Problem 2) $\quad p \odot q = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} p(\tau)\, q(t+\tau)\, d\tau$

$$p(t) = \begin{cases} 0, & t < 0 \\ 1, & 0 < t < 1 \\ 0, & t > 0 \end{cases} \rightarrow \begin{cases} 0, & \tau < 0 \\ 1, & 0 < \tau < 1 \\ 0, & \tau > 0 \end{cases}$$

$$q(t) = \begin{cases} 0, & t < 0 \\ 1-t, & 0 < t < 1 \\ 0, & 0 < t \end{cases} \rightarrow \begin{cases} 0, & t+\tau < 0 \\ 1-t-\tau, & 0 < t+\tau < 1 \\ 0, & 1 < t+\tau \end{cases}$$

case 1)

$t < -1$

no overlap, thus zero!

case 2)

$-1 < t < 0$

$\max(0, -t) = -t$

$\min(1, 1-t) = 1$

$$(p \odot q)(t) = \frac{1}{\sqrt{2\pi}} \int_{-t}^{1} (1-t-\tau)\, d\tau = \frac{1}{\sqrt{2\pi}} \left[ \tau - t\tau - \frac{\tau^2}{2} \right]_{-t}^{1}$$

$$= \frac{1}{\sqrt{2\pi}} \left[ \frac{1}{2} - t + t - t^2 + \frac{t^2}{2} \right]$$

$$= \frac{1-t^2}{2\sqrt{2\pi}}$$

case 3)

$0 \leq t < 1$

$\max(0, -t) = 0$

$\min(1, 1-t) = 1-t$

$$\frac{1}{\sqrt{2\pi}} \int_0^{1-t} (1-t-\tau)\, d\tau = \frac{1}{\sqrt{2\pi}} \left[ \tau - t\tau - \frac{\tau^2}{2} \right]_0^{1-t}$$

$$= \frac{1}{\sqrt{2\pi}} \left[ \underbrace{1-t-t+t^2}_{(1-t)^2} - \frac{(1-t)^2}{2} \right]$$

$$= \frac{1-t^2}{2\sqrt{2\pi}}$$

case 4)

$t > 1 \rightarrow$ no overlap $\rightarrow$ Thus 0.

$$\boxed{(p \odot q)(t) = \frac{1}{2\sqrt{2\pi}} \cdot \begin{cases} 0, & t < -1 \\ 1-t^2, & -1 < t < 0 \\ 1-t^2, & 0 < t < 1 \\ 0, & t > 1 \end{cases}}$$