## GEORGIA INSTITUTE OF TECHNOLOGY SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

## ECE 2026 Spring 2015 Lab #5: Spectrograms, Harmonic Lines, Sampling & Aliasing

Date: 9-12 Feb. 2015

**Pre-Lab:** You should read the Pre-Lab section of the lab and do all the exercises in the Pre-Lab section *before your assigned lab time*.

**Verification:** The Exercise section of each lab must be completed **during your assigned Lab time** and the steps marked *Instructor Verification* must also be signed off **during the lab time**. One of the laboratory instructors must verify the appropriate steps by signing on the **Instructor Verification** line. When you have completed a step that requires verification, simply raise your hand and demonstrate the step to the TA or instructor. Turn in the completed verification sheet to your TA when you leave the lab.

**Lab Homework Questions:** The Lab-Homework Sheet has a few lab related questions that can be answered at your own pace. The completed Lab-HW sheet is due at the beginning of the next lab.

Forgeries and plagiarism are a violation of the honor code and will be referred to the Dean of Students for disciplinary action. You are allowed to discuss lab exercises with other students, but you cannot give or receive any written material or electronic files. In addition, you are not allowed to use or copy material from old lab reports from previous semesters. Your submitted work must be your own original work.

#### 1 Overview

Please read through the information below prior to attending your lab. The objective of this lab is to introduce more complicated signals that are related to the basic sinusoid. These signals which implement frequency modulation (FM) and amplitude modulation (AM) are widely used in communication systems such as radio and television. In addition, they can be used to create interesting sounds that mimic musical instruments. There are a number of demonstrations on the CD-ROM that provide examples of these signals for many different conditions. The resulting signal can be analyzed to show its time-frequency behavior by using the *spectrogram*.



This lab studies signal synthesis for AM and FM signals, and their time-frequency content as shown in a spectrogram. An underlying objective of the lab is to learn more about the spectrogram. There are several specific steps that will be considered in this lab:

- 1. Synthesize a beat-note signal with a MATLAB M-file, and display its spectrogram.
- 2. Study the frequency resolution of the spectrogram for two closely spaced sinusoids.
- 3. Spectrogram: Make empirical observations of the spectrogram as the section length is changed.
- 4. Synthesize a linear-FM chirp with a MATLAB M-file, and display its spectrogram.
- 5. Spectrogram: Create a spectrogram that displays negative frequencies, as well as positive ones.
- 6. Synthesize a frequency-modulated (FM) signal to match a given spectrogram. i.e, match specific time-frequency spectral content.

#### 2 Pre-Lab

We have spent a lot of time learning about the properties of sinusoidal waveforms of the form:

$$x(t) = A\cos(2\pi f_0 t + \varphi) = \Re\left\{ \left( A e^{j\varphi} \right) e^{j2\pi f_0 t} \right\}$$
 (1)

In this lab, we will extend our treatment of sinusoidal waveforms to more complicated signals composed of sums of sinusoidal signals, or sinusoids with changing frequency, i.e., frequency-modulated sinusoids.

#### 2.1 Review: Chirp, or Linearly Swept Frequency

A linear-FM *chirp* signal is a sinusoid whose instantaneous frequency changes linearly from a starting value to an ending one. The formula for such a signal can be defined by creating a complex exponential signal with a quadratic angle function  $\psi(t)$ . Mathematically, we define  $\psi(t)$  as



$$\psi(t) = 2\pi\mu t^2 + 2\pi f_0 t + \varphi \tag{2}$$

The derivative of  $\psi(t)$  yields an instantaneous cyclic frequency that changes linearly versus time.

$$f_i(t) = 2\mu t + f_0 \quad \text{(hertz)} \tag{3}$$

The slope of  $f_i(t)$  is equal to  $2\mu$  and its t=0 intercept is  $f_0$ . The frequency variation in (3) produced by the time-varying angle function is called *frequency modulation*, so these signals are called FM signals. Finally, since the linear variation of the frequency (3) can produce an audible sound similar to a siren or a bird chirp, linear-FM signals are also called *chirps*.

