University of Illinois at Urbana-Champaign

ECE 311: Digital Signal Processing Lab

LAB 5: SOLUTIONS

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Problem 1
The MATLAB code for this problem is on the course website.

(a) The poles of and zeros of $H_1(z)$ are computed as shown below. The function `dpzplot` is used to plot the poles and zeros.

```
>> a1 = [1 -0.9 0.81];

>> b1 = [1 0 0 ];

zs1 = roots(b1);
ps1 = roots(a1);
poles =
   0.4500 + 0.7794i
   0.4500 - 0.7794i
>> zeros = roots(b1)
zeros =

     0
     0

>> dpzplot(b1,a1);
```

The pole-zero plot of $H_1(z)$ is shown in Fig. 1.

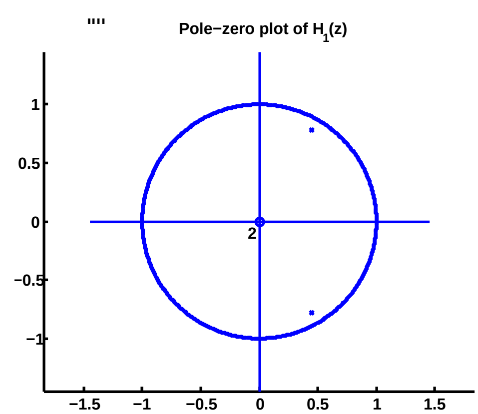

Figure 1: Pole-zero plot of $H_1(z)$

(b) The following code is used for Part (b)

```
omega = [0:511]*pi/256;
unitcirc = exp(i*omega);

temp1 = ps1*ones(1,512);
```

```
polevectors1 = ones(2,1)*unitcirc - ps1*ones(1,512);

polelength1 = abs(polevectors1);
poleangle1 = atan2(imag(polevectors1),real(polevectors1));
```

(c) The MATLAB code for this part is shown below,

```
zerovectors1 = ones(2,1)*unitcirc - zs1*ones(1,512);

zerolength1 = abs(zerovectors1);
zeroangle1 = atan2(imag(zerovectors1),real(zerovectors1));
```

(d) The plots of `polelength1`, `poleangle1`, `zerolength1`, `zeroangle1` is shown in Figs. 2 and 3. The MATLAB code is shown below

```
subplot(2,1,1),plot(omega/pi,polelength1)
xlabel('\omega, (\times \pi)'),ylabel('polelength1'),title('polelength for Problem 1')
subplot(2,1,2),plot(omega/pi,poleangle1)
xlabel('\omega, (\times \pi)'),ylabel('poleangle1'),title('poleangle for Problem 1')

figure
subplot(2,1,1),plot(omega/pi,zerolength1)
xlabel('\omega, (\times \pi)'),ylabel('zerolength1'),title('zerolength for Problem 1')
subplot(2,1,2),plot(omega/pi,zeroangle2)
xlabel('\omega, (\times \pi)'),ylabel('zeroangle1'),title('zeroangle for Problem 1')
```

