Name:____

MATH 321 MIDTERM

This exam will be due on Thursday, October 12, at 4 pm. You may use your textbook, your class notes, and your homework for MATH 321, Fall 2017. You may not use other books, notes, or the Internet for this exam. You may not discuss any part of this exam with any other person besides myself until the exam has been handed in.

Please sign below to signify that you have abided by the above rules:

Signature:____

(1) **(5 points)** Negate the following: for all $a, b \in \mathbb{R}$ satisfying $a < b$, there exists $n \in \mathbb{N}$ such that $a + \frac{1}{n} < b$.

(2) **(20 points)** Prove formally: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(3) **(18 points)** Prove that a function $f : A \to B$ is 1-to-1 if and only if, for all $y \in B$, $f^{-1}(\{y\})$ contains at most one point.

(4) **(17 points)** Let
$$A = \Big\{ n + \frac{(-1)^n}{n} : n \in \mathbb{N} \Big\}.$$
Compute $\sup A$ and $\inf A$, if they exist, and prove your answers.

(5) **(20 points)** Given a sequence (a_n) and a natural number $k \in \mathbb{N}$, let (b_n) be the sequence defined by $b_n = a_{n+k}$. That is, the terms in (b_n) are the same as the terms in (a_n) once the first k terms have been skipped. Prove that (a_n) converges if and only if (b_n) converges, and if they converge, show that $\lim a_n = \lim b_n$. Thus the convergence of a sequence is not affected by omitting (or changing) a finite number of terms.

(6) **(20 points)** In what follows, if S and T are any two sets, we write $S \sim T$ if S and T have the same cardinality and let $P(S)$ denote the power set of S. Prove that if A and B are sets such that $A \sim B$, then $P(A) \sim P(B)$. **Hint**: Assuming that $f : A \to B$ is a 1-to-1 correspondence, construct a 1-to-1 correspondence $g : P(A) \to P(B)$.

(7) **Bonus (15 points).** Prove that the set of subsequences of any (infinite) sequence has the same cardinality as the real numbers.