MATH 321 DAY 1: INTRODUCTION TO THE COURSE

KENAN INCE

1. Logistics

1.1. Introduction: about me.

- Name/pronouns/PhD from Rice, research in how hard it is to untie mathematical knots
  - Please feel free to talk to me about my research at any time during the semester.
- My primary goal in teaching math is to communicate that math is a creative, logical field with immense importance to society. A lot of what we're teaching is logic, which is like a prosthetic you attach to your reasoning to make it more powerful.

1.2. [open on computer] Syllabus - read in your own time.

1.2.1. *About the class.*

- A proof-based class in which many of the results assumed in Calculus are proven. This course does not "build out" from what you learned in calculus, but rather "builds under", rigorously undergirding many known facts about the real numbers and discovering some surprising, unintuitive facts along the way! Topics include point-set topology of the real numbers, a rigorous treatment of limits for sequences and functions, continuity, and differentiability. There will be an emphasis on rigorous and clear proof-writing.

1.2.2. *Office and office hours.*

- My office is Foster 311 (go up the stairs to the third floor, turn right; I'm at the end of the hall.)
- My office hours this semester will be MW 1:30pm-3pm; TTH 2:30-3:30pm. Anyone who can't make those hours? I'll usually be around the office 12:30-1:45pm MTWTh and (maybe) 11-noon Friday as well. Make appointments at least 24 hours in advance!
- Tentative course outline: linked in Canvas under Syllabus

1.2.3. *Canvas.*

- Sign up now for Canvas notifications
- I can't guarantee I'll always remember to announce assignments in-class; you're expected to check Canvas before and after every class period to determine what assignments you have.
- I will post all of your grades in Canvas.
- We take the Title IX and disability provisions extremely seriously! If you require any academic accommodations related to a disability, please contact Karen Hicks ASAP.

1.2.4. *Study groups.* Under "graded discussions" in Canvas.

2. Setting the Stage

Get in groups of size 3–4. Group members should introduce themselves - name, pronouns, hobbies outside school, goals for the semester/year, reason they're taking the course. For each of the questions that follow, I will ask you to:

(1) Think about a possible answer on your own.
(2) Discuss your answers with the rest of your group.
(3) Share a summary of each group's discussion.

2.0.1. *Questions.*

(1) What are the goals of a liberal arts education?
(2) How does a person learn something new?
(3) What is the value of making mistakes in the learning process?
(4) How do we create a safe environment where risk taking is encouraged and productive failure is valued?

- I will be teaching this course using IBL methods for all the reasons you gave, as well as because research shows IBL improves student understanding of mathematics more than traditional lecture-based teaching methods.

  "Any creative endeavor is built on the ash heap of failure."—Mike Starbird

Claim 1. An education must prepare a student to ask and explore questions in contexts that do not yet exist. That is, we need individuals capable of tackling problems they have never encountered and to ask questions no one has yet thought of.

If we really want students to be independent, inquisitive, & persistent, then we need to provide them with the means to acquire these skills.

- Pass around Day 1 sign-in sheet.docx

2.1. **Group work.**

- What rules should we set for groups?
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    * Whose voices are valued?
    * What philosophy do we have about who can make contributions to mathematics and the value of those contributions?
    * How does this connect to the values we espouse for our democracy?
- [slide] Why group work/Group roles?
- [slide] Assign roles to each group by first letter of last name; they'll rotate every day of class.
- Anyone can answer questions posed to the whole class, but if your group came up with an idea that hasn't been shared yet, it **someone** in your group's responsibility to share your answer so that everyone can learn from you!

2.2. **Introductory problem-solving.**

- To get used to the format of this course and get the creative group-work juices flowing, get into groups and try the following problems.

Exercise 2. Solve the following problems in your groups. (Pick one or two depending on time and experience; ask students how experienced they are with induction and how hard of a problem they want. Definitely give them Collatz' Conjecture.)

(1) [slide] There is an island upon which a tribe resides. The tribe consists of 1000 people, with various eye colours. Yet, their religion forbids them to know their own eye color, or even to discuss the topic; thus, each resident can (and does) see the eye colors of all other residents, but has no way of discovering his or her own (there are no reflective surfaces). If a tribesperson does discover his or her own eye color, then their religion compels them to commit ritual suicide at noon the following day in the village square for all to witness. All the tribespeople are highly logical and devout, and they all know that each other is also highly logical and devout (and they all know that they all know that each other is highly logical and devout, and so forth). Of the 1000 islanders, it turns out that 100 of them have blue eyes and 900 of them have brown eyes, although the islanders are not initially aware of these statistics (each of them can of course only see 999 of the 1000 tribespeople).
One day, a blue-eyed foreigner visits to the island and wins the complete trust of the tribe.
One evening, he addresses the entire tribe to thank them for their hospitality.
However, not knowing the customs, the foreigner makes the mistake of mentioning eye color in his address, remarking "how unusual it is to see another blue-eyed person like myself in this region of the world".
What effect, if anything, does this faux pas have on the tribe?

(a) Hint A: what happens if there is only one blue-eyed person and 999 brown-eyed people?
(b) Hint B: now suppose there are two blue-eyed people and that you're one of the blue-eyed people. What happens (or doesn't happen) at noon the next day? Now prove this always happens by induction.
(c) **Solution**: 100 days after the address, all the blue-eyed people commit suicide. Proof by induction:

Proposition 3. *Suppose that the tribe had n blue-eyed people for some positive integer n. Then n days after the traveler's address, all n blue-eyed people commit suicide.*

Proof. We induct on n. When $n = 1$, the single blue-eyed person realizes that the traveler is referring to them, and thus commits suicide on the next day. Now suppose inductively that n is larger than 1. Each blue-eyed person will reason as follows: "If I am not blue-eyed, then there will only be $n - 1$ blue-eyed people on this island, and so they will all commit suicide $n - 1$ days after the traveler's address". But when $n - 1$ days pass, none of the blue-eyed people do so (because at that stage they have no evidence that they themselves are blue-eyed). After nobody commits suicide on the $(n - 1)^{st}$ day, each of the blue eyed people then realizes that they themselves must have blue eyes, and will then commit suicide on the n^{th} day. □

(2) Given a positive integer n, if it is odd then calculate $3n + 1$. If it is even, calculate $n/2$. Repeat this process with the resulting value. For example, if you begin with 1, then you obtain

$$1, 4, 2, 1, 4, 2, 1, \ldots$$

which will repeat forever in this way. If you start with 5, then you obtain the sequence $5, 16, 8, 4, 2, 1, \ldots,$ and now find yourself in the previous case. If you start from any positive integer, does this process always end by cycling through $1, 4, 2, 1, 4, 2, 1, \ldots$? If so, prove it. If not, give a positive integer which does not terminate by cycling through $1, 4, 2$.

- This is an unsolved problem called the Collatz Conjecture!
- Why do you think I gave you an unsolved problem?
  - What do you think mathematicians spend most of their time doing?
  - How do you define "success" when working on an unsolved problem?
    * Making sense of a problem
    * Increasing depth of understanding (what methods DON'T work?)
    * How would you describe your process of engagement on this problem?
  - Failure is completely normal!
    * Even on problems that are solved, it often took years or even millennia to solve them!
    * So don't feel ashamed or worried if you don't get something immediately.

2.3. Homework for next class.