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Experiment 9: Confirming the Band Gap Energy of Germanium

Kaimi Kahihikolo, Colleen Lau

Abstract

The temperature dependence of the conductivity of un-doped Germanium has been studied. By measuring the voltage, current, and temperature of a Germanium sample connected to a voltage source, we were able to successfully calculate the band gap energy of Germanium, E_G, near room temperature. The value of E_G is determined to be 6.730 ± 0.002 [eV], which is in agreement with the accepted theoretical value of 6.70 [eV]—with a z-score of 1.5σ.

1 Introduction

By understanding, and quantifying the energy band gaps of semiconductors, the world has experienced significant leaps in technological advancement. The development of fundamental technologies such as transistors and LEDs, which are utilized in almost every modern device, is attributed to an understanding of solid-state physics.

2 Theory

Recall quantum mechanics predicts that the allowed energy levels of electrons are discrete, or quantized. The area between said energy levels are known as a band gap. The size of this band gap is intrinsic to the material, however band gap size is also a function of temperature.

For conductors in general, the band gap is sufficiently small such that electrons can "jump" to a higher energy level. For a semi-conductor, an electron can make the transition if sufficient energy, such as thermal energy, is absorbed. Superconductors have such small band gaps that electrons essentially flow freely from one material to another. However in insulated materials, the band gap is so large almost no electrons can "jump" to another energy level—as the applied thermal energy would most likely destroy the system first. Thus the band gap energy of a material is directly related to it's conductivity. In this experiment the conductivity of a Germanium sample is measured as a function of temperature, which leads to an estimate of the band gap energy of Germanium.

Recall that Ohm's Law states that,

$$\vec{E} = \rho\vec{J} \tag{1}$$

where $\vec{E}$ is the electric field, $\vec{J}$ is the current density, and ρ is the resistivity—which is intrinsic to the material. A materials resistance, R, is related to the geometric structure of the material as well as the resistivity.

Suppose a rectangle plate has length, ℓ, and area, A. Along the length of this plate, the electric field is uniform, that is to say,

$$V = |\vec{E}|\ell \tag{2}$$

Similarly, the current density is perpendicular to the area of this plate and is uniform, that is to say,

$$I = |\vec{J}|A \tag{3}$$

By solving Eq. 2 for E and Eq. 3 for J, Eq. 1 cab be rewritten as,

$$V = \left(\frac{\rho\ell}{A}\right) \cdot I \tag{4}$$

Recall that the conductivity, σ, is defined as the inverse of the resistivity,

$$\sigma = \frac{I\ell}{VA} \tag{5}$$

The conductivity has a temperature dependence given by,

$$\sigma = \sigma_0 \text{exp}\left(-\frac{E_g}{2k_B T}\right) \tag{6}$$

where E_g is the band gap energy, k_B is the Boltzmann constant, and T is temperature in Kelvin. However, the temperature from the sensor utilized in this experiment is given in $[\mu V]$, so the temperature is converted using the following relationship,

$$T = \left(\frac{1}{40}\frac{k}{\mu V}\right) R + T_{room} \tag{7}$$

where R is the temperature reading from the sensor in $[\mu V]$. Eq. 6 can be linearized with respect to T^{-1},

$$ln(\sigma) = \left(-\frac{E_g}{2k_B}\right)\frac{1}{T} + ln(\sigma_0) \tag{8}$$

which is of the form $y = mx + b$. Therefore, by plotting the $ln(\sigma)$ against T^{-1}, one can determine the band gap of Germanium, where M is the slope,

$$E_G = -(M) \cdot (2k_B) \tag{9}$$

3 Procedure

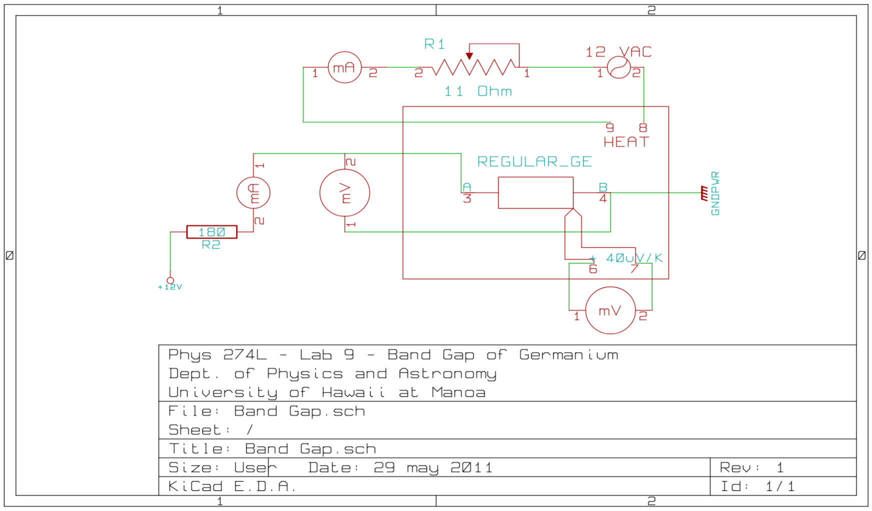

Figure 1: Experiment Apparatus

Begin by measuring the length, width, and thickness of the Germanium sample with calipers. Also, take note of the room temperature. Then assemble driver board and multi-meters according to the circuit diagram in Fig. 1.

