Section 1:

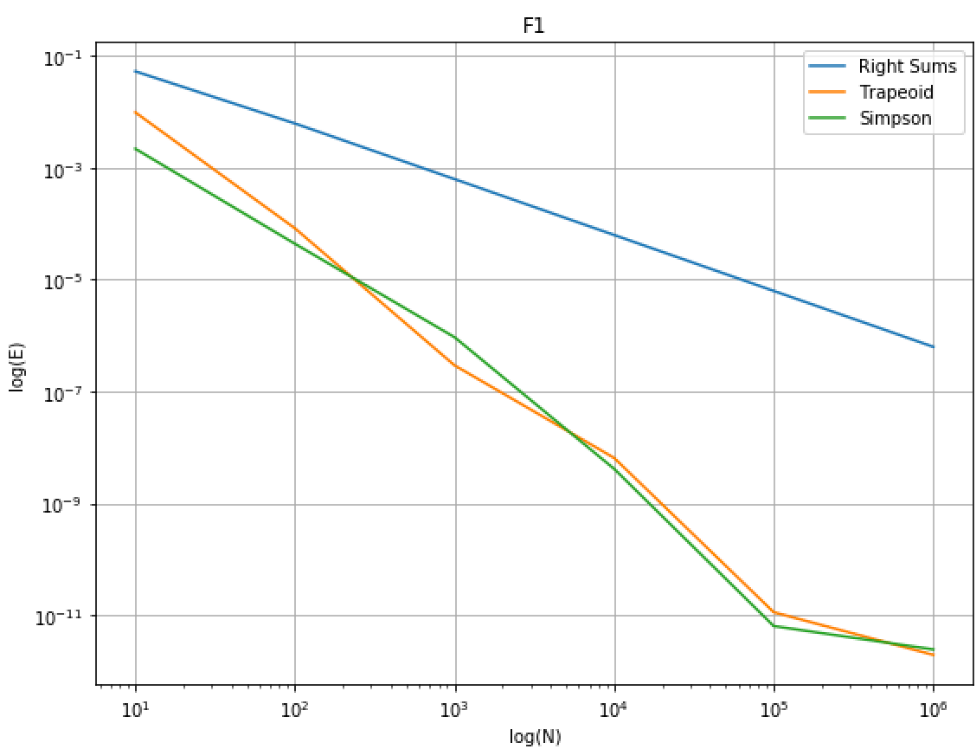
In this homework assignment, we had to study the rates of convergence of numerical approximations to

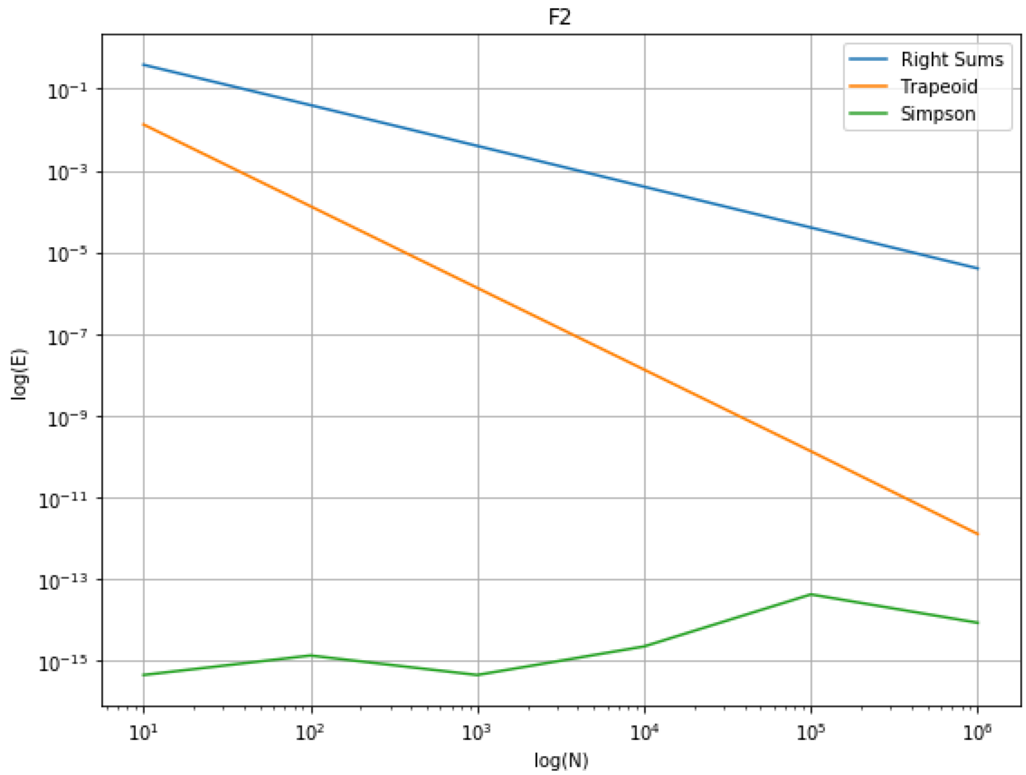
$$\int_{-1}^{1} F_k(x)\,\mathrm{d}x \approx \cdots$$