If the signal starts at time  $t = t_1$  s with a frequency of  $f_1$  Hz, and ends at time  $t = t_2$  s with a frequency of  $f_2$  Hz, then the slope of the line in (3) will be

$$SLOPE = 2\mu = \frac{f_2 - f_1}{t_2 - t_1} \tag{4}$$

Note that if the signal starts at time t = 0 s, then  $f_1 = f_0$  is also the starting frequency. Otherwise,  $f_0 = ?$ 

#### 2.2 Review: MATLAB Synthesis of Chirp Signals

In MATLAB signals can only be synthesized by evaluating the signal's defining formula at discrete instants of time. These are called *samples* of the signal. For the chirp we use the following:

$$x(t_n) = A\cos(2\pi\mu t_n^2 + 2\pi f_0 t_n + \varphi)$$

where  $t_n$  is the  $n^{\text{th}}$  time sample. The following MATLAB code will synthesize a linear-FM chirp:

```
%-Number of time samples per second
  fSamp = 8000;
  dt = 1/fSamp;
  tStart = 0;
  tStop = 1.5;
  tt = tStart:dt:tStop;
  mu = 600;
  fzero = 400;
  phi = 2*pi*rand;  %-- random phase
%
                 %%
    psi = ????;
  cc = real(7.7*exp(j*psi));
   soundsc(cc, fSamp); %-- uncomment to hear the sound
  plotspec( cc+j*1e-12, fSamp, 256 ), colorbar, grid on %-- with negative frequencies
```

- (a) Determine the total duration of this synthesized signal in seconds, and also the length of the tt vector. Use MATLAB's size command to check that the signal vector cc has the expected size.
- (b) Determine the range of frequencies (in hertz) that will be synthesized by the MATLAB script above, i.e., determine the minimum and maximum frequencies (in Hz) that will be heard. This will require that you relate the parameters  $\mu$ ,  $f_0$ , and  $\varphi$  to the minimum and maximum frequencies. Examine the MATLAB spectrogram to determine the instantaneous (cyclic) frequency  $f_i(t)$  versus time. Zoom in to verify the correct starting and ending frequencies.
- (c) The spectrogram usually shows only the frequency components for  $f \geq 0$ , but with the "tiny imaginary part" trick plotspec will show the negative frequency components. We will called this a two-sided spectrogram. Since the chirp signal is real-valued, the spectrum must have conjugate symmetry, so the magnitudes of the negative frequency components are a mirror image of those in the positive frequency region.
- (d) Use soundsc() to listen to the signal in order to determine whether the signal's frequency content is increasing or decreasing. Notice that soundsc() needs to know two things: the vector containing the signal samples, and the rate at which the signal samples are to be played out. This rate should be the same as the rate at which the signal values were created (fSamp in the code above). For more information do help sound and help soundsc in MATLAB.
- (e) The test case above generates a chirp sound whose frequency starts low and chirps up. Modify the parameters so that the chirp starts at 3500 Hz and chirps down to 500 Hz.

#### 2.3 More Spectrogram: Decibels and Section Length

The common log scale used in engineering is decibels (dB), which is defined as  $20 \log_{10}(A)$  where A is amplitude. The built-in MATLAB spectrogram M-file uses a dB scale for amplitude when displaying its spectrogram image. The decibel has two notable features:

1. Ratios become Differences: On a dB scale, a numbers are represented with logarithms, so the ratio P/Q becomes

$$20\log_{10}(P/Q) = 20\log_{10}(P) - 20\log_{10}(Q)$$

If  $A_2 = (1/10)A_1$  then  $A_2$  is 20 dB lower than  $A_1$ , because with logs, we get

$$20\log_{10}(A_2) = 20\log_{10}((1/10)A_1) = 20\log_{10}(A_1) + 20\log_{10}(0.1) = 20\log_{10}(A_1) - 20\,\mathrm{dB}.$$

Since ratios become differences, dB is most often used to compare the relative size of values.