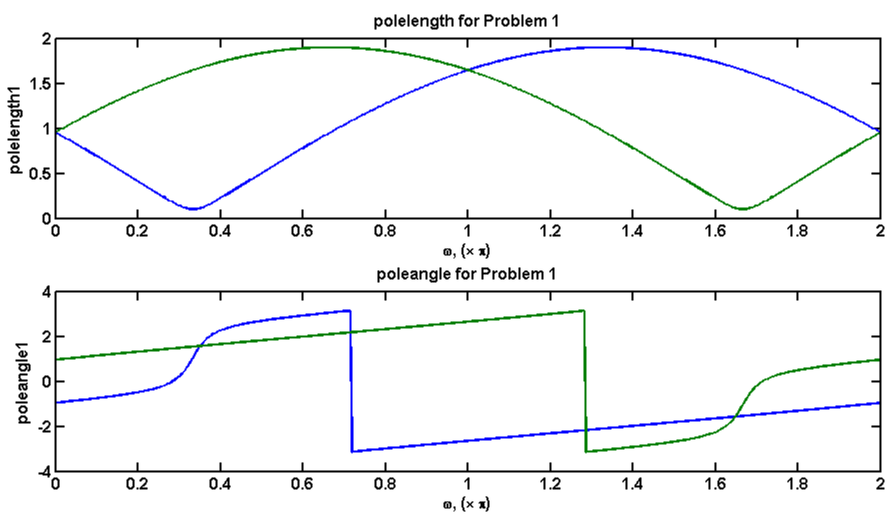

Figure 2: The magnitude and phase plots of the poles of $H_1(z)$

Since there are no zeros the magnitude response will be minimum when the product of the pole lengths is maximum and will be maximum when the product is minimum. From Fig. 2 the product is maximum when $\omega \approx 0.35\pi$ and maximum when $\omega \approx \pi$.

(e) The phase and magnitude response of $H_(z)$ using geometric arguments and by using `freqz` is shown in Figs. 4 and 5 respectively. The MATLAB code is shown below,

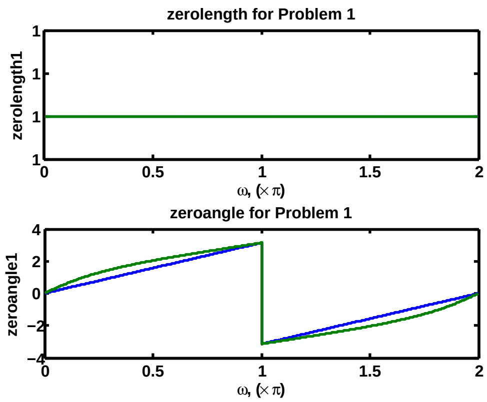

Figure 3: The magnitude and phase plots of the zeros of $H_1(z)$

```
geomH1mag = (zerolength1(1,:).*zerolength1(2,:))./(polelength1(1,:).*polelength1(2,:));
geomH1phase=ones(1,2)*zeroangle1-ones(1,2)*poleangle1;

subplot(2,1,1);
plot(omega/pi,20*log10(geomH1mag));

xlabel('Frequency (\times \pi) '); ylabel('Magnitude(dB)'); title('Magnitude response of H_1(z) -
 using Geometric Technique');
grid on;
subplot(2,1,2);
plot(omega/pi,unwrap(geomH1phase));
xlabel('Frequency  (\times \pi)'); ylabel('Phase(degrees)'); title('Phase response for H_1(z) -
 using Geometric Technique');
```

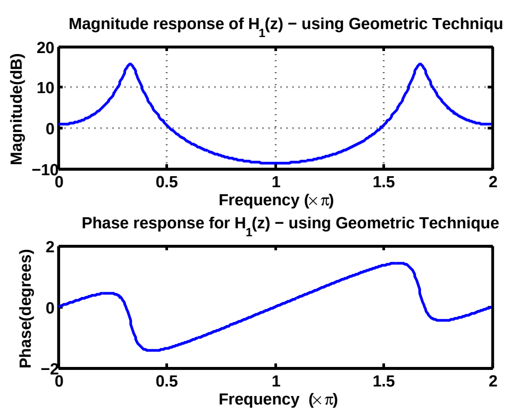

Figure 4: The magnitude and phase response $H_1(z)$ using geometric arguments

Problem 2

(a) The poles of and zeros of $H_2(z)$ are computed as shown below. The function `dpzplot` is used to plot the poles and zeros.

```
>> a2=[1 -0.9 0.81];

>> b2=[1 -0.5 0] ;
```

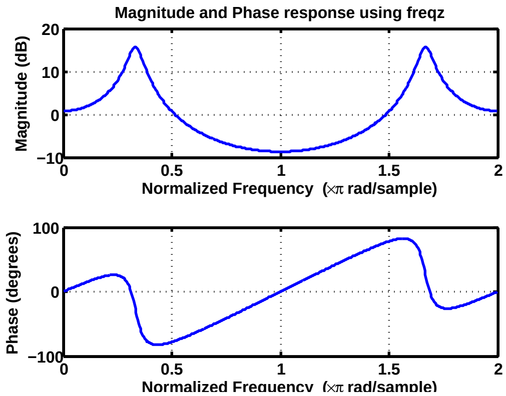

Figure 5: The magnitude and phase response $H_1(z)$ using freqz

```
>> dpzplot(b2,a2);

>> zs2 = roots(b2);
>> ps2 = roots(a2);
```

