

A Review on Optimal Power Flow Problems: Conventional and Metaheuristic Solutions

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Abstract— Optimal power flow (OPF) of power system consists in solving an optimization problem in relation to a criterion (economic or technic), called fitness function, while respecting equality and inequality constraints. The selected optimization criterion in this paper is the fuel cost of generation. For this, we are going to present a review of most conventional and metaheuristic solution methods used in literature. In order to demonstrate the advance of metaheuristic methods compared to conventional ones for OPF problem solution, we propose an application example using both interior point method (IPM) and the particle swarm optimization (PSO). The review study allowed us a better understanding of OPF problem and method solutions given in literature.

Keywords— OPF Problem, Optimization, conventional solution methods, Metaheuristic solution methods, PSO, IPM.

I. INTRODUCTION

The demand of electrical energy throughout the world is increasing very rapidly. In the world, 70% of energy production is based on thermal power plant. So, there is large requirement of fuel and hence rapid increase in fuel cost occurs [1].

The rapid increase in demand and the evolution of power systems, as well as the transit of long-distance power flows have contributed to the search for efficient methods to meet either economic or technical criteria, or both economic and technical criteria. It is this problem which is referred to in the literature as OPF.

Optimal power flow (OPF) is one of the most important problems for power system planners and operators [2]. Its main aim is to minimize the fitness function and simultaneously satisfying equality and inequality constraints.

In the literature, many methods are used to solve OPF problems. There are conventional and metaheuristic methods. Conventional methods have been developed and improved to solve linear and non-linear system optimization problems. Conventional methods are not always appropriate and cannot guarantee the overall solution. To solve these drawbacks, researchers have developed new methods called meta-heuristic methods. These are a set of optimization algorithms to solve difficult problems. They are very effective in the field of single-objective optimization [3].

This paper aims to provide a state of the art of the different optimization methods and applying Interior point method (IPM) and Particle swarm optimization on IEEE-30 bus system for a comparative study of the OPF.

II. OPF PROBLEM FORMULATION

The aim of OPF problem is to minimize the fitness function and simultaneously satisfying the equality and inequality constraints. The standard OPF problem can be written in the following form:

$$\begin{cases} \text{Min} : J(x, u) \\ \text{Subject to} : f_i(x, u) = 0, \quad i = 1, \dots, p \\ \text{and} : g_j(x, u) \leq 0, \quad j = 1, \dots, q \end{cases} \quad (1)$$

Where $J(x, u)$ represents the fitness function, $f_i(x, u)$ represents the equality constraints, $g_j(x, u)$ represents the inequality constraints, x represents the state variables, u represents the control variables, that is those which can be adjusted for proper operation of the system, p and q represent respectively the number of equality and inequality constraints.

The fitness function the most used for OPF problem is the minimization of the total cost of active power generation. It is given by quadratic function of eq.2:

$$\text{Minimize} : J(x, u) = \sum_{i=1}^{N_g} (a_i P_{gi}^2 + b_i P_{gi} + c_i) \quad (2)$$

Where (x, u) is state and control variables vector, P_{gi} is the active power generated, N_g is the number of generation bus and a_i , b_i and c_i are the fuel cost of i th unit, expressed respectively in \$/h, \$/MWh and \$/MW²h.

Fitness function is subjected to various constraints:

- Equality constraints characterize the equilibrium condition of the power system

$$P_{gi} - P_{di} - \sum_{j=1}^n V_i V_j [G_{ij} \cos(\theta_{ij}) + B_{ij} \sin(\theta_{ij})] = 0 \quad (3)$$

$$Q_{gi} - Q_{di} - \sum_{j=1}^n V_i V_j [G_{ij} \sin(\theta_{ij}) - B_{ij} \cos(\theta_{ij})] = 0 \quad (4)$$

- The inequality constraints that characterize the operating limits of the power system structures. These are:
 - a) Limits on active power generation P_{gi} excepted slack bus;
 - b) Limits on voltage generation V_{gi} ;
 - c) Limits on Transformer tap setting T_i ;
 - d) Limits on Shunt VAR compensation Q_c ;
 - e) Limits on the transit of apparent power in the lines S_{li} .
 Mathematically, these constraints are written:

$$\left\{ \begin{array}{l} P_{gi}^{min} \leq P_{gi} \leq P_{gi}^{max} \\ Q_{gi}^{min} \leq Q_{gi} \leq Q_{gi}^{max} \\ V_i^{min} \leq V_i \leq V_i^{max} \\ T_{ij}^{min} \leq T_{ij} \leq T_{ij}^{max} \\ Q_{ci}^{min} \leq Q_{ci} \leq Q_{ci}^{max} \\ |S_{li}| \leq |S_{li}^{max}| \end{array} \right. \quad (5)$$

III. CONVENTIONAL METHODS FOR OPF

A. Linear Programming (LP)

Linear Programming (LP) method deals with situations where maximum or minimum of a certain set of linear function is decided [4]. LP method is used when all the constraints are linear. This method is reliable and has a good convergence characteristic; however the main shortage is it could be trapped in local minima [5].

