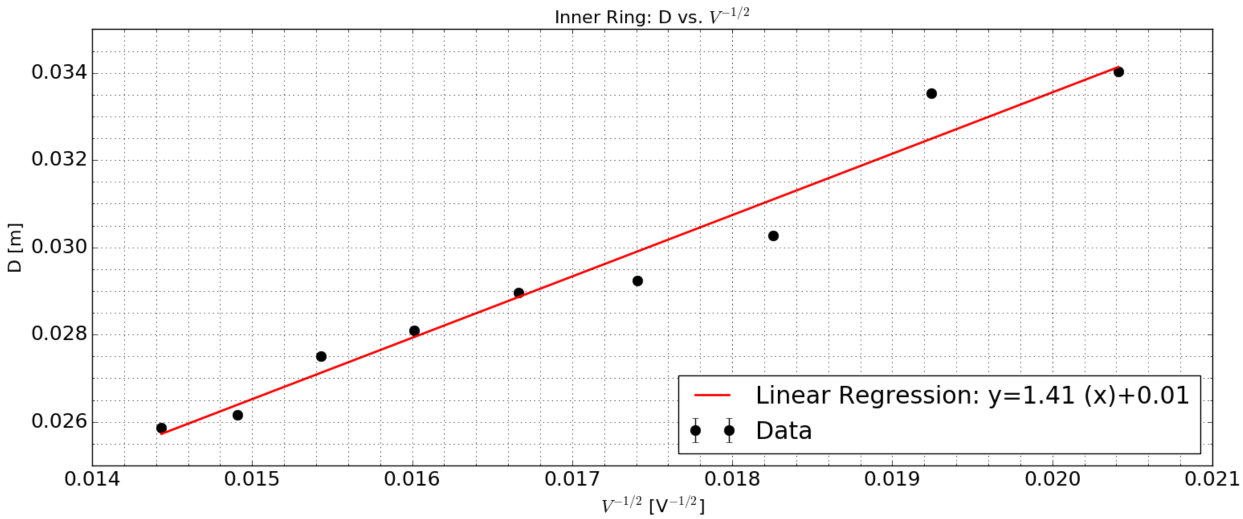


Figure 3: Inner Ring: Plot of D against $V^{-1/2}$
Slope: $1.407 \pm 0.104$ [m $V^{-1/2}$] ; Intercept: $0.0054 \pm 0.0018$ [m] ; $\chi^2$: $2.49e-05$

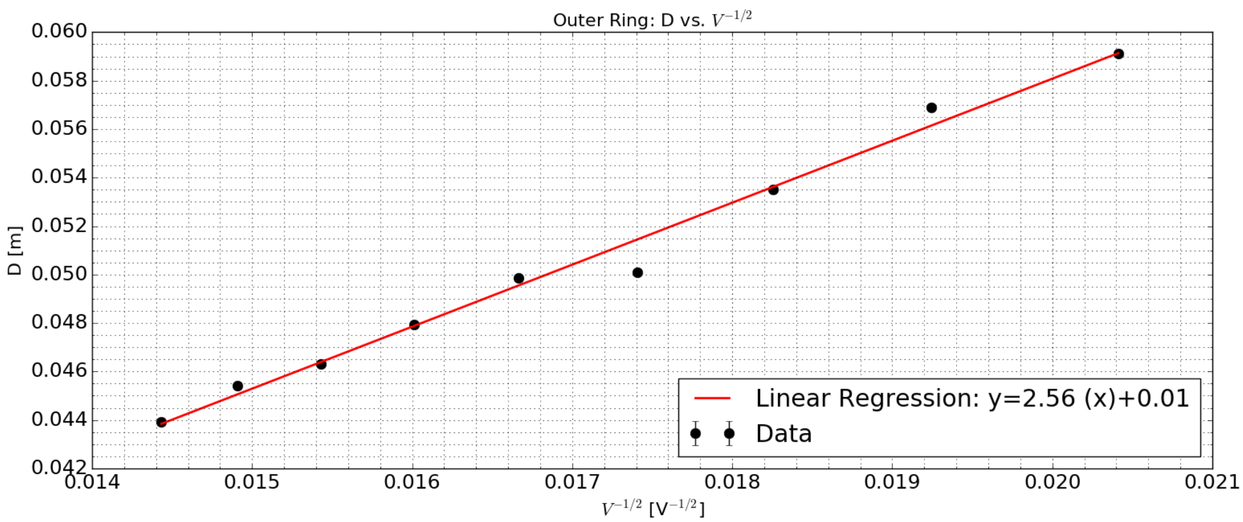


Figure 4: Outer Ring: Plot of D against $V^{-1/2}$
Slope: $2.558 \pm 0.107$ [m $V^{-1/2}$] ; Intercept: $0.0069 \pm 0.0012$ [m] ; $\chi^2$: $2.62e-05$

## 5 Results

The atomic plane spacing associated with the inner ring, $d_1$, is determined to be $227.0 \pm 17.0$ [pm]. When compared the theoretical value of 213.0 [pm], we find that the two values are in agreement with a z-score of $0.80\sigma$. Similarly, for the outer ring, $d_2$, is determined to be $125.0 \pm 6.0$ [pm], which is an agreement with the theoretical value, 123.0 [pm], with a z-score of $0.29\sigma$. The errors were calculated using the standard error propagation procedure.

## 6 Discussion

Although our results are in high agreement with the theoretical, there were some factors which contributed to large error. For example, while conducting the experiment, my partner and I discovered the electron gun was producing an additional diffraction pattern than what was predicted—which appeared as a straight line across the screen. Another contribution to the error is that my partner and I decided to measure from the outsides of the rings, which is a systematic error which causes our measurements to be slightly larger than desired—which can explain the overestimates for plane spacing. In terms of errors in calculation, the small angle approximation was invoked twice which carries significant error on this scale (picometer).

## 7 Conclusion

In summary, my partner and I were able to successfully recreate the Davission and Germer experiment, which showed that electrons have both particle-like and wave-like properties. Using this we were able to calculate the spacing of atomic planes to be $d_1 = 227.0 \pm 17.0$ [pm] and $d_2 = 125.0 \pm 6.0$ [pm]. When compared to the theoretical values of 213 [pm] and 123.0 [pm], respectively, we find that the calculated values are statistically indistinguishable from the theoretical. If this experiment were repeated, we would address the list of sources of error discussed in Section 6.