The Proof Part of 'Optimal Throughput of The Full-duplex Two-way Relay System with Energy Harvesting'

I. Proof of Proposition 1

Substitute (13) into $p(\gamma_s^{\mathsf{p}_1} \geq \gamma_0)$, we have

$$\begin{aligned} p(\gamma_s^{\mathsf{p}_1} \geq \gamma_0) &= p\Big(\frac{\eta\alpha P_2 |g_1|^2 |g_2|^2}{\eta\alpha|\hat{f}_2|^2 |g_1|^2 \frac{\alpha\eta P_2 |g_2|^2}{1-\alpha} + \eta\alpha |g_1|^2 \sigma^2 + (1-\alpha)\sigma^2 + |\hat{f}_2|^2\eta\sigma^2} \geq \gamma_0\Big) \\ &= p\Big(|g_2|^2 \geq \frac{(1-\alpha)\sigma^2\gamma_0}{(1-\alpha)P_2 - |\hat{f}_2|^2\alpha\eta P_2} + \frac{(1-\alpha)^2\sigma^2\gamma_0 + (1-\alpha)|\hat{f}_2|^2\eta\sigma^2\gamma_0}{\Big((1-\alpha)\alpha\eta P_2 - (\alpha\eta)^2|\hat{f}_2|^2 P_2\Big)|g_1|^2}\Big) \\ &= \int_{z=0}^{\infty} f_{|g_1|^2}(z) p\Big(|g_2|^2 \geq \frac{(1-\alpha)\sigma^2\gamma_0}{(1-\alpha)P_2 - |\hat{f}_2|^2\alpha\eta P_2} + \frac{(1-\alpha)^2\sigma^2\gamma_0 + (1-\alpha)|\hat{f}_2|^2\eta\sigma^2\gamma_0}{\Big((1-\alpha)\alpha\eta P_2 - (\alpha\eta)^2|\hat{f}_2|^2 P_2\Big) z}\Big) dz \\ &= \frac{1}{\lambda_{g_1}} \exp\Big(-\frac{(1-\alpha)\sigma^2\gamma_0}{(1-\alpha)P_2\lambda_{g_2} - |\hat{f}_2|^2\alpha\eta P_2\lambda_{g_2}}\Big) * \\ &\quad \int_{z=0}^{\infty} \exp\Big(-\frac{z}{\lambda_{g_1}} - \frac{(1-\alpha)^2\sigma^2\gamma_0 + (1-\alpha)|\hat{f}_2|^2\eta\sigma^2\gamma_0}{\Big((1-\alpha)\alpha\eta P_2 - (\alpha\eta)^2|\hat{f}_2|^2 P_2\Big)\lambda_{g_2} z}\Big) dz, \end{aligned} \tag{I.1}$$

since g_1 and g_2 follow Rayleigh distribution, $|g_1|^2$ and $|g_2|^2$ are exponential random variables with mean values of λ_{g_1} and λ_{g_2}, respectively. On the other hand, $\hat{f}_2$ is the residual gain of the loop-back interference channel, which is a small constant value. Furthermore, these equations: $f_{|g|^2}(z) \triangleq \frac{1}{\lambda_g} e^{-z/\lambda_g}$ and $F_{|g|^2}(z) \triangleq p(|g|^2 < z) = 1 - e^{-z/\lambda_g}$ are used.

II. Proof of Proposition 2

In order to show that $\tau_{upper}^{\mathsf{p}_1}$ is log-concave with respect to α_1, we need to show that $\log \tau_{upper}^{\mathsf{p}_1}$ is concave. Given the objective function of (33), we have

$$\begin{aligned} \log \tau_{upper}^{\mathsf{p}_1} &= \log f(\alpha_1) g(\alpha_1)(1-\alpha_1) + \log U \\ &= \log f(\alpha_1) + \log g(\alpha_1) + \log(1-\alpha_1) + \log U. \end{aligned} \tag{II.1}$$

Thus, (II.1) is concave if $\log f(\alpha_1) + \frac{1}{2}\log(1-\alpha_1)$ and $\log g(\alpha_1) + \frac{1}{2}\log(1-\alpha_1)$ are concave, respectively. Functions $f(\alpha_1)$ and $g(\alpha_1)$, given in (31) and (32), can be recast as

$$f(\alpha_1) = a\sqrt{\frac{b(1-\alpha_1)}{\alpha_1}} K_1\Big(\sqrt{\frac{b(1-\alpha_1)}{\alpha_1}}\Big), \tag{II.2}$$

and

$$g(\alpha_1) = a^{'}\sqrt{\frac{b^{'}(1-\alpha_1)}{\alpha_1}} K_1\Big(\sqrt{\frac{b^{'}(1-\alpha_1)}{\alpha_1}}\Big), \tag{II.3}$$

where $a = \exp(-\frac{\sigma^2\gamma_0}{P_2\lambda_{h_2}})$, $a^{'} = \exp(-\frac{\sigma^2\gamma_0}{P_1\lambda_{h_1}})$, $b = \frac{4\sigma^2\gamma_0}{\eta P_2 \lambda_{g_1}\lambda_{g_2}}$ and $b^{'} = \frac{4\sigma^2\gamma_0}{\eta P_1 \lambda_{h_1}\lambda_{h_2}}$, respectively. First, Considering the function $\log f(\alpha_1) + \frac{1}{2}\log(1-\alpha_1)$, it is equivalent to

$$\log a + \log\Big(\sqrt{\frac{b(1-\alpha_1)}{\alpha_1}}\Big) K_1\Big(\sqrt{\frac{b(1-\alpha_1)}{\alpha_1}}\Big) + \frac{1}{2}\log(1-\alpha_1). \tag{II.4}$$