M. Olofsson et al. [8] developed LP to solve OPF problem. To improve the convergence, he applied a method based on a second order sensitivity approximation of the active power losses of the total system. Numerical simulations were first applied on a railway network and then, on IEEE-14, 57 and 118 bus system.

H. Habibollahzadeh [9] used LP to solve OPF of linear systems. His method has been applied to minimize the generation cost of the thermal system and focused on active and reactive power problem. Optimization of the objective function is performed in two steps:

- The optimization is achieved using the piecewise linear approximation of the thermal generation curves;
- The nonlinear objective with quadratic approximation of generation costs for the thermal plants is used to obtain further improvements.

LP method has been developed and tested on IEEE 39 bus system. It has been particularly efficient for constrained OPF problems.

B. Nonlinear Programming (NLP)

NLP is a method which deals with optimization problems involving nonlinear functions and nonlinear constraints. The NLP method is often used to solve the convergence problem posed by the LP method.

D. Pudjianto et al. [10] used LP and NLP to solve OPF problems. Method application consisted to locate reactive power support among competing generators in a deregulated environment. The statistical analysis of the performance characteristics of the reactive OPF has been carried out on modified IEEE-118 bus system.

H. Habibollahzadeh et al. [9] used for accurate active and reactive OPF models. In the hydraulic modeling of the system, author considered an important part of the hydraulic production.

C. Quadratic Programming (QP)

The Quadratic programming method (QPM) is considered as a natural transition from linear to nonlinear programming. Indeed, most of the methods developed for QP are direct extensions of the LP method. This method has different form of NLP and its constraints are linear.

Gerald F. Reid and Lawrence Hasdorff [11] used QP to solve Economic dispatch problems. The method is tested on IEEE-5, 14, 30, 57 and 118 bus systems. The

convergence of the method is obtained after three iterations for systems considered and solution time is small enough to allow the method to be used for on-line dispatching at practical time intervals.

N. Grudin [12] used QP method to solve reactive PF problems. Method was developed and applied on IEEE-30 bus system and 278-bus power system. It has been observed that developed versions of modified successive programming method provide reliability of optimization. The effectiveness of the method is confirmed by the extensive testing and application of developed for solving reactive power flow problems in question.

D. Interior Point Method (IPM)

Interior Point Method (IPM) was developed in 1984 by Karmarkar to solve linear optimization problems. That method is very efficient for solving linear and nonlinear convex optimization problems at large scale. It is the most widely used of the classical methods for the following reasons:

- It offers a great simplicity in the treatment of inequality constraints by the Logarithmic Barrier Function;
- It converges quickly to the optimal solution;
- It does not require a choice of starting point;

Edgardo D. Castronuovo et al. [6] used IPM to solve OPF problems. During the iterative process to solve load flow equations, the research of the optimal solution is based on combination of two directions: the affine-scaling and the centralization. Author showed that the suitable combination of these directions can increase the potential of IPM in terms of speed and reliability. That method was developed and applied on IEEE 30 and 118 buses system and two real networks from Brazil. Author implemented the Predictor-Corrector Primal-Dual algorithm, the Centralized IPM and Largest Step Path Following algorithm versions to reduce the computational effort requested for the solution of the OPF problem.

S. Mouassa [7] used IPM to minimize generation cost. The method was implemented on IEEE 30 bus system. Results were compared with the lambda iterative method. They are very close. However, there is a slight difference in the active losses, i.e. 11% between two methods.

E. Gradient Method (GM)

The Gradient method (GM) belongs to a large class of numerical methods called descent methods.

E. P. de Carvalho et al. [13] used GM to solve OPF problems. The method is applied on IEEE-14 and 30 bus system. Results obtained with GM showed the viability of the use of augmented Lagrangian function combined with Log-barrier and the reduced GM to solve the OPF problems.

