Page :
Date : / /

WiDS Assignment
Week 1

Q1. $P(A)$ = Prob it rains today, $P(B)$ = Prob. it rains Tomorrow

(a) $1-0.3$ ($\because 0.3 = P$(it will not rain on either day))

$P(A \cup B)$

$= 0.7$

(b) Using $P(A\cap B) = P(A) + P(B) - P(A\cup B)$

$= 0.6 + 0.5 - 0.7$

$= 0.4$

(c) $P(A\cap B') = P(A) \cancel{+P(B')} - P(A\cup B')$

$= 0.6 \cancel{+0.5} - \cancel{0.3}\ 0.4$

$= 0.2$

(d) $P(A) + P(B) - 2P(A\cap B)$

$= 0.6 + 0.5 - (0.4)\times 2$

$= 0.3$

Q2.

Ans. $S = \{(i,j)$ $\forall$ all $i, j \in [1,6]$

$A = X_1 + X_2 = 8$

$= (2,6), (6,2), (5,3), (3,5), (4,4)$

Sample space $= 6\times 6 = 36$

so $P(A) = \frac{5}{36}$

Q3. A : All n children are girls

B : Randomly chosen child is a girl

$P(A) = \left(\frac{1}{2}\right)^n$ $P(B) = \frac{1}{2}$

$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{\left(\frac{1}{2}\right)^n}{1/2} = \left(\frac{1}{2}\right)^{n-1}$

Q4. $X =$

1. $F_X(x) = P(X \leq x) = p F_d(x) + (1-p) F_c(x)$

2. $P(X = x_0) = p\, P(X_d = x_0) \longrightarrow$ Discrete

$f_X(x) = (1-p) f_c(x) \longrightarrow$ continuous

3. $E[X] = \cancel{p}\; E[p X_d] + E[(1-p) X_c]$

$= p E[X_d] + (1-p) E[X_c]$

~~4. As per law of total variance~~

~~$\mathrm{Var}(X) = \cancel{p}\; p \mathrm{Var}(X_d) + (1-p) \mathrm{Var}(X_c)$~~

4. ~~Var~~ As per law of total variance

$\mathrm{Var}(X) = E[\mathrm{Var}(X \mid \text{coin})] + \mathrm{Var}(E[X \mid \text{coin}])$

$X \mid H = X_d, \quad X \mid T = X_c$

$E[\mathrm{Var}(X \mid \text{coin})] = p \mathrm{Var}(X_d) + (1-p) \mathrm{Var}(X_c)$

$E[X \mid \text{coin}] = \begin{cases} E[X_d], & H \\ E[X_c], & T \end{cases}$

$\mathrm{Var}(E[X \mid \text{coin}]) = p(1-p)(E[X_d] - E[X_c])^2$

$\Rightarrow \mathrm{Var}(X) = p \mathrm{Var}(X_d) + (1-p) \mathrm{Var}(X_c) + p(1-p)(E[X_d] - E[X_c])^2$

Q5.

Ans. $cov(Z, W) = cov(X + XY^2, X)$ [$\because$ removing constants]

$= cov(X, X) + cov(XY^2, X)$

$cov(X, X) = 1$ . . . (i)

$cov(XY^2, X) = E[X^2 Y^2] - E[XY^2]E[X]$

$= E[X^2]E[Y^2]$

$= 1$ . . . (ii)

Summing up both (i) & (ii)

$cov(Z, W) = 2$

Q6. $P(\text{he gets no offers}) = (0.8)^4$

$= 0.4096$

$P(\text{he gets atleast } 1) = 1 - P(\text{he gets no offers})$

$= 0.5904$

So hes wrong its not a 90% chance he'll get an offer

Q7. X = no. of errors

Ans. $X \sim$ Binomial distribution$(1000, 0.1)$

Since sample is large we can approximate

$\mu = np = 100, \sigma^2 = np(1-p) = 90$

~~$P(X > 120) = P($~~

$$Z = \frac{X - \mu}{\sigma} = \frac{120 - 160}{\sqrt{90}} = 2.108$$

~~Approx~~ $P(~~x~~ Z > 120) = 0.0174$ approx
(using 2.11 z score)

Q 8.

