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Experiment 3: Determining the Refractive Index of Air Using an Interferometer

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Abstract

Using a Michelson Interferometer equipped with a variable pressure gas chamber, my partner and I sought to determine the refractive index of air. When air pressure is changed within the chamber, the incoming light passing through said chamber will experience a phase shift due to a change in refractive index. By measuring the number accumulated diffraction fringes as a function of pressure, one can measure the phase shift. It is determined that $n_{exp} = 1.000179 \pm 0.000004$, which is in strong agreement—less than 1σ—with the accepted, theoretical value $n_{theo} = 1.00027 \pm 0.00075$.

1 Introduction

Before the turn of the 20th century, light was believed to propagate through a medium called the aether (or ether). The aether was theorized to explain the wave-like properties of light—before the discovery of wave-particle duality—specifically, it's ability to propagate throughout space, without a medium. However, the concept of an aether was disproved by Albert A. Michelson and Edward W. Morley in the famous Michelson-Morley experiment in 1887. This experiment utilized a Michelson Interferometer, which looked for variations in the speed of light through the stationary aether. This experiment was awarded the Nobel prize in 1907.

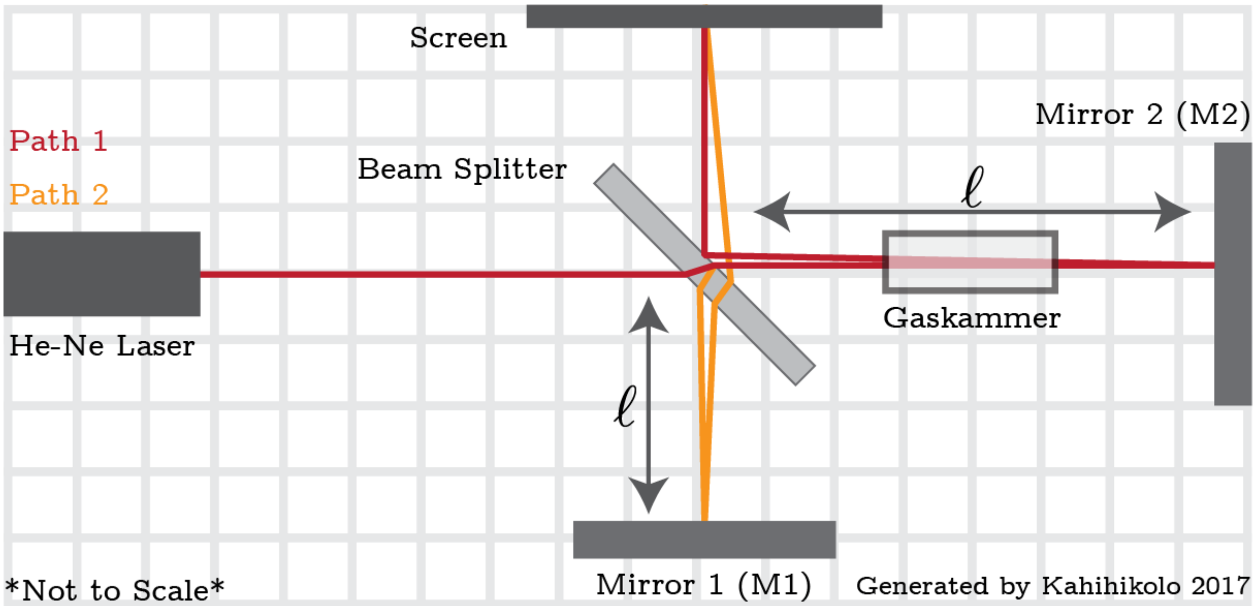

Figure 1: Experiment Apparatus:
Path 1: Light travels through the splitter and is directed through a gas chamber. Then it is reflected back by Mirror 2 through the gas chamber and refracted onto a screen. Path 2: Light is reflected by the splitter onto Mirror 1 which reflects it back through the splitter onto a screen.

