

## Optional Lab: Gradient Descent for Logistic Regression

### Goals

In this lab, you will:

- update gradient descent for logistic regression.
- explore gradient descent on a familiar data set

```
In [1]: import copy, math
import numpy as np
%matplotlib widget
import matplotlib.pyplot as plt
from lab_utils_common import dlc, plot_data, plt_tumor_data, sigmoid, compute_cost_logistic
from plt_quad_logistic import plt_quad_logistic, plt_prob
plt.style.use('./deeplearning.mplstyle')
```

### Data set

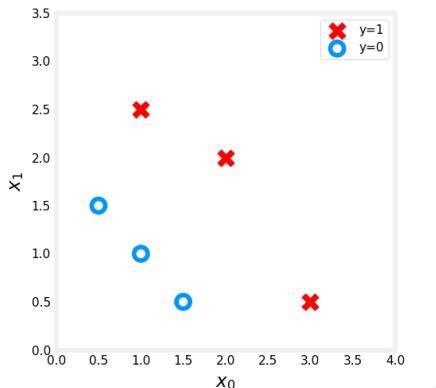
Let's start with the same two feature data set used in the decision boundary lab.

```
In [2]: X_train = np.array([[0.5, 1.5], [1, 1], [1.5, 0.5], [3, 0.5], [2, 2], [1, 2.5]])
y_train = np.array([0, 0, 0, 1, 1, 1])
```

As before, we'll use a helper function to plot this data. The data points with label  $y = 1$  are shown as red crosses, while the data points with label  $y = 0$  are shown as blue circles.

```
In [3]: fig,ax = plt.subplots(1,1,figsize=(4,4))
plot_data(X_train, y_train, ax)

ax.axis([0, 4, 0, 3.5])
ax.set_ylabel('$x_1$', fontsize=12)
ax.set_xlabel('$x_0$', fontsize=12)
plt.show()
```



### Logistic Gradient Descent

Recall the gradient descent algorithm utilizes the gradient calculation:

$$\begin{aligned} \text{repeat until convergence: } & \\ w_j &= w_j - \alpha \frac{\partial J(\mathbf{w}, b)}{\partial w_j} \quad \text{for } j := 0..n-1 \\ b &= b - \alpha \frac{\partial J(\mathbf{w}, b)}{\partial b} \\ &\} \end{aligned}$$

Where each iteration performs simultaneous updates on  $w_j$  for all  $j$ , where

$$\frac{\partial J(\mathbf{w}, b)}{\partial w_j} = \frac{1}{m} \sum_{i=0}^{m-1} (f_{\mathbf{w}, b}(\mathbf{x}^{(i)}) - y^{(i)}) x_j^{(i)}$$

$$\frac{\partial J(\mathbf{w}, b)}{\partial b} = \frac{1}{m} \sum_{i=0}^{m-1} (f_{\mathbf{w}, b}(\mathbf{x}^{(i)}) - y^{(i)})$$

- $m$  is the number of training examples in the data set
- $f_{\mathbf{w}, b}(\mathbf{x}^{(i)})$  is the model's prediction, while  $y^{(i)}$  is the target
- For a logistic regression model  
 $z = \mathbf{w} \cdot \mathbf{x} + b$   
 $f_{\mathbf{w}, b}(x) = g(z)$   
 where  $g(z)$  is the sigmoid function:

#### Gradient descent for logistic regression

$$\begin{aligned} \text{repeat } & \left[ \text{logistic like linear regression!} \right] \\ w_j &= w_j - \alpha \left[ \frac{1}{m} \sum_{i=1}^m (f_{\mathbf{w}, b}(\mathbf{x}^{(i)}) - y^{(i)}) x_j^{(i)} \right] \\ b &= b - \alpha \left[ \frac{1}{m} \sum_{i=1}^m (f_{\mathbf{w}, b}(\mathbf{x}^{(i)}) - y^{(i)}) \right] \\ &\} \text{ simultaneous updates} \end{aligned}$$

Linear regression  $f_{\mathbf{w}, b}(\mathbf{x}) = \mathbf{w} \cdot \mathbf{x} + b$

Logistic regression  $f_{\mathbf{w}, b}(\mathbf{x}) = \frac{1}{1 + e^{-(\mathbf{w} \cdot \mathbf{x} + b)}}$

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- Monitor gradient descent (learning curve)
- Vectorized implementation
- Feature scaling

$$g(z) = \frac{1}{1+e^{-z}}$$

## Gradient Descent Implementation

The gradient descent algorithm implementation has two components:

- The loop implementing equation (1) above. This is `gradient_descent` below and is generally provided to you in optional and practice labs.
- The calculation of the current gradient, equations (2,3) above. This is `compute_gradient_logistic` below. You will be asked to implement this week's practice lab.

