MATH 321 PRACTICE MIDTERM

KENAN INCE

(1) Negate the following: given $\epsilon > 0$ there exists $y \in \mathbb{N}$ such that, for all $x \in \mathbb{R}$ such that $xy \leq \epsilon$, $x \in A$ or $y \in B$.

(2) For each of the following statements, say whether it is true or false. If the statement is true, prove it. If it's false, give a counterexample.
   (a) Every bounded set contains its supremum.
   (b) A decreasing sequence is always bounded.
   (c) Every sequence contains a convergent subsequence.
   (d) Every bounded sequence converges.
   (e) If A and B are sets, then $P(A) \cup P(B) \subseteq P(A \cup B)$. (Here $P(S)$ denotes the power set of S for any set S.)
   (f) If A and B are sets, then $P(A \cup B) \subseteq P(A) \cup P(B)$.
   (g) If A and B are sets so that $P(A) \subseteq P(B)$, then $A \subseteq B$.
   (h) If A and B are sets so that $A \subseteq B$, then $P(A) \subseteq P(B)$.
   (i) If A, B, C are sets, then $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$.
   (j) If A, B, C are sets, then $(A \setminus B) \cup C = (A \cup C) \setminus (B \cup C)$.
   (k) If A and B are sets, $A \subseteq B \iff A \setminus B = \emptyset$.
   (l) If A and B are sets, $A = B \iff A \setminus B = \emptyset$.
   (m) $(-\infty, \infty) \sim (-\frac{\pi}{2}, \frac{\pi}{2})$.
   (n) The set of irrational numbers $\mathbb{I}$ is countable.
   (o) The set of rational numbers $\mathbb{Q}$ is countable.
   (p) If $(x_n) \to 0$, then $((-1)^n x_n) \to 0$ as well.

(3) Define the sequence (x_n) recursively by setting

$$x_1 = \sqrt{2}$$
$$x_{n+1} = \sqrt{2 + x_n} \text{ for all } n \in \{1, 2, 3, \dots\}$$

   (a) Show that the sequence (x_n) converges.
   (b) Let $\lambda = \lim x_n$. Show that $\lambda^2 - \lambda - 2 = 0$.

(4) Does the sequence $(\frac{2n-3}{5n})$ converge or diverge? Prove your answer.

(5) Find the supremum and infimum of each of the following sets. No proofs are necessary.
   (a) $A = \{x \in \mathbb{I} : x^2 < 2\}$
   (b) $B = \{x \in \mathbb{Q} : x^2 < 2\}$.
   (c) $C = \{1 - \frac{1}{n} : n \in \mathbb{N}\}$
   (d) $D = \{n + \frac{(-1)^n}{n} : n \in \mathbb{N}\}$
   (e) $E = \{\frac{1}{n} : n \in \mathbb{N} \text{ and } n \text{ is prime}\}$

(6) What can you say about a non-empty subset A of the real numbers for which $\sup A = \inf A$? Prove your answer.

(7) Using the definition of convergence of a sequence, prove that

$$\lim \left(\frac{n-1}{n+1}\right) = 1.$$