Homework 8

Problem 1)

a)

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

$$\mathbb{F}[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} e^{-iw\cancel{x}}\, dx$$

$$= \frac{1}{2\pi\sigma} \int_{-\infty}^{\infty} e^{\frac{-(x-\mu)^2}{2\sigma^2} - iwx}\, dx$$

$$= \cancel{\frac{1}{2\pi\sigma}} \quad \text{let } x-\mu = u \Rightarrow x = u+\mu$$

$$dx = du$$

$$= \frac{1}{2\pi\sigma} \int_{-\infty}^{\infty} e^{-\frac{u^2}{2\sigma^2} - iw(u+\mu)}\, du$$

$$= \frac{e^{-iw\mu}}{2\pi\sigma} \int_{-\infty}^{\infty} e^{\frac{-u^2}{2\sigma^2} - iwu}\, du$$

$$= \frac{e^{-iw\mu}}{2\pi\sigma} \sqrt{2\sigma^2\pi}\; e^{-\frac{\sigma^2 w^2}{2}}$$

$$\boxed{= \frac{1}{\sqrt{2\pi}} e^{-iw\mu} e^{-\frac{\sigma^2 w^2}{2}}}$$

Homework 8 continued

Problem 1 continued)

b)

$f(t) = \sin(w_0 t)$, w_0 constant

$$\mathcal{F}[f(t)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sin(w_0 t) e^{-iwt} dt$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(\frac{e^{iw_0 t} - e^{-iw_0 t}}{2i} \right) e^{-iwt} dt$$

$$= \frac{1}{\sqrt{2\pi} \cdot (2i)} \left[\underbrace{\int_{-\infty}^{\infty} e^{-(w-w_0)it} dt}_{2\pi\delta(w-w_0)} - \underbrace{\int_{-\infty}^{\infty} e^{-(w+w_0)it} dt}_{2\pi\delta(w+w_0)} \right]$$

Dirac Delta Function

$$= \frac{2\pi}{\sqrt{2\pi} \cdot (2i)} \left[\delta(w-w_0) - \delta(w+w_0) \right] = \boxed{\frac{\sqrt{2\pi}}{i} \left(\delta(w-w_0) - \delta(w+w_0) \right)}$$

c) $f(x) = e^{-a|x|}$, $a > 0$

$$\mathcal{F}[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a|x|} e^{-iwx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{0} e^{ax} e^{-iwx} dx + \int_{0}^{\infty} e^{-ax} e^{-iwx} dx \right]$$

$$\boxed{= \frac{1}{\sqrt{2\pi}} \left[\frac{1}{a-iw} + \frac{1}{a+iw} \right]}$$

Homework 8 continued

Problem 1 continued)

d)

$f(t) = \delta(t)$ distribution

$\delta(t)$ is zero everywhere except ~~at~~ @ $t = 0$

$$\mathcal{F}[f(t)] = \mathcal{F}[\delta(t)] = \cancel{\frac{1}{2\pi}} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \delta(t) e^{-i\omega t} dt$$

$$= \frac{1}{\sqrt{2\pi}} \cdot e^{-i\omega \cdot 0}$$

$$\boxed{= \frac{1}{\sqrt{2\pi}}}$$