Let $f_{new}(\alpha_1)$ denote $\sqrt{\frac{b(1-\alpha_1)}{\alpha_1}}) K_1(\sqrt{\frac{b(1-\alpha_1)}{\alpha_1}})$. We could derive its second order derivative:

$$-\frac{b}{\alpha_1^3} K_0\Big(\sqrt{\frac{b(1-\alpha_1)}{\alpha_1}}\Big) + \frac{1}{2}\frac{b}{\alpha_1^2} K_1\Big(\sqrt{\frac{b(1-\alpha_1)}{\alpha_1}}\Big) \frac{b/\alpha_1^2}{2\sqrt{\frac{b(1-\alpha_1)}{\alpha_1}}}, \tag{II.5}$$

where we used these equations: $\frac{d}{dx}K_0(x) = -K_1(x)$; $K_0(x) + K_2(x) = -2\frac{d}{dx}K_1(x)$ and $K_2(x) = \frac{2}{x}K_1(x) + K_0(x)$ [1]. Furthermore, the second order derivative of (II.4) is

$$\frac{f_{new}^{''}(\alpha_1) f_{new}(\alpha_1) - [f_{new}^{'}(\alpha_1)]^2}{[f_{new}(\alpha_1)]^2} - \frac{1}{2(1-\alpha_1)^2}, \tag{II.6}$$

where $f_{new}^{''}(\alpha_1)$ and $f_{new}^{'}(\alpha_1)$ are the first and second order derivatives of $f_{new}(\alpha_1)$, respectively.

In order to show that (II.4) is concave, we need to show that (II.6) is less than 0, which is equivalent to show that

$$\frac{f_{new}^{''}(\alpha_1) f_{new}(\alpha_1)}{[f_{new}(\alpha_1)]^2} - \frac{1}{2(1-\alpha_1)^2} < 0. \tag{II.7}$$

Moreover, from (II.5), we could find that (II.7) is less than 0 when

$$\frac{\frac{b^2}{4\alpha_1^4}\Big(K_1\big(\sqrt{\frac{b(1-\alpha_1)}{\alpha_1}}\big)\Big)^2}{[f_{new}(\alpha_1)]^2} - \frac{1}{2(1-\alpha_1)^2} < 0. \tag{II.8}$$

Meanwhile, substituting $f_{new}(\alpha_1)$ into (II.8), it becomes

$$\frac{\frac{2b^2(1-\alpha_1)^2}{4\alpha_1^4}\Big(K_1\big(\sqrt{\frac{b(1-\alpha_1)}{\alpha_1}}\big)\Big)^2 - \frac{b(1-\alpha_1)}{\alpha_1}\Big(K_1\big(\sqrt{\frac{b(1-\alpha_1)}{\alpha_1}}\big)\Big)^2}{2(1-\alpha_1)^2 \frac{b(1-\alpha_1)}{\alpha_1}\Big(K_1\big(\sqrt{\frac{b(1-\alpha_1)}{\alpha_1}}\big)\Big)^2}. \tag{II.9}$$

Furthermore, (II.9) is less than 0 when $\frac{2b^2(1-\alpha_1)^2}{4\alpha_1^4} - \frac{b(1-\alpha_1)}{\alpha_1} < 0$, which is recast as $\frac{b(1-\alpha_1)}{\alpha_1^3} < 2$. Therefore, $\log f(\alpha_1) + \frac{1}{2}\log(1-\alpha_1)$ is concave when α_1 satisfies the condition $\frac{b(1-\alpha_1)}{\alpha_1^3} < 2$. Similarly, $\log g(\alpha_1) + \frac{1}{2}\log(1-\alpha_1)$ is concave when α_1 satisfies the condition $\frac{b^{'}(1-\alpha_1)}{\alpha_1^3} < 2$. Thus, $\log \tau_1^{\mathsf{p}_1}$ is a concave function when both $\frac{b(1-\alpha_1)}{\alpha_1^3} < 2$ and $\frac{b^{'}(1-\alpha_1)}{\alpha_1^3} < 2$ are satisfied, i.e., $\tau_{upper}^{\mathsf{p}_1}$ is log-concave with respect to α_1.

REFERENCES

[1] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series and products*. 7th ed. ACADEMIC Press, 2007.