Part 1:

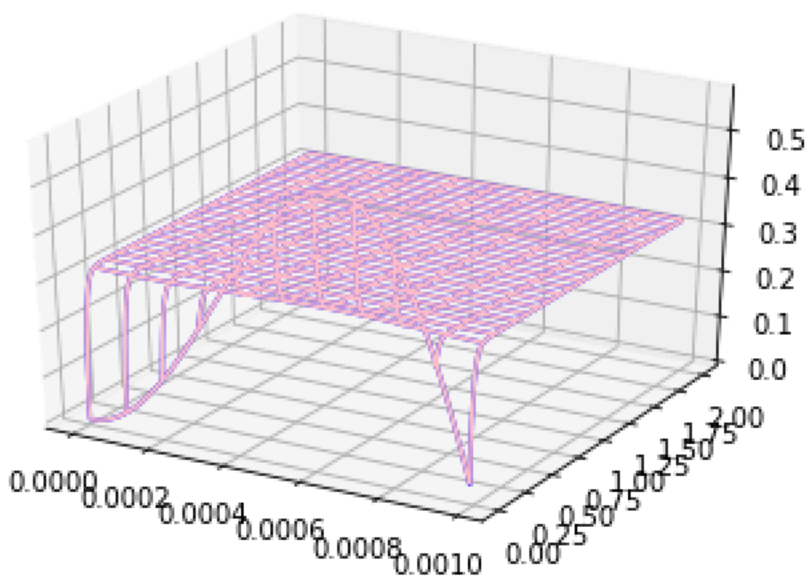
The minimum length that the FE array can be is 40001. At δ = 0.00005, we get that the length of the first section of the array using forward euler, is 40001. Any time I make the δ even a bit smaller than that, it will not graph anything due to some infinite error. The graphs for what each δ iteration look like are below.

When the δ is set to 0.00005, the graphs are no longer the same by "eyeball norm". For all values to up that, they are identical, I created two graphs on one chart, one with a pink line and the other with a blue line. The pink line is produced after the blue line, so when they are identical, you cannot see any blue lines peeking through.

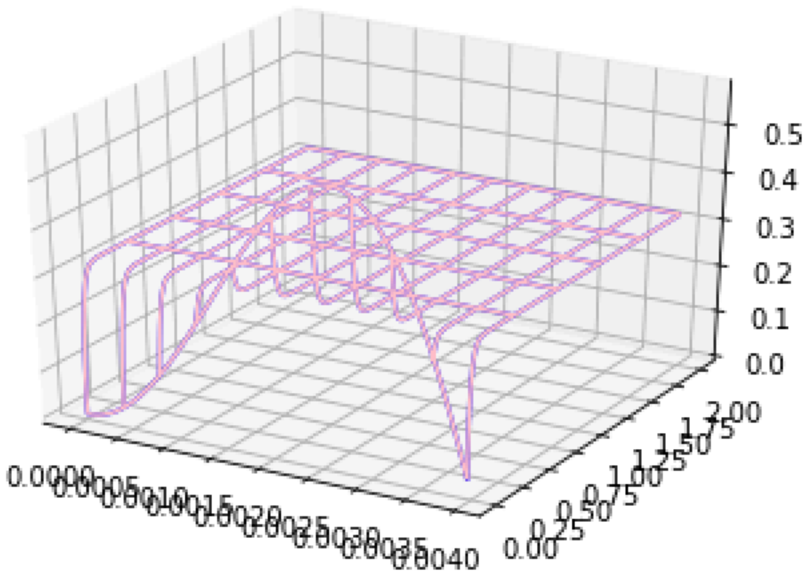
At 0.00004999, you can start to see blue showing through the graphs indicating that the graphs are not converging anymore.

