Homework 8 continued

Problem 4)

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$$

a) $\tau(k,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} T(x,t) e^{-ikx} dx$

$$\frac{\partial \mathcal{F}[T]}{\partial t} = \kappa \frac{\partial^2 \mathcal{F}[T]}{\partial x^2}$$

$$\frac{\partial \tau}{\partial t} = -\kappa k^2 \tau$$

$$\tau(k,t) = \tau(k,0) e^{-\kappa k^2 t}$$

↓ Inverse FFT ↓

$$\boxed{T(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tau(k,0) \cdot e^{-\kappa k^2 t} \cdot e^{ikx} dk \quad , \quad \tau(k,0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} T(x,0) e^{-ikx} dx}$$

b)

zero everywhere else.

$$\tau(k,0) = \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} 100 e^{-ikx} dx = \boxed{\frac{200 \sin(k)}{\sqrt{2\pi} \cdot k}}$$

$$T(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{200 \sin(k)}{\sqrt{2\pi} \cdot k} \cdot e^{-1000k^2 t} \cdot e^{ikx} dk \quad , \quad \kappa = 10^3 \ m^2/s$$

$$\boxed{T(x,t) = \frac{100}{\pi} \int_{-\infty}^{\infty} \frac{\sin(k)}{k} \cdot e^{ikx - 1000k^2 t} dk}$$