Homework 4

Problem 1)

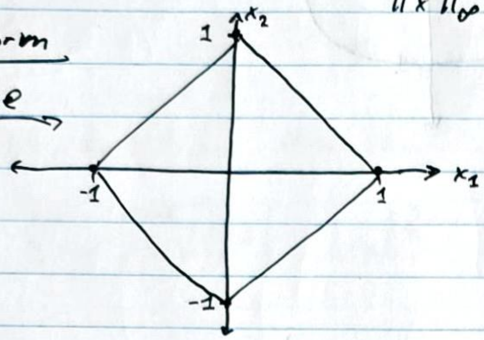
$$\boxed{A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}}$$

Want to graph $\|x\|_1$, $\|x\|_2$, $\|x\|_\infty$ such that:

$$\|x\|_1 = 1$$
$$\|x\|_2 = 1$$
$$\|x\|_\infty = 1$$

1-norm

base



$$\|(x_1, x_2)\|_1 = 1$$

$$|x_1| + |x_2| = 1$$

$x_2 = 1 \pm x_1 \Rightarrow x_2 = 0, \quad x_1 = \pm 1$

$x_1 = 1 \pm x_2 \Rightarrow x_1 = 0, \quad x_2 = \pm 1$

Apply A to each point in set: $\{(0,1), (0,-1), (1,0), (-1,0)\}$

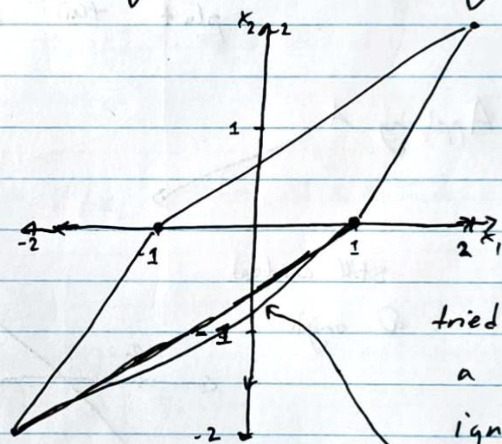
$$\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

New graph after applying A:

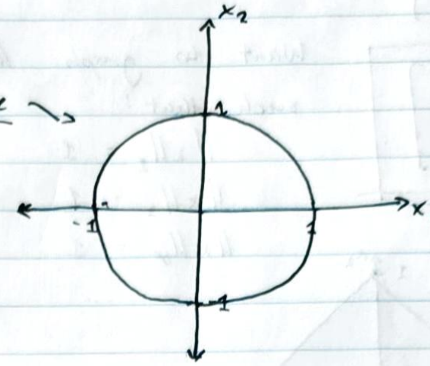


tried to draw a straight line, ignore this line.

Homework 4 continued

Problem 1 continued)

$A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ 2-norm   base



$\|(x_1, x_2)\|_2 = 1$

$(x_1^2 + x_2^2)^{1/2} = 1$

↳ this is a circle of radius 1! centered @ the origin!

We can parametrize this to be: $x = (\cos(\theta), \sin(\theta))$ s.t. $0 \leq \theta \leq 2\pi$

$x_1 = \cos(\theta)$

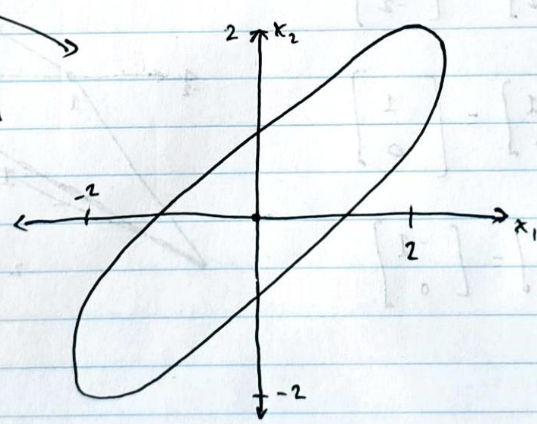
$x_2 = \sin(\theta)$

$\Rightarrow A \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \cos(\theta) + 2\sin(\theta) \\ 2\sin(\theta) \end{bmatrix}$

plot this for $\theta = [0, 2\pi]$ to get ellipse, which after using python, looks like this:

Applying A

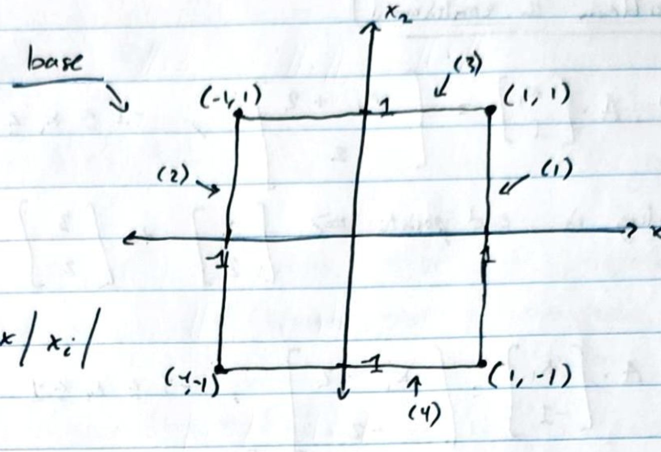
still centered @ origin



Homework 4 continued

Problem 1 continued)

$A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ $\qquad$ ∞-norm



$\|x\|_\infty = 1$

$\|x\|_\infty = \max_i |x_i|$

(1) let $x_1 = 1$, then x_2

$\max_i \{|1|, |x_2|\} = 1$

$\Rightarrow \forall |x_2| \leq 1 \rightarrow -1 \leq x_2 \leq 1$

(2) let $x_1 = -1$, then x_2

$\max_i \{|-1|, |x_2|\} = 1 \rightarrow \forall |x_2| \leq 1 \rightarrow -1 \leq x_2 \leq 1$

(3) let $x_2 = 1$, then x_1

$\max_i \{|x_1|, |1|\} = 1 \rightarrow \forall |x_1| \leq 1 \rightarrow -1 \leq x_1 \leq 1$

(4) let $x_2 = -1$, then x_1

$\max_i \{|x_1|, |-1|\} = 1 \rightarrow \forall |x_1| \leq 1 \rightarrow -1 \leq x_1 \leq 1$

Applying A to this idea:

~~let $x_2 = t \Rightarrow A \cdot \begin{bmatrix} 1 \\ t \end{bmatrix} = \begin{bmatrix} 1 + 2t \\ 2t \end{bmatrix}$~~

(1) $A \cdot \begin{bmatrix} 1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 + 2x_2 \\ 2x_2 \end{bmatrix}$, $-1 \leq x_2 \leq 1$

plug in end points (-1 and 1) $\Rightarrow \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ to $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ ← line segment

(2) $A \cdot \begin{bmatrix} -1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 + 2x_2 \\ 2x_2 \end{bmatrix}$, $-1 \leq x_2 \leq 1$

plug in end points $\Rightarrow \begin{bmatrix} -3 \\ -2 \end{bmatrix}$ to $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Homework 4 continued

Problem 1 continued)

(3) $A \cdot \begin{bmatrix} x_1 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 + 2 \\ 2 \end{bmatrix}$, $-1 \le x_1 \le 1$

plug in end points $\Rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ to $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

(4) $A \cdot \begin{bmatrix} x_1 \\ -1 \end{bmatrix} = \begin{bmatrix} x_1 - 2 \\ -2 \end{bmatrix}$, $-1 \le x_1 \le 1$

plug in endpoints $\Rightarrow \begin{bmatrix} -3 \\ -2 \end{bmatrix}$ to $\begin{bmatrix} -1 \\ -2 \end{bmatrix}$

New graph