1. In this quiz, you will practice calculating the multivariate chain rule for various functions. **1 point**

For the following functions, calculate the expression $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt}$ in matrix form, where $\mathbf{x} = (x_1, x_2)$.

$f(\mathbf{x}) = f(x_1, x_2) = x_1^2 x_2^2 + x_1 x_2$

$x_1(t) = 1 - t^2$

$x_2(t) = 1 + t^2$

- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [2x_1^2 x_2 + x_1, 2x_1 x_2^2 + x_2]\begin{bmatrix} 2t \\ -2t \end{bmatrix}$
- ◉ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [2x_1 x_2^2 + x_2, 2x_1^2 x_2 + x_1]\begin{bmatrix} -2t \\ 2t \end{bmatrix}$
- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [2x_1 x_2^2 + x_2, 2x_1^2 x_2 + x_1]\begin{bmatrix} 2t \\ -2t \end{bmatrix}$
- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [2x_1^2 x_2 + x_1, 2x_1 x_2^2 + x_2]\begin{bmatrix} -2t \\ 2t \end{bmatrix}$

2. For the following functions, calculate the expression $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt}$ in matrix form, where $\mathbf{x} = (x_1, x_2, x_3)$. **1 point**

$f(\mathbf{x}) = f(x_1, x_2, x_3) = x_1^3 cos(x_2) e^{x_3}$

$x_1(t) = 2t$

$x_2(t) = 1 - t^2$

$x_3(t) = e^t$

- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [3x_1^2 cos(x_2)e^{x_3}, -x_1^3 sin(x_2)e^{x_3}, x_1^3 sin(x_2)e^{x_3}]\begin{bmatrix} 2 \\ 2t \\ e^t \end{bmatrix}$
- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [3x_1^2 cos(x_2)e^{x_3}, -x_1^3 cos(x_2)e^{x_3}, x_1^3 cos(x_2)e^{x_3}]\begin{bmatrix} 2 \\ 2t \\ e^t \end{bmatrix}$
- ◉ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [3x_1^2 cos(x_2)e^{x_3}, -x_1^3 sin(x_2)e^{x_3}, x_1^3 cos(x_2)e^{x_3}]\begin{bmatrix} 2 \\ -2t \\ e^t \end{bmatrix}$
- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [3x_1^2 cos(x_2)e^{x_3}, x_1^3 cos(x_2)e^{x_3}, x_1^3 sin(x_2)e^{x_3}]\begin{bmatrix} 2 \\ 2t \\ -e^t \end{bmatrix}$

3. For the following functions, calculate the expression $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt}$ in matrix form, where $\mathbf{x} = (x_1, x_2)$ and $\mathbf{u} = (u_1, u_2)$. **1 point**

$f(\mathbf{x}) = f(x_1, x_2) = x_1^2 - x_2^2$

$x_1(u_1, u_2) = 2u_1 + 3u_2$

$x_2(u_1, u_2) = 2u_1 - 3u_2$

$u_1(t) = cos(t/2)$

$u_2(t) = sin(2t)$

- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [2x_1, 2x_2]\begin{bmatrix} 2 & -3 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} sin(t/2) \\ 2cos(2t) \end{bmatrix}$
- ◉ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [2x_1, -2x_2]\begin{bmatrix} 2 & 3 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} -sin(t/2)/2 \\ 2cos(2t) \end{bmatrix}$
- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [2x_1, 2x_2]\begin{bmatrix} 2 & -3 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} -cos(t/2)/2 \\ 2sin(2t) \end{bmatrix}$
- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [-2x_1, -2x_2]\begin{bmatrix} -2 & 3 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} -sin(t/2)/2 \\ 2cos(t) \end{bmatrix}$

4. For the following functions, calculate the expression $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt}$ in matrix form, where $\mathbf{x} = (x_1, x_2)$ and $\mathbf{u} = (u_1, u_2)$. **1 point**