We had three methods that we were comparing this to and they were right sums, trapezoidal rule, and Simpson's method. These three methods are very similar in their approximation and they eventually all converge to the same number with a slight different in error. We examined three equations over the intervals from -1 to 1, each one, examined over the interval split into N equal parts. The N values I chose in this assignment were 10, 100, 1000, 10000, 100000, and 1000000.

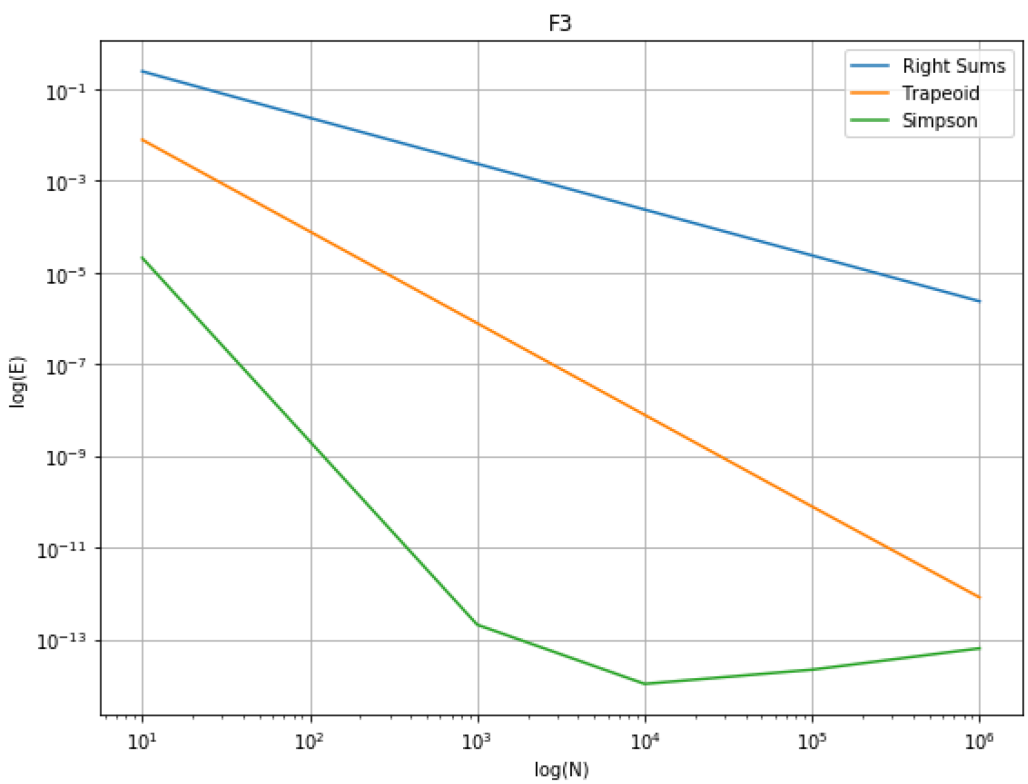
The three equations we evaluated are $F1(x) = abs\left(x - \frac{pi}{10}\right)$, $F2(x) = (x-1)^2$, $F3(x) = e^x$

The first thing I did, was translate each of the methods into a function in python that would output what the algorithm would spit out. Each of the functions took in four values, the function, the lower bound, the upper bound, and the number of intervals. I then used the functions to solve for the values of the convergences at each of the interval (N) sizes. Once I had those, I plotted the outputs of each of the three funtions onto one graph. The results are shown below:





F2
Right Sums
Trapeoid
Simpson
log(E)
log(N)



F3
Right Sums
Trapeoid
Simpson
log(E)
log(N)

As you can see, the lines on each of the graphs are pretty similar in shape. This is because although the methods are different, they are all finding the numerical approximations of the same functions and are converging to the same number. They are not identical however, because what you see graphed above is not the actual output we got for each of the functions and interval sizes, but rather it is a graph of the natural log of the intervals [log(N)] by the natural log of the errors [log(E)].

The difference you see on each of the graphs above, is how large the errors are for each convergence approximation with respect to how large the intervals are. The reason the errors get larger is because the interval sizes increase so there is more room for the errors to get larger.