(b) The code for part (b) is on the website. The plots are shown in Figs 6 - 7 There is an additional zero at $z = 0$. The magnitude

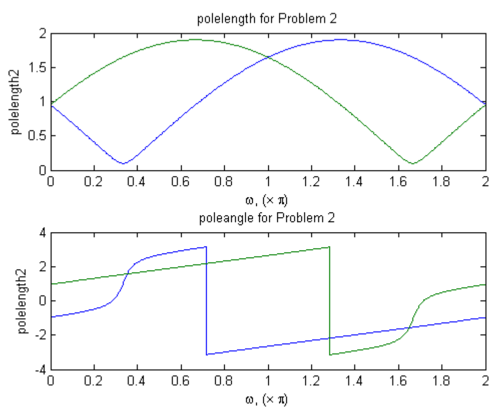

Figure 6: The magnitude and phase plots of the poles of $H_2(z)$

response will be lower at $\omega = 0$ compared to the magnitude response of $H_1(z)$ in problem 1. The zero length is maximum at $\omega = \pi$ which will cause the magnitude response to be higher at $\omega = \pi$.

(c) The phase and magnitude response of $H_2(z)$ using geometric arguments and by using `freqz` is shown in Figs. 8 and 9 respectively. As discussed in part (b) teh magnitude response is lower at $\omega = 0$ and higher at $\omega = \pi$.

Problem 3 The MATLAB code for this problem is on the website.

(a) The poles and zeros can be computed as shown below. The pole-zero plot is shown in Fig. 10. The magnitude of the zeros is four of the poles.

```
>> a3=[1 -sqrt(3)/2 1];
>> b3=[0.25 -sqrt(3)/2 1] ;
>> zs3 = roots(b3)
zs3=
```