- 2. The dB range must be restricted because  $20\log_{10}(0) = -\infty$ . If we want to map the linear amplitude range [0,1] into dB, we must define a minimum dB level. Since  $20\log_{10}(1) = 0$  dB is the maximum, other dB values for [0,1] will be negative. The minimum dB level will chop off the bottom of [0,1] and make it equal to  $[\epsilon,1]$  where  $\epsilon$  is very small. For example, a dB range of 80 dB would define the minimum to be -80 dB, so  $\epsilon = 10^{-80/20} = 10^{-4} = 0.0001$ .
- Express the numbers 0.1, 1, 2, 5, 10 and 100 in dB.
- Convert  $-6 \, dB$ ,  $-60 \, dB$ , and  $-80 \, dB$  to numbers.

Another topic is displaying spectrogram of signals is section length which determines the time and frequency resolution. Figure 1 shows a sinusoid analyzed with a constant section length overlapping 50% in time expressed in different colors and its corresponding spectrogram. It is clear that at the signal transition regions the frequency content is not just a single spectral line.

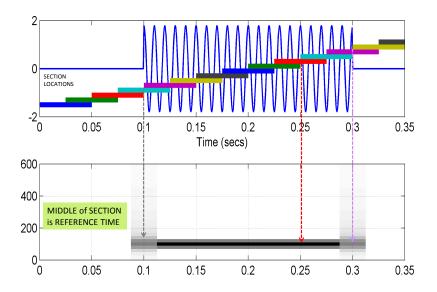


Figure 1: An illustration of the edge effect in spectrogram display.

#### 2.4 Beat Note Spectrograms and Frequency Resolution

Beat notes as as a sum of two sinusoids provide an interesting way to investigate the time-frequency characteristics of spectrograms. Although some of the mathematical details require further study beyond this course, it is not difficult to appreciate the following issue: there is a fundamental trade-off between knowing which frequencies are present in a signal's spectrum and knowing how those frequencies vary with time. As discussed previously, a spectrogram estimates the frequency content over short sections of the signal; this is the *Section Length* parameter.<sup>1</sup> If we make the section length very short we can track rapid changes in the signal, usually changes in the frequency content. The tradeoff, however, is that shorter sections may not provide enough data to do an accurate frequency measurement. On the other hand, long sections allow the spectrogram to perform excellent frequency measurements, but fail to track sudden frequency changes. For example, if a signal is the sum of two sinusoids whose frequencies are nearly the same, a very long section length is needed to "resolve" the two sinusoidal components. This trade-off between the section length (in time) and the frequency resolution is akin to Heisenburg's Uncertainty Principle in physics. We can summarize this discussion by stating the following hypothesis:

The frequency resolution of the spectrogram is inversely proportional to the Section Length. In other words, when the true spectrum has two lines (at  $f_1$  and  $f_2$ ) these two lines will be visible as distinct lines in the spectrogram if  $|f_1 - f_2| \approx C/T_{\text{SECT}}$  where C is a proportionality constant and  $T_{\text{SECT}}$  is the section duration in secs.

Note: When using plotspec(xx,fs,Lsect), the section length in samples is an input argument to the spectrogram function. We can use the sampling rate to convert to duration,  $T_{\rm SECT} = L_{\rm SECT}/f_s$ .

We will use beat note signals which consist of two closely spaced spectral lines to confirm this hypothesis. A beat note signal may be viewed as a single frequency signal whose amplitude varies with time, *or* as the sum of two sinusoidal signals with different constant frequencies. Both views can be used to explain the effect of (window) section length when finding the spectrogram of a beat signal.

<sup>&</sup>lt;sup>1</sup>The section length is often called the window length; the two terms are used interchangeably in DSP.

