ECE4580 Homework #7

Due: Mar. 02, 2017

Problem 1. (20 pts) Many times in computer vision, it is very difficult to find the optimal value of a function by plotting the function because the vector to optimize has high dimensions. The only values one has access to are the gradients. This problem puts you in the same kind of predicament, as the optimal value is desired even though is it not visualizable. Enter gradient descent/ascent.

I have created a 2D function whose minimal value is to be estimated using gradient descent. You know that the domain of the function is limited to $x \in [0, 10]$ and $y \in [0, 10]$. Load the file `gradient.mat` and use the function **gradF** which gives the gradient of the function $F : \mathbb{R}^2 \to \mathbb{R}^2$ to find the minimal value of $F(\vec{x})$ on the domain $[0, 10] \times [0, 10]$. Pick an initial guess and see where the gradient descent takes it. Turn in your initial guess and the final estimated minimum. For fun, feel free to turn in the trajectory followed (this would be the sequence of incrementally better estimates).

Note: The argument to `gradF` *is a 2x1 position vector. The return value is a 2x1 motion vector.*

Problem 2. (35 pts) Implement the Laplacian smoothing (partial) differential equation derived in class. This partial differential equation is also called the isotropic diffusion equation or the heat equation (so many names!). As a differential equation applied to the image $I : \mathbb{R}^2 \to \mathbb{R}$, it is:

$$\dot{I} = \Delta I, \quad \text{with } \Delta I|_{\partial D} = 0.$$

where ∂D is the image boundary, and the zero boundary conditions on the Laplacian are to have a well-posed evolution, as per the gradient variational update derivation. The homework stub is called `isodiffuse.m` and should be in the zip-file.

1. Apply the algorithm to the image to diffuse, and to some of the other images in the homework Matlab file. Turn in the before and after, with the after such that it is obvious that diffusion has occurred. Do so for the boring rectangular blob image and another.

2. Now, the heat equation is supposed to look like Gaussian smoothing due to the link between the motion of particles and Gaussian motion. In particular, the relationship between the amount of time an image is smoothed and the equivalent standard deviation of Gaussian smoothing is $\sigma = \sqrt{2t}$. Show that if you smooth for a total time of $t = k \cdot \Delta t$, then the image resembles a Gaussian smoothing image with $\sigma = \sqrt{2t}$.

   To do this, use the blurme image from the previous homework and apply the heat equation to it for times $0.25, 0.5, 0.75, 1, 1.5, 2$ and show that it gets closer to $\sigma = 2$ as time progresses. Hopefully it does so! A good way to test how close the images are is to take the difference between the heat smoothed and the Gaussian smoothed image then use the `sumsqr`, which computes the sum of squared values of the argument and returns a scalar. The function `sumsqr` is a great way to compute the error energy that we've been using in class. The error should get smaller as the diffusion gets closer in time to 2. Much luck.

   Note: It might help to plot the `log` *of the energy since it may be huge. The error energy should go more negative in the log space. Also, Matlab has a function called* `fspecial` *that can be used to create the Gaussian kernel matrix. Make sure to make it big enough in size, usually some reasonable factor bigger than the standard deviation argument (say a factor of 2.5 rounded to nearest odd number).*

3. In class, we also discussed how the image would tend to the average image value. Compute the average value, and show that this value remains roughly constant as the smoothing progresses (plot the average values versus the number of iterations). If you run it for a long time, you should see the image slowly becoming one uniform blob, so it does tend to the average value (it probably takes longer than you really care if you have an image that starts off non-smooth).

Problem 3. (20 pts) Move on to the Week #4 activities of the learning module.

The group submission should reflect the work of the group, and should also be submitted individually with the name of your partner in the document. If submitting video or links to video for the pair, then only one member need to do so, while the other member should just note as much. The prior expectaction for deliverables continues to hold.