2 Theory

The Michelson Interferometer has been utilized in many experiments (i.e; LIGO), however this instrument is being used to measure the index of refraction of air as a function of pressure. As light travels through a medium, it's direction of motion as well as its speed are altered—known as dispersion.

The index of refraction varies with material, and the index of refraction for gases depend on the pressure. Using a Michelson Interferometer, the pressure inside of a gas chamber will be varied to determine the refractive index of air—see Fig. 1 for diagram. Theoretically, index of refraction as a function of pressure should support the following relationship:

$$n(P) = \alpha P + \beta \tag{1}$$

where n is the index of refraction, P is the pressure, and α/β are constants.

2.1 Derivation of Index/Pressure Relationship

The Michelson Interferometer apparatus contains a monochromatic light source. The light will approach a splitter which will direct the light along 2 separate paths, see Fig. 1 for description of light paths.

When light is recombined on the screen, a series of concentric circles can be observed. The projection contains bright and dark bands corresponding to interference patterns. These dark regions—destructive interference—occurs when $\Delta\phi = (2n+1)\pi\,, n \in \mathbb{Z}^+$—where $\Delta\phi$ is the phase difference.

Suppose that Mirror 1 and the beam splitter are separated by a distance ℓ; suppose the same for the screen and the beam splitter. Recall, $\lambda = 2\ell/m$, where m is the number of wavelengths along the path. Along path 1 and path 2,

$$m_1 = 2\ell/\lambda_1 \quad m_2 = 2\ell/\lambda_2$$

where λ is the wavelength of light in that medium.

$$\begin{aligned}\Delta m = m_1 - m_2 &= 2\ell\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)\\ &= 2\ell\left(\frac{n_1 f}{c} - \frac{n_2 f}{c}\right)\end{aligned}$$

where n is the index of refraction, and f is the frequency of light,

$$\Delta m = \frac{2\ell f}{c}(n_1 - n_2) \tag{2}$$

Now, by substituting in the the theoretical relationship (Eq. 1) for n,

$$\begin{aligned}
\Delta m &= \frac{2\ell f}{c}[(\alpha P_1 + b) - (\alpha P_2 + b)] \\
&= \frac{2\ell f}{c}\alpha(P_1 - P_2) \qquad (3) \\
\Rightarrow P_2 &= \frac{-c}{2\ell f\alpha}\Delta m + P_0 \qquad (4)
\end{aligned}$$

where P_0 is atmospheric pressure. It can be seen from Eq. 4 that a plot of pressure in the gas chamber against accumulated fringe shifts yields,

$$\begin{aligned}
slope &= \frac{-c}{2\ell f\alpha} \qquad (5) \\
\Rightarrow \alpha &= \frac{-c}{2\ell f \cdot (slope)} \qquad (6)
\end{aligned}$$

Substituting in Eq. 6 into Eq. 1, one can solve for the index of refraction for air,

$$n(P) = \left(\frac{-c}{2\ell f \cdot (slope)}\right) P + P_0 \qquad (7)$$

3 Procedure

This experiment utilizes a Michelson interferometer with a gas chamber located in front of one of the mirrors. Using a hand pump, the pressure within the chamber can be varied. As light from the Helium-Neon laser passes through the chamber it experiences a phase shift due to a change in refractive index.

1. Using a caliper, measure the length of the gas chamber. Take note of the wavelength of the Helium-Neon laser, as well as the temperature and atmospheric pressure in the room.

2. Record about 10 measurements of the accumulated fringe shifts on the screen with varying pressure inside the chamber.

3. Plot the recorded gauge pressures against the number of accumulated fringe shifts.

4. Plot the pressure in the chamber against accumulated fringe shifts $(76.0\,[cmHg] - P_{Gauge})$. Using the slope, calculate α and n_{air} using Eq. 6 and 7, respectively.