- (a) Use the MATLAB code written in earlier lab to create and plot a beat signal defined via:  $b(t) = 10\cos(2\pi(f_\Delta)t + \varphi_\Delta)\cos(2\pi(1024)t + \varphi_c),$  with a duration of 5 s, and a sampling rate of  $f_s = 8000$  samples/sec. The frequency  $f_\Delta$  should be set to 4 Hz, but will be varied in later parts. The phases can be random.
- (b) When  $f_{\Delta} = 4$  determine the locations of the two spectrum lines that you expect to see in the spectrogram. In other words, derive (mathematically) the spectrum of the signal defined in part (a).
- (c) Make the spectrogram of b(t) using a (window) section length of  $L_{\rm SECT}=256$  using the commands<sup>2</sup>: plotspec(xx,fSamp,256); colorbar, grid on, zoom on Comment on what you see. Are there two spectral lines, i.e., (horizontal lines across the spectrogram)? If necessary, use the zoom tool (in the MATLAB figure window), or zoom on, to examine the important regions of the spectrogram.
- (d) It should not be possible to see both spectrum lines with  $L_{\rm SECT}=256$ . In order to get both lines a longer section length is needed, so try doubling the section length. Try  $L_{\rm SECT}=512$ , then  $L_{\rm SECT}=1024$ , and so on until you can discern two spectrum lines.<sup>3</sup> Then reduce the value of  $L_{\rm SECT}$  little by little to get the smallest  $L_{\rm SECT}$  that will work. Getting a value of  $L_{\rm SECT}$  to the nearest 500 is sufficient.

As before, use zooming to examine the important regions of the spectrogram. Once you have two spectrum lines, record the value of  $L_{\rm SECT}$  and determine whether the frequencies present in the spectrogram are correct. In addition, convert  $L_{\rm SECT}$  to the section duration in seconds,  $T_{\rm SECT}$ .

#### 2.5 Fourier Series

Another objective of this lab is to demonstrate usage of the fseriesdemo GUI. If you have installed the *SP-First* Toolbox, you will already have this demo on the matlabpath.

In this MATLAB GUI, you can choose the signal from a few pre-determined signal types and change the signal period. You can also decide the number of Fourier coefficients to compute and the corresponding magnitude and phase will be displayed accordingly.

Figure 2 shows the interface for the fseriesdemo GUI.

#### 2.6 Triangle Wave and Its Fourier Series

The periodic triangular wave has a known Fourier Series. After consulting the text in Section 3.6.2 for the new Chapter 3 which can be downloaded from the class site, we can write:

$$a_k = \begin{cases} \frac{-2}{\pi^2 k^2} & \text{for } k \text{ odd} \\ \frac{1}{2} & \text{for } k = 0 \\ 0 & \text{for } k \text{ even} \end{cases}$$
 (5)

• Evaluate the coefficients for k = 1, 3, 5 and 15. Then compute the ratios  $a_3/a_1, a_5/a_1$  and  $a_{15}/a_1$ .

Comment: Here's a general question: are the Fourier Series coefficients independent of time scaling? First of all, y(t) = x(bt) is scaling. For example, a plot of x(3t) will be squeezed along the horizontal axis by a factor of b=3.

<sup>&</sup>lt;sup>2</sup>Use plotspec instead of specgram in order to get a linear amplitude scale rather than logarithmic.

<sup>&</sup>lt;sup>3</sup>Usually the window (section) length is chosen to be a power of two, because a special algorithm called the FFT is used in the computation. The fastest FFT programs are those where the FFT length is a power of 2.

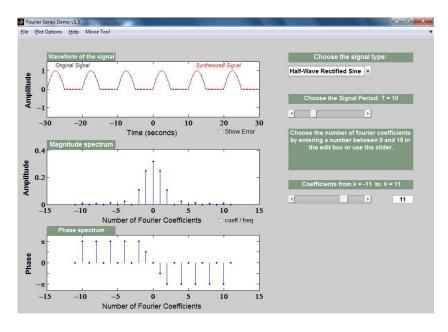


Figure 2: The fseriesdemo MATLAB GUI interface.

What are the Fourier coefficients of y(t)=x(bt) in terms of the Fourier coefficients of x(t)? If x(t) has a period equal to T, then the period of y(t) is T/b because x(t) is squeezed by b. Thus, the fundamental frequency of y(t) is  $\frac{2\pi}{(T/b)}=b(\frac{2\pi}{T})$ , i.e., it is scaled by b.