### Calculating the Gradient, Code Description

Implements equation (2),(3) above for all  $w_j$  and  $b$ . There are many ways to implement this. Outlined below is this:

- initialize variables to accumulate `dj_dw` and `dj_db`
- for each example
  - calculate the error for that example  $g(\mathbf{w} \cdot \mathbf{x}^{(i)} + b) - y^{(i)}$
  - for each input value  $x_j^{(i)}$  in this example,
    - multiply the error by the input  $x_j^{(i)}$ , and add to the corresponding element of `dj_dw`. (equation 2 above)
    - add the error to `dj_db` (equation 3 above)
- divide `dj_db` and `dj_dw` by total number of examples ( $m$ )
- note that  $\mathbf{x}^{(i)}$  in numpy  $X[i,:]$  or  $X[i]$  and  $x_j^{(i)}$  is  $X[i,j]$

```
In [4]: def compute_gradient_logistic(X, y, w, b):
    """
    Computes the gradient for logistic regression

    Args:
        X (ndarray (m,n)): Data, m examples with n features
        y (ndarray (m,)): target values
        w (ndarray (n,)): model parameters
        b (scalar) : model parameter
    Returns
        dj_dw (ndarray (n,)): The gradient of the cost w.r.t. the parameters w.
        dj_db (scalar) : The gradient of the cost w.r.t. the parameter b.
    """
    m,n = X.shape
    dj_dw = np.zeros((n,))
    dj_db = 0.

    for i in range(m):
        f_wb_i = sigmoid(np.dot(X[i],w) + b)           # (n,) (n,) = scalar
        err_i = f_wb_i - y[i]                          # scalar
        for j in range(n):
            dj_dw[j] = dj_dw[j] + err_i * X[i,j]       # scalar
        dj_db = dj_db + err_i                         # scalar
    dj_dw = dj_dw/m                                # (n, )
    dj_db = dj_db/m                                # scalar

    return dj_db, dj_dw
```

Check the implementation of the gradient function using the cell below.

```
In [5]: X_tmp = np.array([[0.5, 1.5], [1,1], [1.5, 0.5], [3, 0.5], [2, 2], [1, 2.5]])
y_tmp = np.array([0, 0, 0, 1, 1, 1])
w_tmp = np.array([2.,3.])
b_tmp = 1.
dj_db_tmp, dj_dw_tmp = compute_gradient_logistic(X_tmp, y_tmp, w_tmp, b_tmp)
print(f"dj_db: {dj_db_tmp}")
print(f"dj_dw: {dj_dw_tmp.tolist()}")
```

`dj_db: 0.49861806546328574`

`dj_dw: [0.49833393278696, 0.49883942983996693]`

### Expected output

`dj_db: 0.49861806546328574`  
`dj_dw: [0.49833393278696, 0.49883942983996693]`

## Gradient Descent Code

The code implementing equation (1) above is implemented below. Take a moment to locate and compare the functions in the routine to the equations above.

```
In [6]: def gradient_descent(X, y, w_in, b_in, alpha, num_iters):
    """
    Performs batch gradient descent

    Args:
        X (ndarray (m,n)) : Data, m examples with n features
        y (ndarray (m,)) : target values
        w_in (ndarray (n,)): Initial values of model parameters
        b_in (scalar) : Initial values of model parameter
        alpha (float) : Learning rate
        num_iters (scalar) : number of iterations to run gradient descent

    Returns:
        w (ndarray (n,)) : Updated values of parameters
        b (scalar) : Updated value of parameter
    """
    # An array to store cost J and w's at each iteration primarily for graphing later
    J_history = []
    w = copy.deepcopy(w_in) # avoid modifying global w within function
    b = b_in

    for i in range(num_iters):
        # Calculate the gradient and update the parameters
        dj_db, dj_dw = compute_gradient_logistic(X, y, w, b)
```

```

# Update Parameters using w, b, alpha and gradient
w = w - alpha * dj_dw
b = b - alpha * dj_db

# Save cost J at each iteration
if i<100000: # prevent resource exhaustion
    J_history.append( compute_cost_logistic(X, y, w, b) )

# Print cost every 10 times or as many iterations if < 10
if i%math.ceil(num_iters / 10) == 0:
    print(f"Iteration {i:4d}: Cost {J_history[-1]}")

return w, b, J_history #return final w,b and J history for graphing

```

Let's run gradient descent on our data set.

```

In [7]: w_tmp = np.zeros_like(X_train[0])
b_tmp = 0.
alph = 0.1
iters = 10000

w_out, b_out, _ = gradient_descent(X_train, y_train, w_tmp, b_tmp, alph, iters)
print(f"\nupdated parameters: w:{w_out}, b:{b_out}")

Iteration 0: Cost 0.684610468560574
Iteration 1000: Cost 0.1590977666870456
Iteration 2000: Cost 0.08460064176930081
Iteration 3000: Cost 0.05705327279402531
Iteration 4000: Cost 0.042907594216820076
Iteration 5000: Cost 0.034338477298845684
Iteration 6000: Cost 0.028603798022120097
Iteration 7000: Cost 0.024501569608793
Iteration 8000: Cost 0.02142370332569295
Iteration 9000: Cost 0.019030137124109114

updated parameters: w:[5.28 5.08], b:-14.222409982019837