You can see the different graphs below:

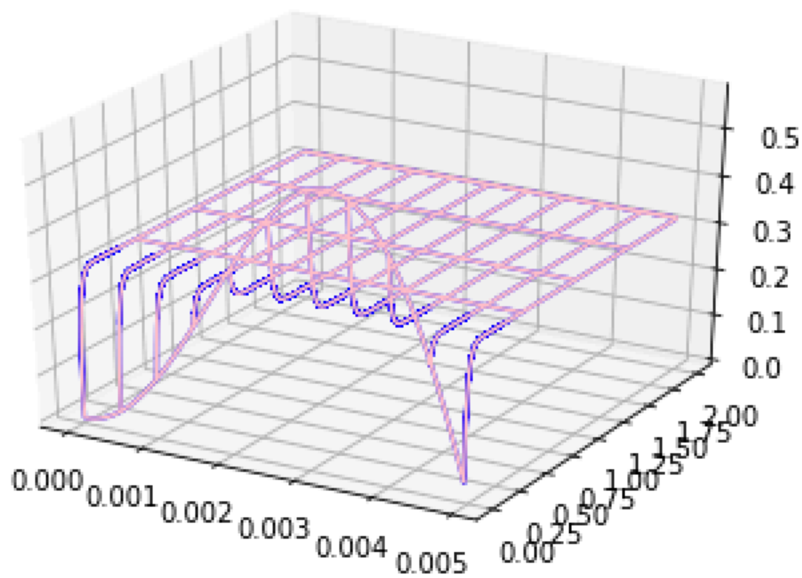
0.00001



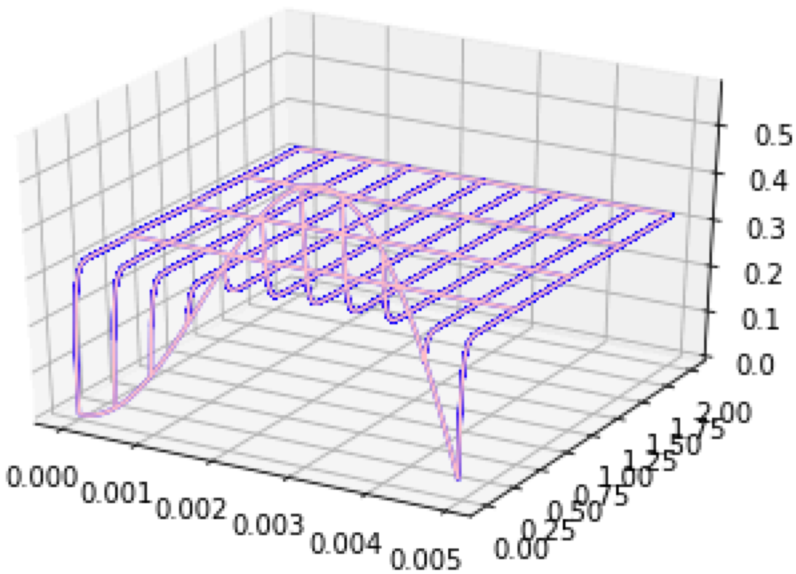
0.00003



0.00004999



0.00005



0.5
0.4
0.3
0.2
0.1
0.0
0.000
0.001
0.002
0.003
0.004
0.005
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

Part 2:

Using the backward Euler method with the symmetric and non-symmetric versions of the second difference operator, I solved the heat equation up to time t=1 and $\delta=\Delta=1/N$.

In order to compare values from matrices of two different sizes, I looped through the larger array and for each number *i* where *i* % (larger matrix size/smaller matrixsize) == 0, I appended that value into a subset array that I was able to use to compare with the smaller array.

I then used the max norm method to calculate the error value between the two matrices to see how it converges as the N values change and the results are posted below.

The magenta line on these graphs is the error of the non-symmetrical D matrix and the cyan line is the error of the symmetrical D matrix. I have labeled the graphs with a title that says what the two different N values are.

For the most part, the graphs are very similar. The non-symmetrical D matrix appears to be of a higher order because it has a larger slope. You can see this in the graph directly below (16 vs 32) where the magenta line starts above the cyan line but end below the cyan line. As the N values change, and they get farther apart, the convergence changes a bit.