Now we write the Fourier Series integral for y(t)

$$(1/(T/b)) \int_{-T/2b}^{T/2b} y(t)e^{-j((2\pi k)/(T/b))t}dt = (b/T) \int_{-T/2b}^{T/2b} x(bt)e^{-j(2\pi k/T)(bt)}dt$$

Make a change of variables:  $\lambda = bt$ , and with  $d\lambda = b(dt)$ , you get

$$b(1/T) \int_{-T/2}^{T/2} x(\lambda) e^{-j(2\pi k/T)\lambda} (1/b) d\lambda = (1/T) \int_{-T/2}^{T/2} x(\lambda) e^{-j(2\pi k/T)\lambda} d\lambda = a_k$$

The scaling factor b cancels to give the RHS. So, time scaling doesn't change the  $\{a_k\}$  coefficients. The Fourier Series coefficients of y(t) are exactly equal to the Fourier Series coefficients of x(t).

Use the GUI to do the following exercises. The parameters of the input signal are its frequency  $f_0$  in Hz, and its phase  $\varphi$  in rads. The amplitude is one. The sampling rate for both the A/D converter and the D/A converter is  $f_s$  in samples/sec.

#### 2.7 Theory of Sampling

In this lab, the short-duration sinusoids and music signals will be created with the intention of playing them out through a speaker. Therefore, it is necessary to take into account the fact that a conversion is needed from the digital samples, which are numbers stored in the computer memory to the actual continuous waveform that will be amplified and heard through the speakers (or headphones).

Chapter 4 treats sampling in detail, but this lab is usually done prior to lectures on sampling, so we provide a quick summary of essential facts here. The idealized process of sampling a signal and the subsequent

reconstruction of the signal from its samples is depicted in Fig. 3. This figure shows a continuous-time input signal x(t), which is sampled by the continuous-to-discrete (C-to-D) converter to produce a sequence of samples  $x[n] = x(nT_s)$ , where n is the integer sample index and  $T_s$  is the sampling period. The sampling

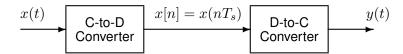


Figure 3: Sampling and reconstruction of a continuous-time signal.

rate (in samples per second) is  $f_s = 1/T_s$ . The discrete-to-continuous (D-to-C) converter creates a continuous output signal y(t) from the samples x[n]. As described in Chapter 4 of the text, the ideal D-to-C converter takes the signal samples  $x(nT_s)$  and interpolates a smooth curve through them. The Sampling Theorem tells us that when the input signal x(t) is a sum of sine waves, the output y(t) will be equal to the input x(t) if the sampling rate is more than twice the highest frequency  $(f_{max})$  in the input, i.e., we need  $f_s > 2f_{max}$ . In other words, if we sample fast enough then there will be no problems resynthesizing the continuous audio signal from x[n].

#### 2.8 A-to-D and D-to-A Conversion

Most computers have a built-in hardware for the analog-to-digital (A-to-D) converter and the digital-to-analog (D-to-A) converter (usually with a sound card or chip). These hardware systems are physical realizations of the idealized concepts of C-to-D and D-to-C converters, respectively, but for purposes of this lab we will assume that the hardware A/D and D/A are perfect realizations.

The digital-to-analog conversion process has many engineering design issues, but in its simplest form the only thing we need to worry about (in this lab) is that the time spacing  $(T_s)$  between the signal samples must correspond to the rate of the D-to-A hardware that is being used. From MATLAB, the sound output is done by the soundsc(xx,fs) function which does support a variable D-to-A sampling rate (fs) to the extent that the hardware on the machine has such variable rate capability. A convenient choice for the D-to-A conversion rate is 11025 samples per second, so  $T_s = 1/11025$  seconds; another common choice is 8000 samples/sec. Both of these rates satisfy the requirement of sampling fast enough as required by the Sampling Theorem. In fact, most piano notes have relatively low frequencies, so sampling rates much lower than 8000 samples/sec could be used. If you are using soundsc(), the vector xx will be rescaled automatically so that its maximum value equals the maximum allowed by the D-to-A converter, but if you are using sound.m, you must scale the vector xx so that it lies between  $\pm 1$ . Consult help sound.