4 Data

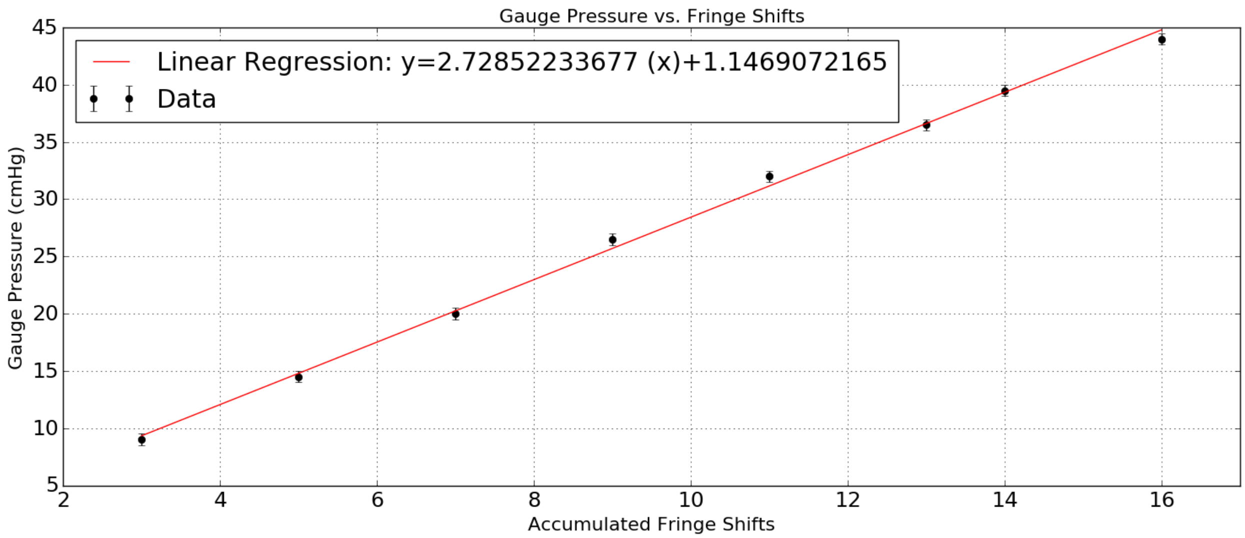

Figure 2: Gauge pressure against accumulated fringe shifts with linear regression. Slope=2.73 ± 0.05 [cmHg], Y-int=1.15 ± 0.54 [cmHg], $\chi^2 = 2.27$.

