# MATH 321 WEEK 2 CLAIMED PROBLEM SOLUTIONS

**1.2.12.** Let $y_1 = 6$, and for each $n \in \mathbb{N}$ define $y_{n+1} = (2y_n - 6)/3$.

(1) Use induction to prove that the sequence satisfies $y_n > -6$ for all $n \in \mathbb{N}$.
   (a) **Base case:** $y_1 = 6 > -6$.
   (b) **Inductive step.** Suppose that $y_n > -6$. Then
   $$y_{n+1} = \frac{2y_n - 6}{3} > \frac{2(-6) - 6}{3} = \frac{-18}{3} = -6$$
   as desired.

(2) Use another induction argument to show the sequence $(y_1, y_2, y_3, \dots)$ is decreasing.
   (a) **Base case:** we note that $y_2 = (2 \times 6 - 6)/3 = 2$, and hence that $y_1 = 6 > y_2$.
   (b) **Inductive step**: Suppose that $y_n > y_{n+1}$. Then
   $$y_{n+2} = \frac{2y_{n+1} - 6}{3} > \frac{2y_n - 6}{3} = y_{n+1}$$
   where the inequality is by the induction assumption.

**1.3.8**. Compute, without proofs, the suprema and infima of the following sets.

(1) $\sup = 1$, $\inf = 0$
(2) $\sup = 1$, $\inf = -1$
(3) $\sup = \frac{1}{3}$, $\inf = 0$ (or $\frac{1}{4}$ if you don't believe that $0 \in \mathbb{N}$)
(4) $\sup = 1$, $\inf = 0$