Using the rheostat, control the current so that the temperature change is slow. Ensure the current does not exceed 3 [mA]. Measure the temperature of the Germanium sample with the temperature sensor. Also measure the voltage and current across of the sample. For higher precision, use a camera to take a picture of the meters to get simultaneous measurements. Take approximately 15 measurements between 2.6 [μV] and 0.5 [μV] on the temperature sensor in increments of 0.2 [μV] to ensure an adequate sampling.

Convert the temperature readings from [μV] to Kelvin using Eq. 7 then take the inverse of those measurements. Using Eq. 6 calculate the natural log of σ. Generate a plot of $ln(\sigma)$ against T^{-1} and fit a linear regression. Using the slope from the regression, use Eq. 9 to calculate the band gap energy of Germanium.

4 Data

Table 1: Miscellaneous Measurements

Length of Germanium [m]	0.0244 ± 0.0001
Width [m]	0.0098 ± 0.0001
Thickness [m]	0.0015 ± 0.0001
Room Temperature [K]	22.0 ± 1.0

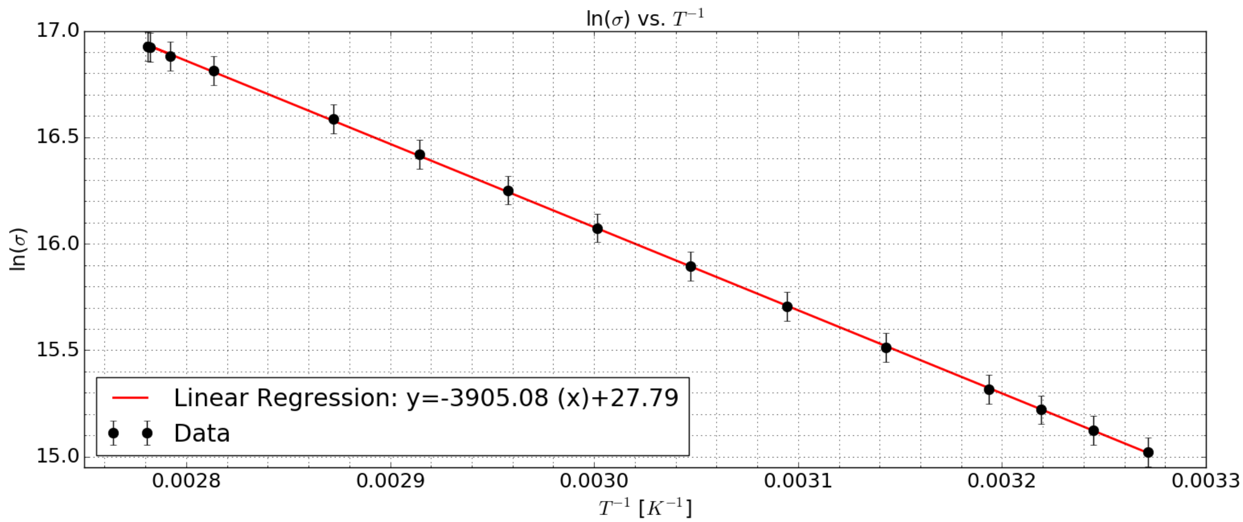

Figure 2: Plot of $ln(\sigma)$ vs. T^{-1} with a linear regression.
Slope $= -3905.08 \pm 9.38$ [K] ; Intercept $= 27.79 \pm 0.03$; $\chi^2 = 0.0004$

5 Results

The slope of Fig. 2 is -3905.08 ± 9.38 [K], using this value, and Eq. 9, the band gap energy of Germanium, E_G, is determined to be 6.730 ± 0.002 [eV]—where the error is determined using the standard error propagation procedure.

6 Discussion

After conducting graphical analyses of the data, the resulting linear regression has a $\chi^2 = 0.0004$, which indicates a very good—but slightly suspicious—fit. The band gap energy of Germanium, E_G, is measured to be 6.730 ± 0.002 [eV]. Compared to the literature value of the band gap energy of 0.67 [eV] [1], the experimental value is in agreement with the theoretical with a z-score of 1.5σ. With such a result, the very low χ^2 from the linear fit is reasonable.

If this experiment were to be conducted again, an improvement would be to control the ambient temperature in the room. Since the band gap energy is temperature dependant, fluctuations in the room temperature will cause the band gap energy of the Germanium to change. Minimizing these outside influences will result in a marginally better fit, thereby yielding a lower z-score.

7 Conclusion

My partner and I were able to successfully confirm the theoretical value for the band gap energy of Germanium found in literature. The experiment yields a energy of 6.730 ± 0.002 [eV], which agrees with the accepted value of 0.67 [eV], with a z-score of 1.5σ—which is statistically indistinguishable from the theoretical. Although the result is quite conclusive, some of the error can be attributed to temperature fluctuations in the room, see Section 6 for a brief explanation.

References

[1] Streetman, Ben G.; Sanjay Banerjee (2000). "Solid State electronic Devices" (5th ed.). New Jersey: Prentice Hall. p. 524