Name:____

MATH 321 TAKE-HOME FINAL EXAM

This exam will be due on Thursday, December 13, at noon, slipped under my office door. You may use your textbook, your class notes, and your homework for MATH 321, Fall 2018. You may not use other books, notes, or the Internet for this exam. You may not discuss any part of this exam with any other person besides myself until the exam has been handed in.

Please sign below to signify that you have abided by the above rules:

Signature:____

In the following questions, $\mathbb{N}$ refers to the set $\{1, 2, 3, \dots\}$ of natural numbers, $\mathbb{I}$ refers to the set of irrational numbers, and $\overline{A}$ refers to the closure of the set A.

(1) True or false? Prove if true, and disprove or give a counterexample if false.
   (a) If (a_n) and (b_n) are sequences and both (a_n/b_n) and (b_n) converge, then (a_n) converges.
   (b) If $(x_n) \to x$, then $(|x_n|) \to |x|$.
   (c) If $K_1, K_2, \dots, K_n$ are compact sets, then $\bigcup_{i=1}^n K_i$ is compact.
   (d) If $C_1, C_2, \dots, C_n$ are connected sets, then $\bigcap_{i=1}^n C_i$ is connected. [**Hint:** is $\emptyset$ connected? You may assume that if $A \subseteq B$, then $\overline{A} \subseteq \overline{B}$.]
   (e) The set
$$S := \{\sqrt{n} : n \in \mathbb{N}\}$$
   is uncountable.
   (f) The set S from part (e) is perfect.
   (g) If E is a nonempty subset of $\mathbb{R}$ that is bounded below, then $\inf E + 1 \in E$.

(2) Let $a \in \mathbb{R}$ with $a > -1$. Prove that $(1+a)^n \geq 1 + na$ for all $n \in \mathbb{N}$.

(3) Compute
$$\lim_{n\to\infty} \left(-\frac{3}{2} + \frac{3}{4} - \frac{3}{8} + \frac{3}{16} \mp \cdots + (-1)^n \frac{3}{2^n} \right).$$

(4) **(Bonus)** Let T be the set of all $x \in [0, 1]$ whose decimal expansion contains only the digits 3 and 8. Answer the following questions regarding T and prove why or why not.
   (a) **(3 points)** Is T countable?
   (b) **(5 points)** Is T compact?
   (c) **(5 points)** Is T perfect?