Ans. ~~For~~ X_i = Sandwiches i guest needs

$P(X_i = 0) = \frac{1}{4}$, $P(X = 1) = \frac{1}{2}$, $P(X = 2) = \frac{1}{4}$

$E[X_i] = 1 \times \frac{1}{2} + \frac{1}{4} \times 2$

$= 1$

$E[X_i^2] = 4 \times \frac{1}{4} + 1 \times \frac{1}{2} = \frac{3}{2}$

$Var(X) = \frac{1}{2}$

Total sandwiches needed, $S = \sum_{i=1}^{64} X_i$

$E[S] = \sum_{i=1}^{64} E[X_i] = 64$

$Var[S] = 64 \times \frac{1}{2} = 32$

As per Central Limit Theorem,

$S \approx N(64, 32)$

$Z_{0.95} = 1.645$

$$Z = \frac{S - \mu}{\sigma}$$

$$\Rightarrow 1.645 = \frac{S - 64}{\cancel{5}\sqrt{32}}$$

$\cancel{= 64}$

$\Rightarrow S = 73.3$

since we need ~~for~~ whole number values

$= 74$ sandwiches

Q9.

Ans.

1. $E[X] = 0$, $E[Y] = 0$ [$\because$ vector given]
2. $Var(X) = 1$, $Var(Y) = 1$, [$\because$ diagonal entries of matrix]
3. $Cov(X, Y) = \rho$

Q10.

Ans. $E[X] = 1$, $E[Y] = 2$, $Cov(X, Y) = 1$

$Var(X) = 4$, $Var[Y] = 3$

$y = X | Y$

<u>Mean</u>

$\cancel{E[Z]}$ $E[X|Y=y] = E[X] + \frac{Cov(X, Y)}{Var(Y)} \cdot (y - E[Y])$

$= 1 + \frac{1}{3}(y - 2)$

Variance

$$Var(X|Y) = Var(X) - \frac{Cov(X,Y)^2}{Var(Y)}$$

$$= 4 - \frac{1^2}{3}$$

$$= \frac{11}{3}$$

$$\boxed{X|Y=y \sim N\left(1+\frac{(y-2)}{3}, \frac{11}{3}\right)}$$

Q11.

Ans. ~~$E[Z] = E[$~~ $Z = 3X - 2Y$

$$E[Z] = 3E[X] - 2E[Y]$$
$$= 0$$

$$Var(Z) = 9\,Var(X) + 4\,Var(Y) + 2(3)(-2)\,Cov(X,Y)$$
$$= 9 + 16 - 12$$
$$= 13$$

$$Z \sim N(0, 13)$$

~~Correl~~ $Corr(Z,X) = \frac{Cov(Z,X)}{\sqrt{Var(Z)\,Var(X)}} = \frac{Cov(Z,X)}{\sqrt{13}}$

$$Cov(Z,X) = Cov(3X-2Y, X) =$$
$$= 3\,Var(X) - 2\,Cov(Y,X)$$
$$= 3 - 2 = 1$$
$$= 1$$

So $Corr(Z,X) = \frac{1}{\sqrt{13}}$

Q12.

Ans. From matrix

$$\Sigma_{xx} = 4$$

$$\Sigma_{x,(y,z)} = \begin{pmatrix} 2 & 1 \end{pmatrix}$$

$$\Sigma_{(y,z),(y,z)} = \begin{pmatrix} 3 & -1 \\ -1 & 5 \end{pmatrix}$$

Inverting the (y,z) ~~pl~~ block

determinant $= 14$

Inverse $= \frac{1}{14}\begin{pmatrix} 5 & 1 \\ 1 & 3 \end{pmatrix}$

$$E[X \mid Y = y, Z = z] = \Sigma_{x,(y,z)} \Sigma^{-1}_{(y,z)} \begin{bmatrix} y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \end{bmatrix} \frac{1}{4} \begin{pmatrix} 5 & 1 \\ 1 & 3 \end{pmatrix} = \frac{1}{14}\begin{bmatrix} 11 & 5 \end{bmatrix}$$

$$\Rightarrow E[X \mid Y = y, Z = z] = \frac{11}{14} y + \frac{5}{14} z$$

$$\mathrm{Var}(X \mid Y, Z) = \Sigma_{xx} - \Sigma_{x;(y,z)} \Sigma^{-1}_{(y,z)} \Sigma_{(y,z);x}$$

$$= 4 - \frac{1}{14}\begin{bmatrix} 2 & 1 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= 29/14$$

$$X \mid (Y = y, Z = z) \sim N\left(\frac{11}{14} y + \frac{5}{14} z \,;\, \frac{29}{14}\right)$$