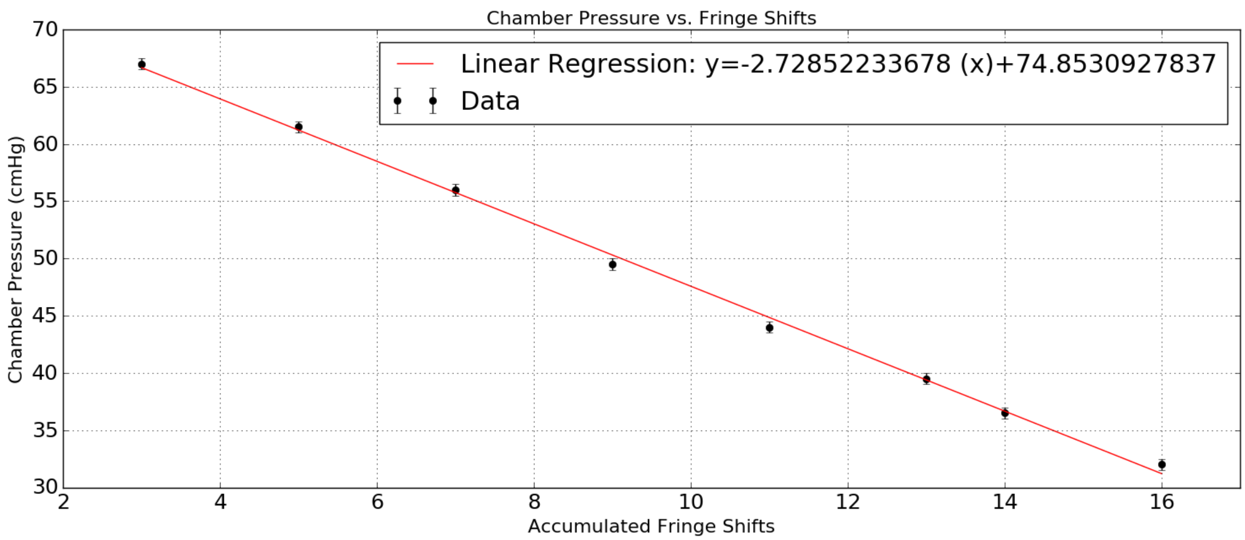

Figure 3: Chamber pressure against accumulated fringe shifts with linear regression. Slope=-2.73 ± 0.05 [cmHg], Y-int=74.85 ± 0.54 [cmHg], $\chi^2 = 2.27$.

5 Results

Table 1: Measurements

Wavelength of He-Ne laser	$632.8 \pm 0.1\,[nm]$
Length of gas chamber	$33.5 \pm 0.1\,[mm]$
Temperature	$19.5 \pm 0.1\,[°C]$
Atmospheric Pressure	$76.073\,[cmHg]$

Table 2: Results

Slope of Fig. 3	$-2.72 \pm 0.05\,[cmHg]$
Y-intercept	$74.85 \pm 0.54\,[cmHg]$
Value of α	$(23.91 \pm 0.45) \times 10^{-8}\,[cmHg^{-1}]$
Experimental n_{air}	1.000179 ± 0.000004
Theoretical n_{air}	1.000277
Z-Score	24.5σ

6 Discussion

After conducting a χ^2 test, the linear regression yields a value of 2.2, which suggests that the linear regression is poorly fit. From inspecting the plot, it can be seen that taking more data points would lead to a better linear regression (therefore a lower χ^2) which will allow for a better estimate for the index of refraction of air. Using the slope in Fig. 3, the results are tabulated in Table 2. The experimental refractive index of air is: $n_{exp} = 1.000179 \pm 0.000004$.

6.1 Temperature Dependant Index of Refraction for Air

Recall that the theoretical relation (Eq. 1) is a function of pressure and does not account for temperature. Using equations found in the CRC handbook [1], one can determine the index of refraction of air in the lab accounting for the non-STP environment.

$$(n-1) \times 10^8 = 8342.13 + 2406030(130 - \sigma^2)^{-1} + 15997(38.9 - \sigma^2)^{-1} \tag{8}$$

where $\sigma = \lambda_{vacuum}^{-1}$. The result from Eq. 8 should be multiplied by the following to yield an estimate for the index of refraction of air,

$$\frac{P \cdot [1 + P \cdot (61.3 - T) \times 10^{-10}]}{96095.4(1 + 0.003661 \cdot T)} \tag{9}$$

Using this relationship and the standard error propagation process, $n_{calculation} = 1.00027 \pm 0.00075$. Which is in agreement with the experimental index of refraction, with a Z-score of approximately 0.1σ—which is in agreement.

7 Conclusion

From the experimental data, it can be seen that the index of refraction of air is 1.000179 ± 0.000004, and using the model found in the CRC Handbook [1], the theoretical refractive index of air is calculated to be 1.00027 ± 0.00075. These values deviate by approximate 0.1σ which suggests that the theoretical relationship, between refractive index and pressure, is in agreement with the experimental data.

If this experiment were conducted again, more measurements should be taken. I think the high χ^2 in the linear regression can be attributed to a poor sampling of data (only 8 data points). Another potential source of error is there was an assumption that distance between the beam splitter and the mirrors are equal. If the instrument were to be misaligned there would be a change in the path length of the laser.

References

[1] "CRC Handbook of Chemistry and Physics, 75th ed.; CRC Press: Boca Raton, FL., 1995; 10-302