#### 3 In-Lab Exercise

For the lab exercise, you will synthesize some signals, and then study their frequency content by using the spectrogram. The objective is to learn more about the connection between the *time-domain* definition of the signal and its *frequency-domain* content.

For the instructor verification, you will have to demonstrate that you understand concepts in a given subsection by answering questions from your lab instructor (or TA).

<sup>&</sup>lt;sup>4</sup>This sampling rate is one quarter of the 44,100-Hz rate used in audio CD players.

#### 3.1 Spectrogram for a Chirp with Aliases

Use the code provided in the pre-Lab section as a starting point in order to write a MATLAB script or function that will synthesize a "chirp" signal. Then use that M-file in this section.

- (a) What happens when we make a signal that "chirps" up to a very high frequency, and the *instantaneous* frequency goes past half the sampling rate? Generate a chirp signal that starts at 900 Hz when t=0 s, and chirps up to 11, 400 Hz, at t=5 s. Use  $f_s=4000$  Hz. Determine the parameters needed in Equation (2).
- (b) Generate the chirp signal in MATLAB and make a spectrogram with a short section length,  $L_{\rm SECT}$ , to verify that you have the correct starting and ending frequencies.<sup>5</sup> For your chosen  $L_{\rm SECT}$ , determine the section duration  $T_{\rm SECT}$  in secs.
- (c) Explain why the instantaneous frequency seen in the spectrogram is goes up and down between zero and  $f_s/2$ , i.e., it does not chirp up to 11,400 Hz. There are two effects that should be accounted for in your explanation.

*Note:* If possible listen to the signal to verify that the spectrogram is faithfully representing the audio signal that you hear.

**Instructor Verification** (separate page)

#### 3.2 Spectrogram of Periodic Full-Wave Rectified Sinusoid

A periodic signal is known to have a Fourier Series, which is usually described as a *harmonic line spectrum* because the only frequencies present in the spectrum are integer multiples of the fundamental frequency. With the spectrogram, it is easy to exhibit this harmonic line characteristic. More detail about the materials below can be found in Lectures 7 & 8, and Section 3.6.3 for the new Chapter 3 which can be downloaded from the class site.

(a) Write a simple MATLAB script that will generate a periodic full-wave rectified sine wave once the period is given. The peak amplitude should be equal to 1. Here is a MATLAB one-liner that can form the basis of this script:

```
tt=0:(1/fs):tStop;xx=Amp*abs(sin(2*pi*tt/T));
The values of fs, tStop, T, Amp will have to be determined.
```

- (b) Generate a full-wave rectified sine wave with T=1 sec, using a sampling rate of  $f_s = 1000$  Hz. The duration should be 5 secs, and Amp = 1.
- (c) Make a spectrogram with a long section duration.<sup>6</sup> It is important to pick a section duration that is equal to an integer number of periods of the periodic full-wave rectified sine waveform created in the previous part. Define  $T_{\text{SECT}}$  to get exactly 5 periods, and then determine the section length  $L_{\text{SECT}}$  (an integer) to be used in plotspec.
- (d) You should expect to see a "harmonic line spectrum" in the spectrogram. Since frequency is along the vertical axis, the harmonic lines will appear as horizontal lines in the spectrogram. Make a list of all the harmonic frequencies that you can see in the spectrogram. Note the Fourier series coefficient can be found as (see Section 3.6.4 of the updated Chapter 3 text):  $a_k = \frac{2}{\pi(1-4k^2)}$ .

<sup>&</sup>lt;sup>5</sup>There is no single correct answer for  $L_{\text{SECT}}$ , but you should pick a value that makes a smooth plot and lets you easily see the changing nature of the instantaneous frequency.