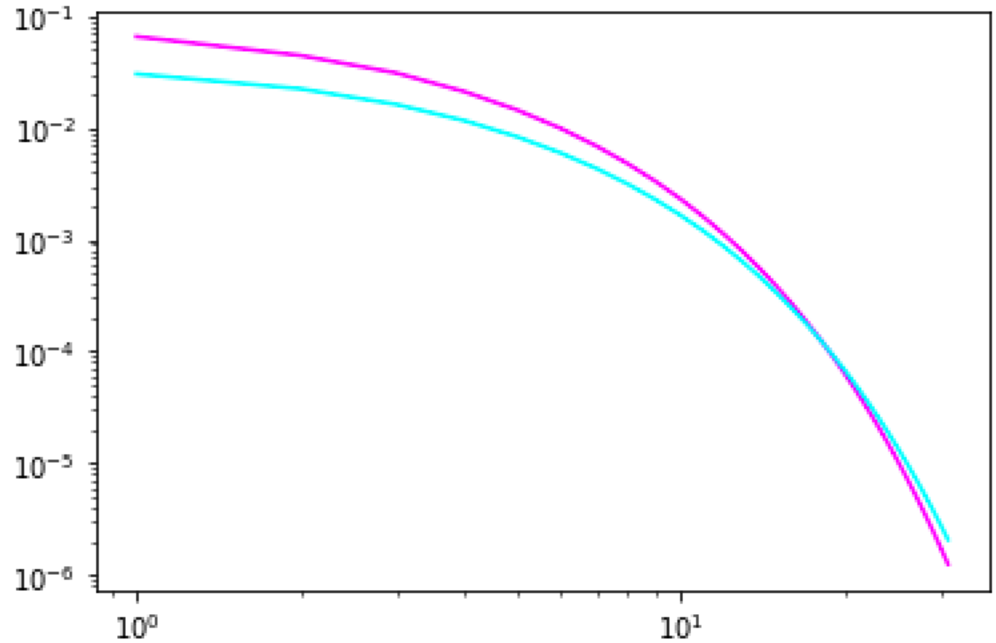
In all of them, they start parallel to each other, but then the non-symmetical D matrix values start to get smaller more rapidly than the symmetrical D matrix values so the lines get closer to each other.

All of the graphs have the same relative shape, they all slope downwards. So at the start of the arrays being compared to each other, they values are larger in difference, but as they loop through the comparisons, the values get closer together and the error is smaller. One thing that is strange though, is that when we get to the 256 vs 512 comparison and N values beyond that, the graph has this downward spike which indicated the error was larger, then rapidly got smaller than got large again before finally decreasing for the remainder. As the different N values get larger, the spike seems to get larger as well.

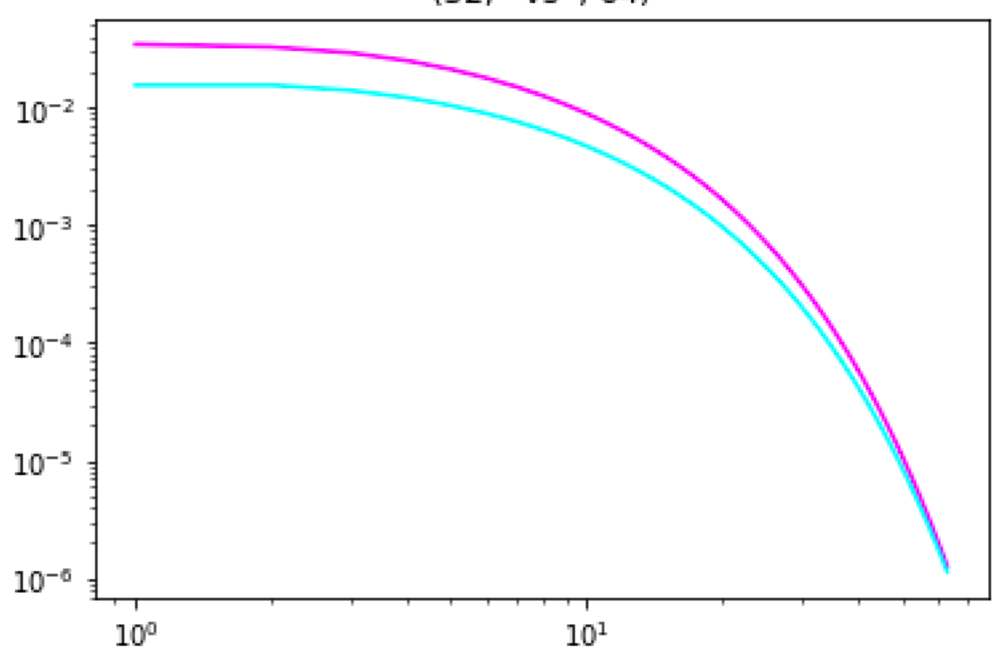
The graphs for the N comparisons are shown below.