A. M. H. Rashed and D. H. Kelly [14] used Lagrangian multipliers and Newton method to solve OPF problems. The method is tested on 5-bus system. It provided good convergence behavior and required less storage than other methods.

F. Newton Method (NM)

Newton's method is very powerful because of the rapid convergence near of the solution. This property is especially useful for power system applications. The NM requires calculation of the second derivative and other constraints.

H. Ambriz-Pérez et al. [5] presented a novel OPF model for the thyristor-controlled series compensator (TCSC). The NM is applied on 5 and 14 bus system. Results obtained TCSC-OPF model are new and more efficient than those currently available in the open literature. It has been shown NM is a powerful tool for solving TCSC-upgraded power networks very reliably and using very few iterations. David I. Sun et al. [16] used NM to solve OPF. The method is tested on 5-bus system. Results showed NM is suitable for solving OPF problems. However, it should be noted that the author mentioned that the effectiveness of his method has not been fully exploited in the prototype codes used in the test.

IV. METAHEURISTIC METHODS FOR OPF

A. Genetic Algorithm (GA)

Genetic algorithms (GA) are stochastic optimization methods that are now well known and are inspired by the mechanisms of natural selection and genetics. They use the principle of survival of the best adapted individuals. A-F. Attia et al. [17] used GA to solve OPF problem. The proposed method has been developed and applied on the IEEE-30 bus system. Results are compared with other techniques of the literature. Comparison showed the effectiveness of the proposed method. S. Kahourzade et al. [18] used three optimization techniques including GA to solve single and multi-objective OPF problem. Proposed technique has been developed and applied on the IEEE-30 bus system.

B. Evolutionary Programming (EP)

Evolutionary programming (EP) is a stochastic optimization which places emphasis on the behavioral linkage between parents and their offspring [27]. S. Kahourzade et al. [18] used three optimization techniques including EP to solve single and multi-objective OPF problem. Proposed technique has been developed and applied on the IEEE-30 bus system. For the single and multi-objective, results were better than others methods. P. Somasundaram et al. [27] used EP to solve OPF problem. The method has been developed and applied on IEEE-30 bus system. The results obtained are compared with those of the conventional security constrained optimal power flow (SCOPF).

C. Particle Swarm Optimization (PSO)

Particle Swarm Optimization (OPF) is a method that was proposed by Kennedy and Eberhart in 1995 as an alternative to standard Genetic Algorithms (GA). These algorithms are inspired by insect swarms and their coordinated movements. Observation shows that to find their food or protect themselves from predators, animals move in groups. N. Kalfallah et al. [20] used PSO for controlling reactive power in western Algerian power system. The use of the method has reduced power losses and improved the voltage profile. It also showed the installation of SCV devices is able to keep the voltages within their limits and to modify significantly the power losses. S. Kahourzade et al. [18] used three optimization techniques including PSO to solve single and multi-objective OPF problem. Proposed technique has been developed and applied on the IEEE-30 bus system.

D. Differential Evolution (DE)

The Differential Evolution (DE) algorithm is inspired by evolutionary strategies and GA. It is applied to continuous variable problems. M. Varadarajan and K.S. Swarup [21] used DE to solve OPF problem. The technique was tested on IEEE-14, 30, 57 and 118 bus system and its results are compared with those of conventional methods. It has been observed that DE method is a simple but a powerful technique to solve OPF problems with nonlinear objectives and constraints. M. Varadarajan and K.S. Swarup [22] used DE to solve OPF problem. The technique was tested on IEEE-14, 30 and 118 bus system and its results are compared first with the PSO, and then with those of a conventional method. It has been observed that results obtained with DE method were more robust than those of PSO and sequential quadratic programming (SQP) algorithms.

E. Artificial Bee Colony (ABC)

The Artificial bee colony is based on the particular communication behavior of the bees. Indeed, the foraging and dance behaviors of honey bees are simulated by the ABC algorithm to get optimal solutions of different optimization problems [23]. M. R. Adaryani and A. Karami [2] used ABC to solve multi-objective problem. The validity and effectiveness of the method was developed and tested on IEEE-9, 30 and 57 bus system. Results have been compared with other methods reported in the literature. The effectiveness and the quality of the results of the method show that it can be applied to solve opf problems in large-scale. K. Ayan et al. [23] used chaotic ABC to solve security and transient stability constrained optimal power flow (STSCOPF) problem. Its method has been developed and applied on IEEE-30 and 39 bus system. Results are compared with those obtained in the literature and showed validity and effectiveness of proposed method.

