MATH 321 PRACTICE FINAL EXAM

KENAN INCE

(1) Let
$$A = \left\{ (-1)^n + \frac{2}{n} : n = 1, 2, 3, \ldots \right\} \text{ and } B = \{x \in \mathbb{Q} : 0 < x < 1\}.$$
Answer the following questions for each set:
 (a) What are the limit points?
 (b) Is the set open? Closed?
 (c) Does the set contain any isolated points?
 (d) Find the closure of the set.

(2) Let (a_n) be an increasing sequence of positive numbers. Prove, using the definition of convergence, that if (a_n) converges, so does $(7a_n^2)$.

(3) Assume K is compact and F is closed. Decide if the following sets are definitely compact, definitely closed, both, or neither.
 (a) $\overline{K \cap F}$
 (b) $\overline{F^c \cup K^c}$
 (c) $\overline{K \setminus F}$
 (d) $\overline{K \cap F^c}$

(4) Which of the following statements about 1-to-1 and onto functions are true? Prove if true, and give a counterexample if false.
 (a) If f and g are onto, then $g \circ f$ is onto.
 (b) If f is onto and g is 1-to-1, then $g \circ f$ is onto.
 (c) If f is onto and g is 1-to-1, then $g \circ f$ is 1-to-1.
 (d) If f and g are 1-to-1, then $g \circ f$ is 1-to-1.
 (e) If f and g are 1-to-1 correspondences, then $g \circ f$ is a 1-to-1 correspondence.

(5) **(Bonus)** Let (a_n) be a bounded sequence and let $S = \{\lim(a_{n_k}) : (a_{n_k}) \text{ is any subsequence of } (a_n)\}$. Prove that S is closed.