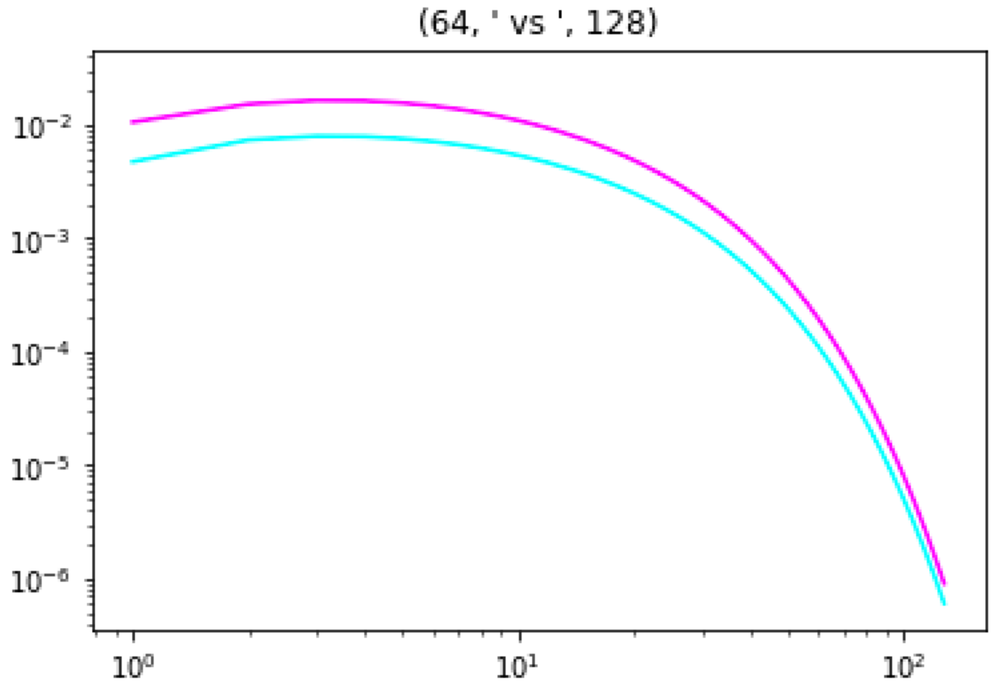
I also included one that is the differenence between N =16 and N = 8192 at the very end and the shape is more similar in shape to the one of 16 and 32. It has a longer period where the error values are not decreasing very quickly before finally decreasing. Im not sure why this one is so different from the 2056 and 4112 graph above it though.

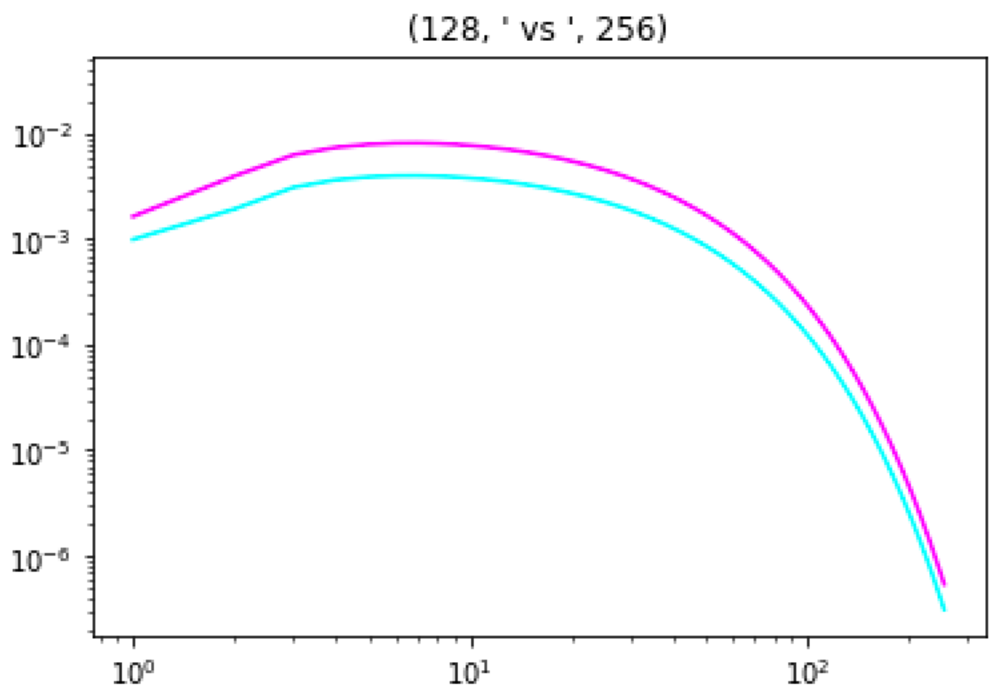
(16, ' vs ', 32)