```

Let's plot the results of gradient descent:

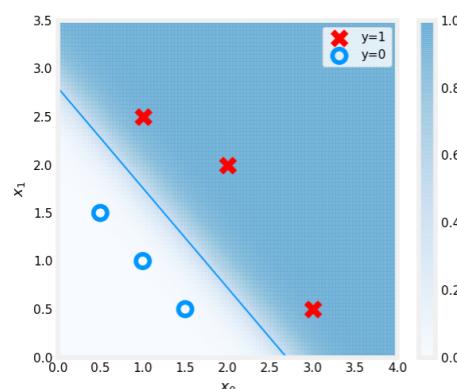
```

In [8]: fig,ax = plt.subplots(1,1,figsize=(5,4))
# plot the probability
plt_prob(ax, w_out, b_out)

# Plot the original data
ax.set_ylabel(r'$x_1$')
ax.set_xlabel(r'$x_0$')
ax.axis([0, 4, 0, 3.5])
plot_data(X_train,y_train,ax)

# Plot the decision boundary
x0 = -b_out/w_out[0]
x1 = -b_out/w_out[1]
ax.plot([x0,x0],[x1,0], c=dlc["dlblue"], lw=1)
plt.show()

```



In the plot above:

- the shading reflects the probability  $y=1$  (result prior to decision boundary)
- the decision boundary is the line at which the probability = 0.5

## Another Data set

Let's return to a one-variable data set. With just two parameters,  $w, b$ , it is possible to plot the cost function using a contour plot to get a better idea of what gradient descent is up to.

```

In [9]: x_train = np.array([0., 1, 2, 3, 4, 5])
y_train = np.array([0, 0, 1, 1, 1])

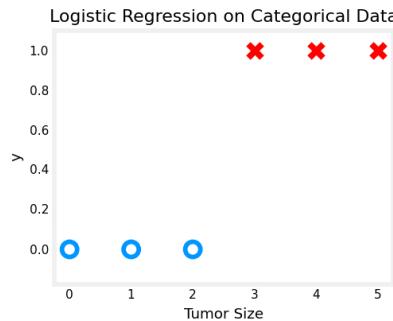
```

As before, we'll use a helper function to plot this data. The data points with label  $y = 1$  are shown as red crosses, while the data points with label  $y = 0$  are shown as blue circles.

```

In [10]: fig,ax = plt.subplots(1,1,figsize=(4,3))
plt_tumor_data(x_train, y_train, ax)
plt.show()

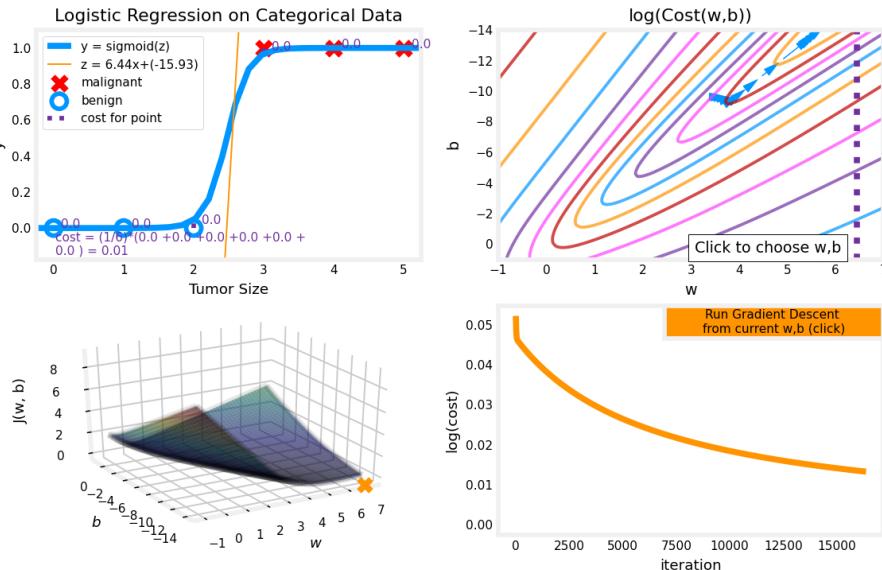
```



In the plot below, try:

- changing  $w$  and  $b$  by clicking within the contour plot on the upper right.
- changes may take a second or two
- note the changing value of cost on the upper left plot.
- note the cost is accumulated by a loss on each example (vertical dotted lines)
- run gradient descent by clicking the orange button.
- note the steadily decreasing cost (contour and cost plot are in  $\log(\text{cost})$ )
- clicking in the contour plot will reset the model for a new run
- to reset the plot, rerun the cell

```
In [11]: w_range = np.array([-1, 7])
b_range = np.array([1, -14])
quad = plt_quad_logistic( x_train, y_train, w_range, b_range )
```



## Congratulations!

You have:

- examined the formulas and implementation of calculating the gradient for logistic regression
- utilized those routines in
  - exploring a single variable data set
  - exploring a two-variable data set

```
In [ ]:
```