<sup>&</sup>lt;sup>6</sup>A long section duration in the spectrogram yields what is called a *narrowband spectrogram* because it will provide excellent resolution of the frequency components of a signal.

- (e) Determine the fundamental frequency for the harmonic lines. Note here the fundamental frequency doubles the original frequency of the sine wave after the full-wave rectification operation.
- (f) Measure the amplitudes of the first and third harmonic lines by using the Fourier Series coefficient:  $a_k = \frac{2}{\pi(1-4k^2)}$ .

**Instructor Verification** (separate page)

#### 3.3 Decibels (dB): Seeing Small Values in the Spectrogram

The periodic full-wave rectified sine wave has a known Fourier series:

$$a_k = \frac{2}{\pi (1 - 4k^2)}$$

Thus, in the spectrogram we should see all harmonic lines. Furthermore, there should be an infinite number of harmonic frequency lines.

Where did all the harmonics go?

The answer is that the higher harmonics have amplitudes that are too small to be seen in a spectrogram that displays values with a *linear amplitude*. Instead, a logarithmic amplitude scale is needed.

Consult Section 3.3 in the Pre-Lab for a discussion of *decibels*, and then answer the following questions about decibels:

- (a) In the language of dB, a factor of two is "6 dB." In other words, if  $B_2$  is 6 dB bigger than  $B_1$ , then it is twice as big (approximately). Explain why this statement is true.
- (b) Determine the dB difference between  $a_1$  and  $a_3$ . In other words,  $a_3$  is how many dB below  $a_1$ . Furthermore, explain why the dB difference.
- (c) Determine (in dB) how far  $a_{15}$  is below  $a_1$  for the periodic full-wave rectified sine wave.

**Instructor Verification** (separate page)

#### 3.4 Spectrogram in dB

A variation of the *SP-First* function plotspec has been written to incorporate the dB amplitude scale. This new function is called plotspecDB, and its calling template is shown below:

```
function him = plotspecDB(xx,fsamp,Lsect,DBrange)
%PLOTSPECDB
              plot a Spectrogram as an image
%
          (display magnitude in decibels)
           him = plotspec(xx,fsamp,Lsect,DBrange)
%
  usage:
%
      him = handle to the image object
%
        xx = input signal
     fsamp = sampling rate
%
%
     Lsect = section length (integer, power of 2 is a good choice)
               amount of data to Fourier analyze at one time
% DBrange = defines the minimum dB value; max is always 0 dB
```

(a) Create a "dB-Spectrogram" for the 5-sec periodic full-wave rectified sine wave generated in Sect. 3.2. Use a dBrange equal to 80 dB. Notice that many more spectrum lines are now visible. List the frequencies of all the harmonic spectrum lines, or give a general formula.

- (b) Generate a second full-wave rectified sine wave by changing the period in 3.2 to T=2 sec. Then make the dB-Spectrogram of this 5-sec full-wave rectified sine wave, being careful to select the section duration as an integer number of periods. From the spectrogram, determine the fundamental frequency and also the frequency of the highest harmonic line. Also, determine the harmonic number for the highest frequency, e.g., the 17<sup>th</sup> or 31<sup>st</sup>, etc.
- (c) Compare the dB difference to those obtained in Section 3.2, part (f).

**Instructor Verification** (separate page)

(d) Change the period to T=0.5 sec and make another dB-Spectrogram. Be careful to select the section duration as an integer number of periods. This period is shorter but the frequency separation of the harmonic lines is greater.

#### 4 Lab Homework: dB Spectrogram of Your Voice

Using the voice signal that you recorded for Lab #1, make a dB spectrogram with a long section length. Since the vowel region of the recording is a signal that is quasi-periodic, the spectrogram should exhibit a harmonic line characteristic during that time. Try different section lengths until the harmonic lines are easy to see in the spectrogram.

1. Turn in a print out of the spectrogram. Annotate the spectrogram to show the vowel region, as well as parameter values and measurements such as:

The values of  $T_{\text{SECT}}$  and  $L_{\text{SECT}}$  used.