F. Gravitational Search Algorithm (GSA)

Gravitational search algorithm (GSA) is an optimization method inspired by the theory of Newtonian gravity in physics [24]. S. Duman et al. [25] used GSA to solve OPF problems. The method has been developed and tested on IEEE-30 and 57 bus system. Results obtained with GSA are compared with other metaheuristic methods reported in the literature. It has been observed that results are effective and robust high quality for solving OPF problems. A. Sharma and S. K. Jain [26] used GSA to solve OPF problems based methods for allocating thyristor controlled series compensator (TCSC) using the congestion rent contribution approach based on the location of marginal price. The method was developed and tested on IEEE-30 and 57 bus system. The performance of GSA is compared with other heuristic methods reported in the literature.

V. APPLICATION EXAMPLES: IPM AND PSO

A. Interior point method

To solve optimization problems through the IPM for NLP, a perturbation parameter is introduced in the complementarily Karush-Kuhn-Tucker (KKT) condition. To transform constrained optimization problem into an unconstrained optimization problem, we refer to the three

following methods: Lagrange's method for equality constraints, Logarithmic barrier method for inequality constraints and Newton's method for Karach-Kuhn-Tucker (KKT) conditions.

With $y = (x+u)^T$, the new formulation of the equation (1) is:

$$\begin{cases} \text{Min} : J(y) - \mu \sum_{i=1}^n \ln s_i \\ \text{Subject to} : f_i(y) = 0, \quad i = 1, \dots, p \\ \text{and} : g_j(y) + s = 0, \quad j = 1, \dots, q \end{cases} \quad (6)$$

From eq.6, the Lagrangian function is then defined:

$$L_\mu = J(y) - \mu \sum_{i=1}^n \ln s_i + \lambda^T f_i(y) + \pi^T (g_j(y) + s) \quad (7)$$

Where λ and π are the variables of the Lagrange multiplier vectors.

According to the KKT condition:

$$\nabla_{y,\lambda,\pi,s} L_\mu = 0 \Leftrightarrow \quad (8)$$

$$\begin{cases} \nabla_y L_\mu = \nabla_y J(y) + \lambda J_f^T(y) + \pi J_g^T(y) = 0 \\ \nabla_\lambda L_\mu = f_i(y) = 0 \\ \nabla_\pi L_\mu = g_j(y) + s = 0 \\ \nabla_s L_\mu = \pi_i - \mu \frac{1}{s_i} = 0 \end{cases} \quad (9)$$

The functions $J_f(y)$ and $J_g(y)$ are Jacobians of $f(y)$ and $g(y)$ functions respectively.

B. Particle swarm optimization

In PSO, the population has n particles that represent the candidate solutions of the optimization problem. Particles change their positions by flying around in an m -dimensional search space until a relatively unchanging position has been encountered, or until the computational limitations are exceeded. Fitness value, called $pbest$ is stocked. Another value that is tracked by the global algorithm is the overall value, called $gbest$. Velocity and position vectors of each particle at iteration $(k+1)$ are given by eq.10, and eq.11.

$$V_{ij}^{k+1} = \omega * v_{ij}^k + c_1 * r_1 (Xpbest_{ij}^k - x_{ij}^k) + c_2 * r_2 (Xgbest_j^k - x_{ij}^k), \quad (10)$$

$$X_{ij}^{k+1} = x_{ij}^k + v_{ij}^{k+1} \quad (11)$$

VI. RESULTS AND DISCUSSIONS

In order to illustrate validity of methods, it has been tested on IEEE-30 bus system.

A. IPM implementation

After numerical simulations we are obtained respectively 802.18 \$/h, 9.45 MW and 0.88 for the generation cost, active losses and voltage deviation.

Fig.1. shows the voltage profile curve for the IEEE-30 network. According the graphic, it has been observed that

the voltage of all bus bars remains within the limits set for the proper operation of the network.

