Homework 5

Problem 1) a)

$\forall a \in \mathbb{Z}$

$a \oplus z = z \oplus a = a \rightarrow$ **$z = 1$ for this to hold**

$a \oplus z = a + z - 1 = a \rightarrow z - 1 = 0 \rightarrow z = 1$ ✓

$z \oplus a = z + a - 1 = a \rightarrow z - 1 = 0 \rightarrow z = 1$ ✓

Problem 1) b)

$\forall a \in \mathbb{Z}, \exists b \in \mathbb{Z} \ni a \oplus b = b \oplus a = z$, Find b.

$z = 1$ from part a) **$b = 2 - a$**

$a \oplus b = a + b - 1 = z = 1$

$a + b - 1 = 1$

$a + b = 2$

$b = 2 - a$ ✓

$b \oplus a = b + a - 1 = 1$

$b + a = 2$

$b = 2 - a$ ✓

Problem 1) c) Show that $\oplus$ commutes.

$c \oplus d = c + d - 1 = d + c - 1 = d \oplus c$

Problem 1) d) $u \in \mathbb{Z}, u \neq z = 1 \ni a \odot u = u \odot a = a, \forall a \in \mathbb{Z}$. Find u.

$a \odot u = a + u - au = a$

$u - au = 0$

$u(1 - a) = 0$

$u = 0$ ✓

$u \odot a = u + a - ua = a$

$u - ua = 0$

$u(1 - a) = 0$

$u = 0$ ✓

$u = 0$