The measured frequency separation of the harmonic lines that are seen in the vowel region.

- 2. From the frequency separation, compute the fundamental frequency in the vowel region. Also determine the fundamental period for this fundamental frequency.
- 3. Recall the value of the pitch period (in secs) measured in Lab #1. Compare the fundamental period determined in the previous part.
- 4. Comment on similarities and differences between the two period measurements.

# ECE-2026 Lab #5 Spring-2015 INSTRUCTOR VERIFICATION SHEET

Turn this page in to your lab grading TA before the end of your scheduled Lab time.

| Name:                     |                           | gtLoginUserName:                   | Date:                |
|---------------------------|---------------------------|------------------------------------|----------------------|
| Part 3.1 Chirp Aliasi     | ng                        |                                    |                      |
| (a) Chirp formula:        | $\cos(\psi(t)) =$         |                                    |                      |
| (b) $L_{	ext{SECT}} =$    | and $T_{\mathtt{SECT}}$ = | =                                  |                      |
| (c) Write explanation     | n of "ups and dow         | rns" in spectrogram:               |                      |
| Verified:                 |                           | Date/Time:                         |                      |
| Part 3.2 Spectrogram      | n of periodic full-wa     | ve rectified sine wave using a lor | ng section duration. |
| (b) $T_{\text{SECT}} =$   | and $L_{\mathtt{SECT}}$ = | =                                  |                      |
| (d) List frequencies      | of visible harmon         | ics =                              |                      |
| (e) Fundamental Fre       | equency =                 |                                    |                      |
| (f) $ a_1  =$             | and $ a_3  =$             |                                    |                      |
| Verified:                 |                           | Date/Time:                         |                      |
| Part 3.3 Questions al     | oout decibels:            |                                    |                      |
| (a) Explain 6 dB.         |                           |                                    |                      |
| (b) dB difference be      | tween $ a_1 $ and $ a_3 $ | =                                  |                      |
| (c) How far is $ a_{15} $ | below $ a_1 $ in dB?      |                                    |                      |
| Verified:                 |                           | Date/Time:                         |                      |
| Part 3.4 dB-Spectrog      | gram of periodic tria     | ngle wave using a long-duration    | section.             |
| (a) List frequencies      | of harmonic lines         | =                                  |                      |
| (b) Fundamental Fro       | equency =                 |                                    |                      |
| (b) Highest Frequen       | cy =                      |                                    |                      |
| (b) Harmonic Numb         | er of Highest Fred        | quency =                           |                      |
| (c) In dB, $a_1 =$        | and $a_3$                 | =                                  |                      |
| Verified:                 |                           | Date/Time:                         |                      |

### Lab #5 **ECE-2026** Spring-2015 LAB HOMEWORK QUESTION

Turn this page in to your lab grading TA at the very beginning of your next scheduled Lab time.

| Name:                      | gtLoginUserName:   | Date:                       |
|----------------------------|--|-----------------------------|
| Part 4: Using the voice    | signal that you recorded for Lab #1, make a dB s           | pectrogram with a long sec- |
| tion length Since the year | ral ragion of the recording is a signal that is guesi pari | adia the spectrogram should |

tion length. Since the vowel region of the recording is a signal that is quasi-periodic, the spectrogram should exhibit a harmonic line characteristic during that time. Try different section lengths until the harmonic lines are easy to see in the spectrogram.

1. Turn in a print out of the spectrogram. Annotate the spectrogram to show the vowel region, as well as parameter values and measurements such as:

The values of  $T_{\text{SECT}}$  and  $L_{\text{SECT}}$  used.

The measured frequency separation of the harmonic lines that are seen in the vowel region.

- 2. From the frequency separation, compute the fundamental frequency in the vowel region. Also determine the fundamental period for this fundamental frequency.
- 3. Recall the value of the pitch period (in secs) measured in Lab #1. Compare the fundamental period determined in the previous part.
- 4. Comment on similarities and differences between the two period measurements.