PSP: WHY BE SO CRITICAL? SOLUTIONS

Note: only grade these questions for accuracy and the rest for completion.

Exercise 1. (Question 5)
(1) The power series $1 + x + x^2 + x^3 + \cdots = \sum_{r=1}^{\infty} x^r$ converges for $|x| < 1$ to $\phi(x) = \frac{1}{1-x}$ (Geometric Series formula)
(2) If $\lim_{x \to 1} \phi(x) \neq \phi(1)$, we say ϕ is **discontinuous** at $a = 1$.

Exercise 2. (Question 6)
(1) Instead of a polynomial, it's a sum of sines evaluated at multiples of x.
(2) For $x = \pi$, this formula says
$$\frac{\pi}{2} = \sin \pi - \frac{1}{2} \sin 2\pi + \frac{1}{3} \sin 3\pi - \ldots$$
but the sine function is 0 at all multiples of π, hence the equation above becomes $\frac{\pi}{2} = 0$.
(3) Differentiating both sides of the series equation, where the series is differentiated term-by-term, yields
$$\frac{1}{2} = \cos x - \cos 2x + \cos 3x - \ldots.$$
Now, again plugging in $x = \pi$, we obtain
$$\frac{1}{2} = \cos \pi - \cos 2\pi + \cos 3\pi - \ldots$$
$$= 1 - (-1) + 1 - (-1) + \ldots$$
$$= \sum_{k=1}^{\infty} 1$$
which is absurd.