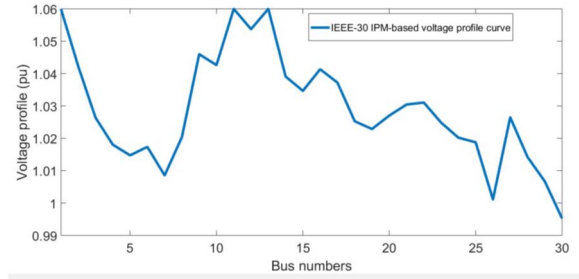


Fig.1. IEEE-30 IPM-based voltage profile curve

B. PSO implementation

The PSO-OPF parameters are:

- Inertia weight range: $\omega_{\max} = 0.9$ and $\omega_{\min} = 0.4$;
- Acceleration coefficient : $c_1 = 2$ and $c_2 = 2$;
- Population size : $N_p = 20$;
- Maximum iterations: 200

After numerical simulations we are obtained respectively 801.84 \$/h, 9.37 MW and 0.96 for the generation cost, active losses and voltage deviation.

Fig.2 and fig.3 show the voltage profile and generation cost curves for the IEEE-30 network, respectively. In fig.2, it has been observed that the voltage of all bus bars remains within the limits set for the proper operation of the network. In the fig.3 we have also noticed that the objective function reached its optimum as early as the 50th iteration.

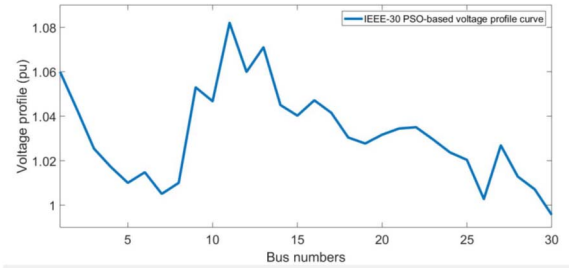


Fig.2. IEEE-30 PSO-based voltage profile curve

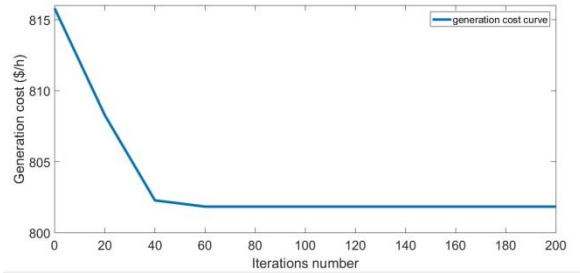


Fig.3. IEEE-30 generation cost curve

Initially, values of the generation cost, active power losses and voltage deviation by the Newton-Raphson method in the literature are respectively 901.95 \$/h, 5.82 MW and 1.15 [30]. The PSO-based OPF reduced the generation fuel cost and the voltage deviation by 11.1% and 16.5%, respectively. Even if losses increase after optimization, but the financial gain remains the most significant because the chosen fitness function is economic.

Tab.1. presents simulation results for both optimization methods.

Tab.1. Simulation results for IPM and PSO

Variables	Methods	
	IPM	PSO
P ₁	176.57	176.69
P ₂	48.85	48.89
P ₅	21.52	21.42
P ₈	22.31	21.68
P ₁₁	12.26	12.11
P ₁₃	11.35	12.00
V ₁	1.06	1.06
V ₂	1.04	1.04
V ₅	1.01	1.01
V ₈	1.02	1.01
V ₁₁	1.06	1.08
V ₁₃	1.06	1.07
Cost (\$/h)	802.18	801.84
Losses (MW)	9.45	9.37
VD	0.88	0.96

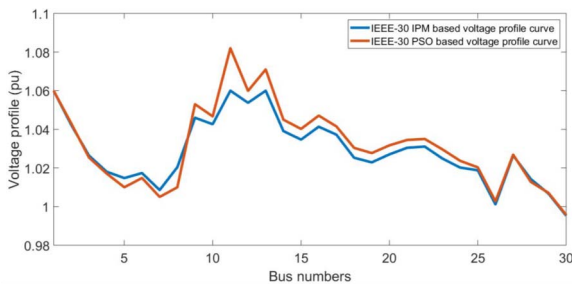


Fig.4. IEEE-30 IPM/PSO-based voltage profile Curves

Differences in the simulation results for both methods are not very significant. They are respectively 0.04, 0.85 and 4.5% for generation cost, active power losses and voltage deviation.

VII. CONCLUSION

In this paper we have reviewed some conventional and metaheuristic optimization methods applied to OPF problems. Since the use of conventional methods (IPM) to solve OPF problems is very complex in terms of modeling and calculation at the risk of converging towards a local optimum, the PSO method was very useful for comparing simulation results. Both methods have been applied on the IEEE-30 network to optimize the fuel cost, which reduces losses and improves the voltage profile. The simulation results for the IEEE-30 network are close to those in the literature.

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