Homework 4 continued

Problem 4)

(1) $xy + 2xz = 5\sqrt{5}$, $(x,y,z) \in \mathbb{R}^3$

(a) $g(x,y,z) = xy + 2xz - 5\sqrt{5} = 0$

$f(x,y,z) = x^2 + y^2 + z^2 = d^2$, where ~~x~~ d is distance

(square root is monotonic, so we can just minimize f the way it is).

Define lagrangian:

$\mathcal{L}(x,y,z,\lambda) = x^2 + y^2 + z^2 + \lambda(xy + 2xz - 5\sqrt{5})$

compute all 4 partials:

(2) $\frac{\partial \mathcal{L}}{\partial x} = 2x + \lambda(y + 2z) = 0 \rightarrow 2x = -\lambda(y + 2z)$

(3) $\frac{\partial \mathcal{L}}{\partial y} = 2y + \lambda(x) = 0 \rightarrow 2y = -\lambda x \rightarrow \lambda = \frac{-2y}{x}$

(x cannot equal 0 from (1) definition)

(4) $\frac{\partial \mathcal{L}}{\partial z} = 2z + \lambda(2x) = 0 \rightarrow z = -\lambda x \rightarrow \lambda = \frac{-z}{x}$

(5) $\frac{\partial \mathcal{L}}{\partial \lambda} = xy + 2xz - 5\sqrt{5} = 0$

Solve System: $\frac{-2y}{x} = \frac{-z}{x} \Rightarrow 2y = z$ → substitute into (5)

→ substitute into (2)

$xy + 4xy - 5\sqrt{5} = 0$

$5xy = 5\sqrt{5}$

$xy = \sqrt{5}$ (6)

substitute $\lambda = \frac{-2y}{x}$ into (2)

$2x = -\lambda(y + 4y)$

$2x = -5\lambda y$

$2x = \frac{10y^2}{x}$

$2x^2 = 10y^2$

$x^2 = 5y^2$

$x = \pm\sqrt{5}\,y$ → plug into (6)

$\pm y^2\sqrt{5} = \sqrt{5}$

$\pm y^2 = 1$

take the +

$y = \pm 1$

Homework 4 continued

Problem 4 continued)

Case 1): $y = 1$

$x = \sqrt{5} \cdot 1 = \sqrt{5}$

$z = 2 \cdot 1 = 2$

point $(\sqrt{5}, 1, 2)$ satisfies $xy + 2xz = 5\sqrt{5}$

Case 2): $y = -1$

$x = -\sqrt{5}$

$z = -2$

point $(-\sqrt{5}, -1, -2)$ satisfies $xy + 2xz = 5\sqrt{5}$

Answer for (a):
points $(-\sqrt{5}, -1, -2)$
$(\sqrt{5}, 1, 2)$

(b) compute distance

$$f(\sqrt{5}, 1, 2) = (\sqrt{5})^2 + (1)^2 + (2)^2 = 10 = d^2$$

distance $d = \sqrt{10}$