$f(\mathbf{x}) = f(x_1, x_2) = cos(x_1)sin(x_2)$

$x_1(u_1, u_2) = 2u_1^2 + 3u_2^2 - u_2$

$x_2(u_1, u_2) = 2u_1 - 5u_2^3$

$u_1(t) = e^{t/2}$

$u_2(t) = e^{-2t}$

- $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [-sin(x_1)cos(x_2), cos(x_1)cos(x_2)] \begin{bmatrix} u_1 & 6u_2 - 1 \\ 2 & -u_2^2 \end{bmatrix} \begin{bmatrix} e^{t^2/2}/2 \\ -2e^{-2t} \end{bmatrix}$
- $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [-sin(x_1)sin(x_2), cos(x_1)cos(x_2)] \begin{bmatrix} 4u_1 & 6u_2 - 1 \\ 2 & -15u_2^2 \end{bmatrix} \begin{bmatrix} e^{t/2}/2 \\ -2e^{-2t} \end{bmatrix}$
- $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [-sin(x_1)cos(x_2), cos(x_1)cos(x_2)] \begin{bmatrix} 41u_1 & 6u_2 - 1 \\ 2 & -15u_2 \end{bmatrix} \begin{bmatrix} e^{t/2}/8 \\ -2e^{2t} \end{bmatrix}$
- $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [-cos(x_1)sin(x_2), cos(x_1)cos(x_2)] \begin{bmatrix} u_1 & 6u_2 - 1 \\ 2 & -15u_2^2 \end{bmatrix} \begin{bmatrix} e^{t} \\ e^{t} \end{bmatrix}$

5. For the following functions, calculate the expression $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt}$ in matrix form, where $\mathbf{x} = (x_1, x_2)$ and $\mathbf{u} = (u_1, u_2)$. 1 point

$f(\mathbf{x}) = f(x_1, x_2, x_3) = sin(x_1)cos(x_2)e^{x_3}$

$x_1(u_1, u_2) = sin(u_1) + cos(u_2)$

$x_2(u_1, u_2) = cos(u_1) - sin(u_2)$

$x_3(u_1, u_2) = e^{u_1 + u_2}$

$u_1(t) = 1 + t/2$

$u_2(t) = 1 - t/2$

- $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} =$

  $[cos(x_1)cos(x_2)e^{x_3}, -sin(x_1)sin(x_2)e^{x_3}, sin(x_1)cos(x_2)e^{x_3}] \begin{bmatrix} cos(u_1) & -sin(u_2) \\ -sin(u_1) & -cos(u_2) \\ e^{u_1+u_2} & e^{u_1+u_2} \end{bmatrix} \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$
- $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} =$

  $[cos(x_1)cos(x_2)e^{x_3}, -sin(x_1)cos(x_2)e^{x_3}, sin(x_1)cos(x_2)e^{x_3}] \begin{bmatrix} cos(u_1) & sin(u_2) \\ -sin(u_1) & -cos(u_2) \\ e^{u_1+u_2} & -e^{u_1+u_2} \end{bmatrix} \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$
- $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} =$

  $[cos(x_1)cos(x_2)e^{x_3}, -sin(x_1)^2 sin(x_2)e^{x_3}, sin(x_1)cos(x_2)e^{x_3}] \begin{bmatrix} sin(u_1) & -sin(u_2) \\ -sin(u_1) & -cos(u_2) \\ 3e^{u_1+u_2} & e^{u_1+u_2} \end{bmatrix} \begin{bmatrix} -1/2 \\ -1/2 \end{bmatrix}$
- $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} =$

  $[cos(x_1)cos(x_2)e^{x_3}, sin(x_1)sin(x_2)e^{x_3}, sin(x_1)cos(x_2)e^{x_3}] \begin{bmatrix} -cos(u_1) & -sin(u_2) \\ -sin(u_1) & -cos(u_2) \\ e^{u_1+u_2} & 2e^{u_1+u_2} \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$