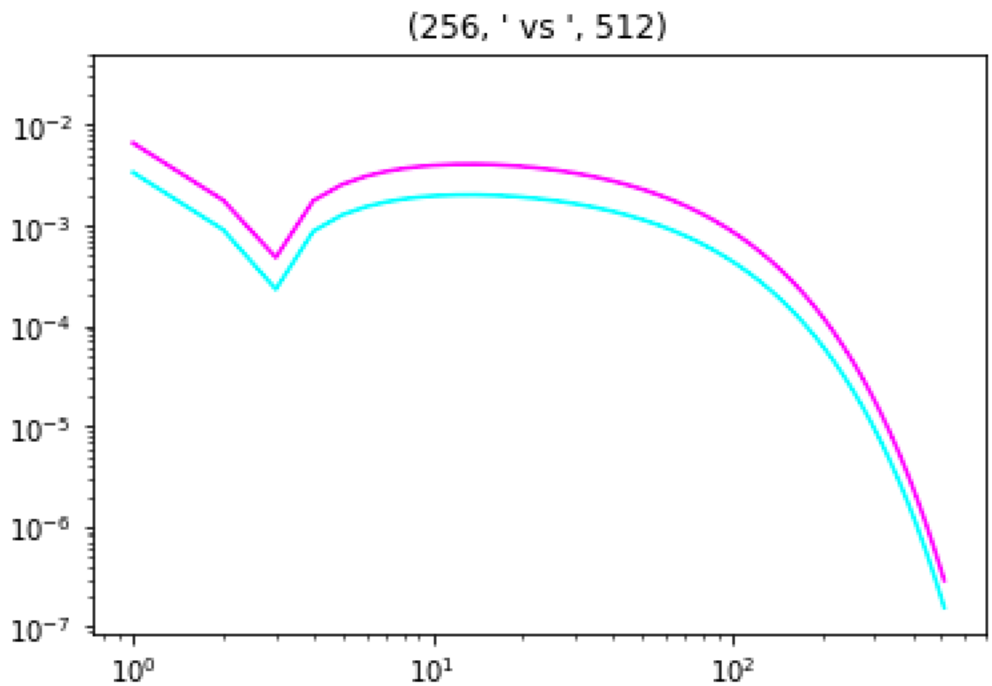


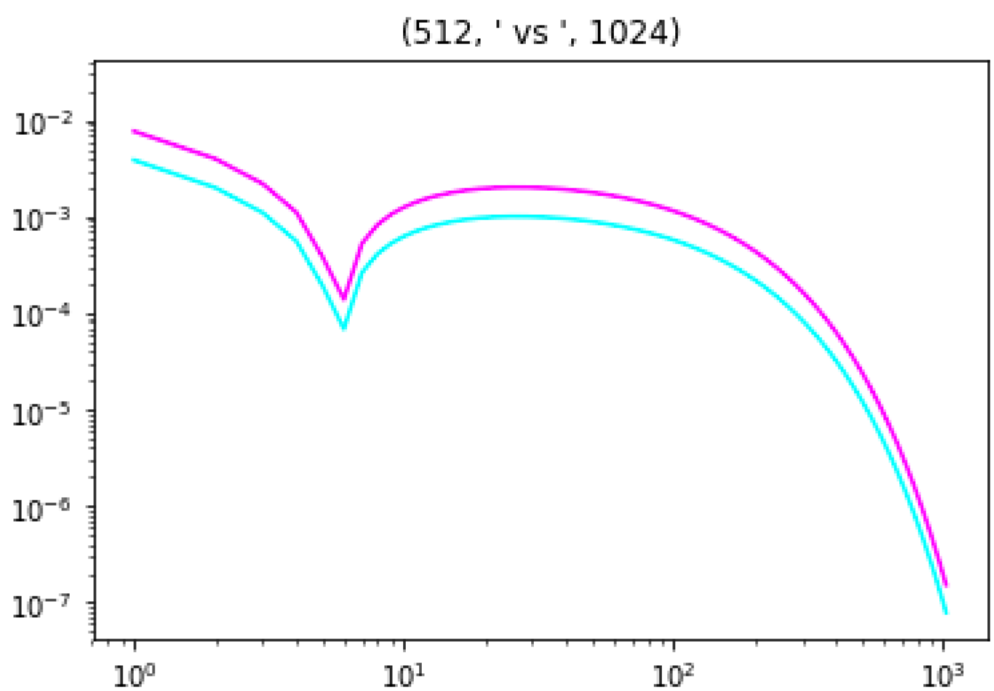
(32, ' vs ', 64)