I also calculated the alpha values for each of the functions in each of the methods. They are show below:

```
('alpha for Right Sums F1   : ', 0.98898227452065424)
('alpha for Right Sums F2   : ', 0.99776880649524646)
('alpha for Right Sums F3   : ', 1.0021604715736911)
('alpha for Trapezoid F1    : ', 2.021724617158791)
('alpha for Trapezoid F2    : ', 2.0036210674501116)
('alpha for Trapezoid F3    : ', 1.9972421058492174)
('alpha for Simpson F1      : ', 1.9322679601607211)
('alpha for Simpson F2      : ', -0.33087879444972251)
('alpha for Simpson F3      : ', 1.6803847243776229)
```

The alpha values for each of the methods are pretty similar with the exception of Simpson of F_2. This is because the slope of the F_2 function using Simpson's method is positive. These alpha values appear to make sense given the graphs that were constructed and the numbers that were outputted by the functions.

Right Sums has the lowest alpha value for each of the functions, being around 1 or less than that. So this one will tend to zero the fastest. The next fastest will be Simpson's method and the last will be Trapezoidal method.

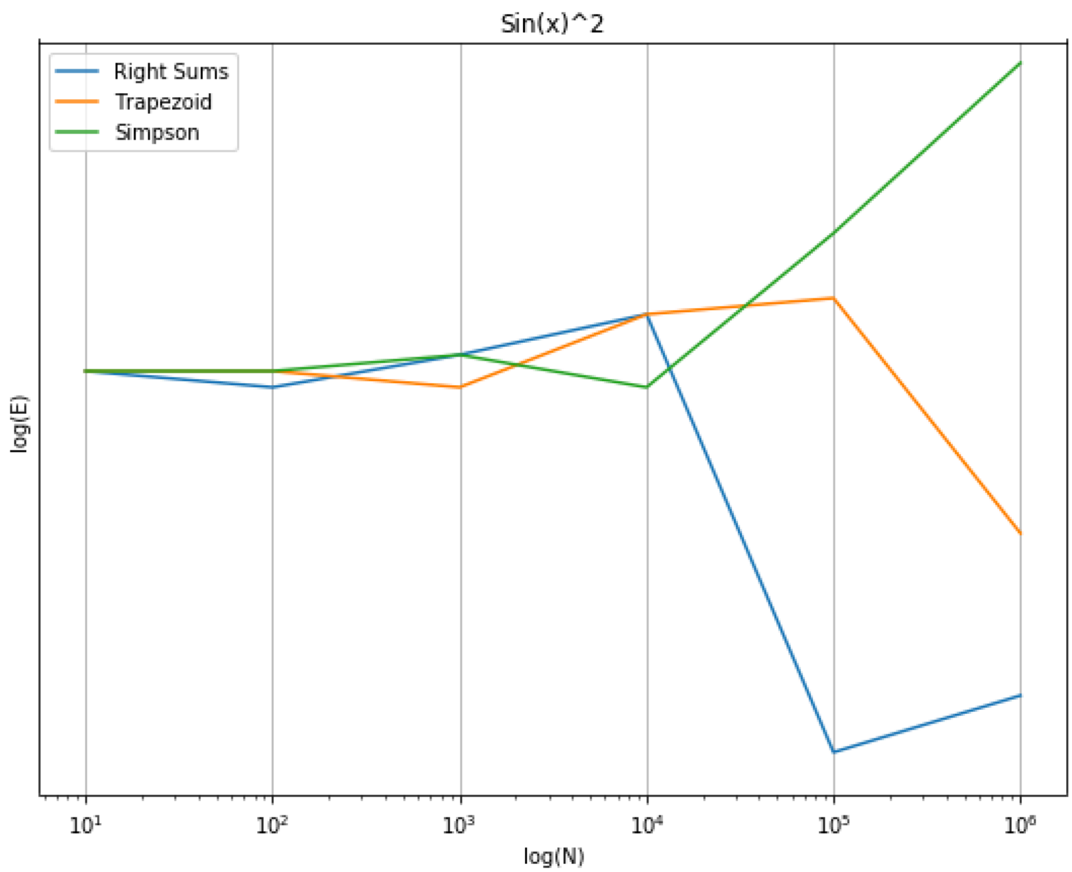
Section 2:

In this problem, we had to consider

$$\int_{-\pi}^{\pi} \sin^2 x \, \mathrm{d}x.$$

We had to compare the convergence rates for the three methods for the equation above. For this one, I noticed that the errors were very similar when the interval rates were small and as the interval sizes increased, the errors did as well and they moved in different directions. They were very similar until about 10^4 and after that interval size, they started to be more erratic and unpredictable. It appears from the picture of the graph that right sums and trapezoid were moving back in the same direction at the end.

This can be see below:



As the number of intervals increases, the error values increase as well. Since there are more data points to get numbers from, if any of them differ along the way, this can cause the error to be larger as the intervals get larger. At 10^1, there are not many values to be considered so it makes sense that the error values will all be pretty similar there.