2)

$$(f(x))^2 = 2025 + \int_0^x \left((f(t))^2 + (f'(t))^2 \right) dt$$

$$\frac{d}{dx}\left[(f(x))^2 \right] = \frac{d}{dx}\left[2025 + \int_0^x \left((f(t))^2 + (f'(t))^2 \right) dt \right]$$

$$2f(x)f'(x) = (f(x))^2 + (f'(x))^2$$

$$0 = (f(x))^2 - 2f(x)f'(x) + (f'(x))^2$$

$$0 = (f(x) - f'(x))^2$$

$$0 = f(x) - f'(x) \Rightarrow f'(x) = f(x) \Rightarrow f(x) = Ce^x \quad , \quad C \in \mathbb{R}$$

$$(f(x))^2 = (Ce^x)^2 = C^2 e^{2x} = 2025 + \int_0^x \left((Ce^t)^2 + (Ce^t)^2 \right) dt$$

$$C^2 e^{2x} = 2025 + 2C^2 \int_0^x e^{2t}\, dt$$

$$C^2 e^{2x} = 2025 + C^2 e^{2x} - C^2 \rightarrow C^2 = 2025 \quad , \quad C = \pm 45$$

$$\boxed{f(x) = \pm 45e^x}$$