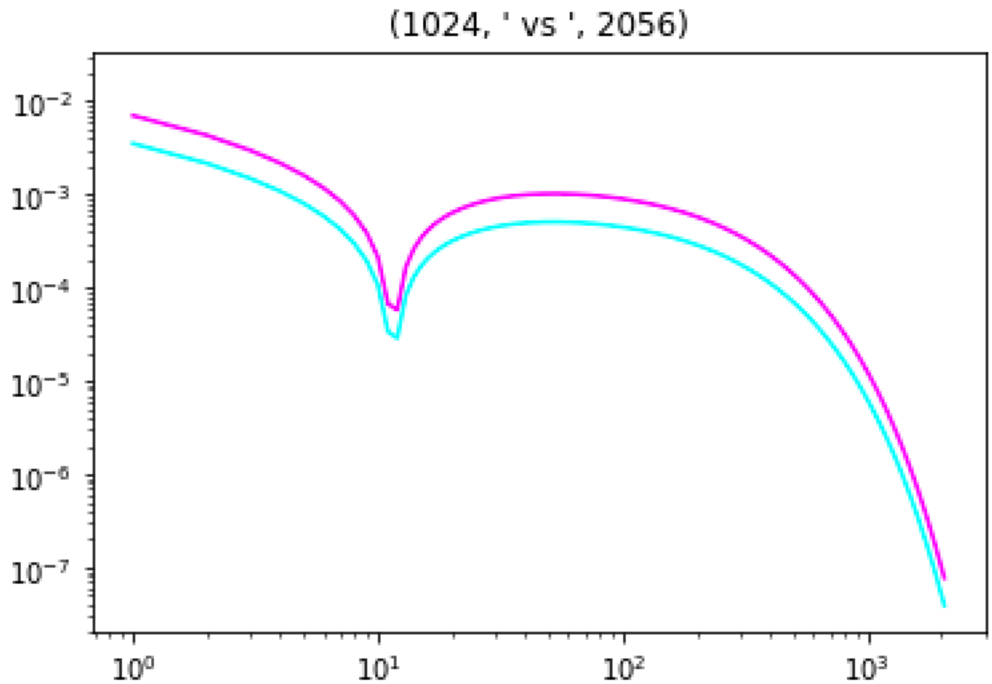


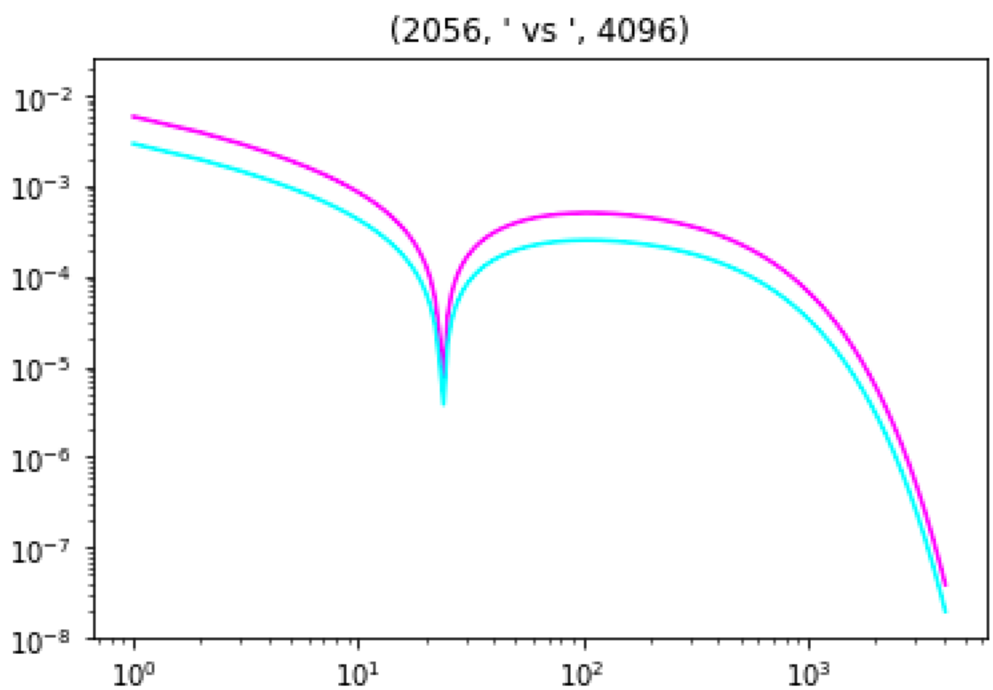

(64, ' vs ', 128)
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{0}
10^{1}
10^{2}


(128, ' vs ', 256)
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{0}
10^{1}
10^{2}

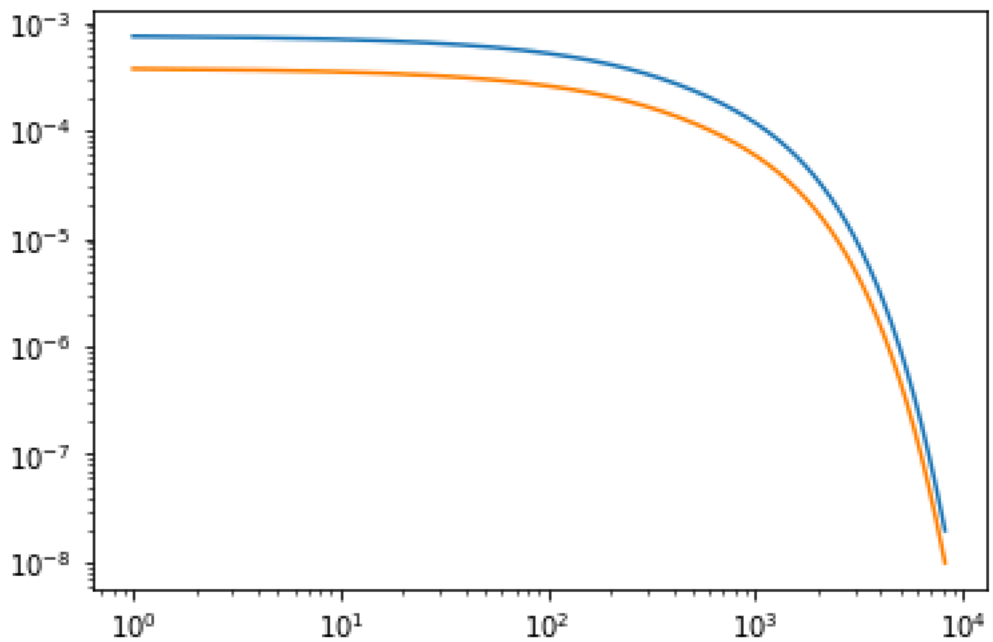

(256, ' vs ', 512)
10−2
10−3
10−4
10−5
10−6
10−7
10⁰
10¹
10²


(512, ' vs ', 1024)
10−2
10−3
10−4
10−5
10−6
10−7
10⁰
10¹
10²
10³


(1024, ' vs ', 2056)
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{0}
10^{1}
10^{2}
10^{3}


(2056, ' vs ', 4096)
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{0}
10^{1}
10^{2}
10^{3}

16 vs 8192



10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}