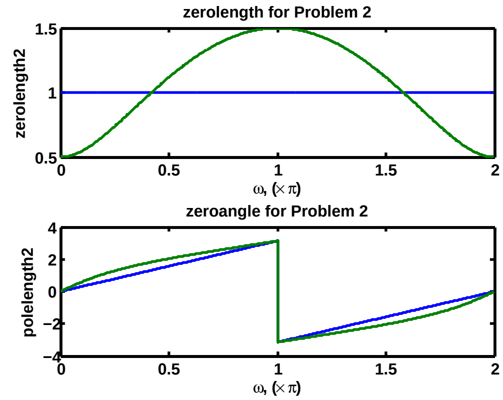

Figure 7: The magnitude and phase plots of the zeros of $H_2(z)$

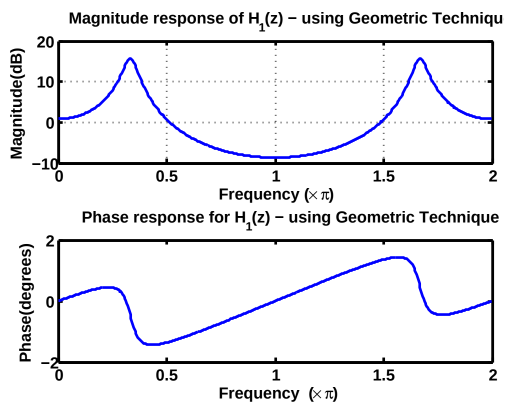

Figure 8: The magnitude and phase response $H_1(z)$ using geometric arguments

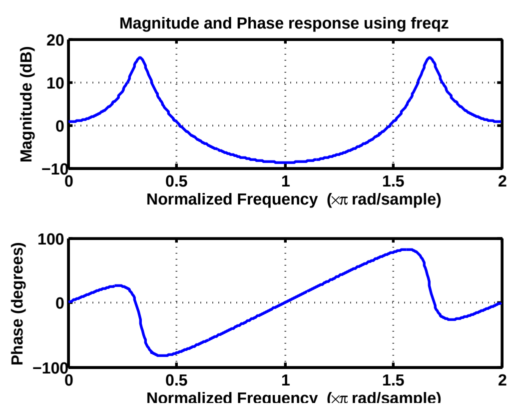

Figure 9: The magnitude and phase response $H_1(z)$ using freqz

```
   1.7321 + 1.0000i
   1.7321 - 1.0000i
>> ps3 = roots(a3)
ps3=
  0.4330 + 0.2500i
  0.4330 - 0.2500i
```

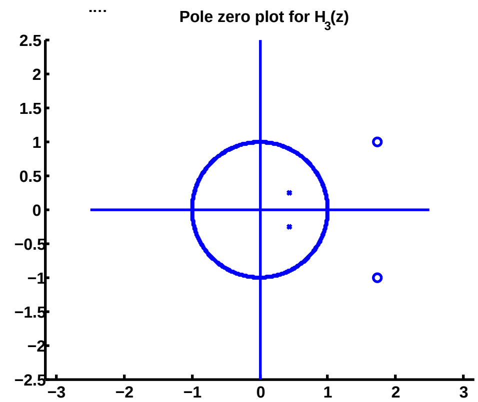

Figure 10: Pole-zero plot for Problem 3(a)

(b) The magnitude and phase of `polelength3` and `zerolength3` are shown in Figs. 11 and 12 respectively. The magnitudes of the poles and zeros looks similar but the vector representing the zero length has twice the length as the poles length for all frequencies. Also note that the gain of the filter is 4. This will make the magnitude response to be a constant at all frequencies.

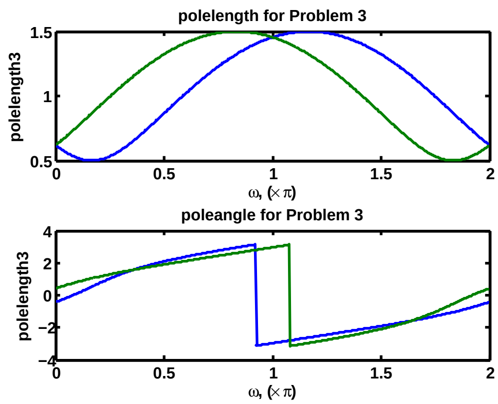

Figure 11: Pole length for Problem 3(b)

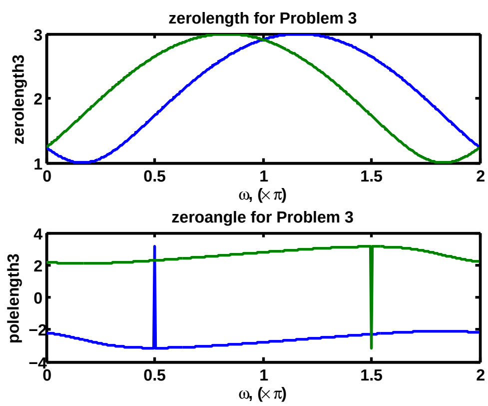

Figure 12: Zero length for Problem 3(b)

Problem 4

The code for this problem is on the course website. The plots are shown in Figs. 13 and 14 respectively.

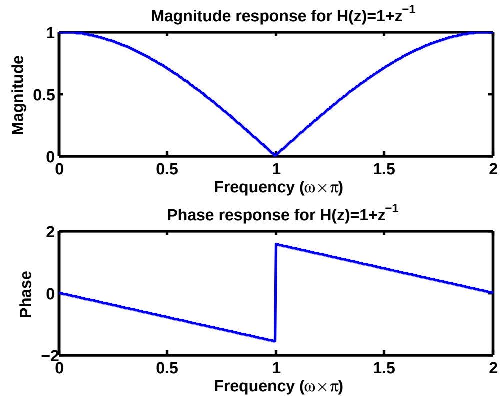

Figure 13: Magnitude and phase response for $H(z) = 1 + z^{-1}$

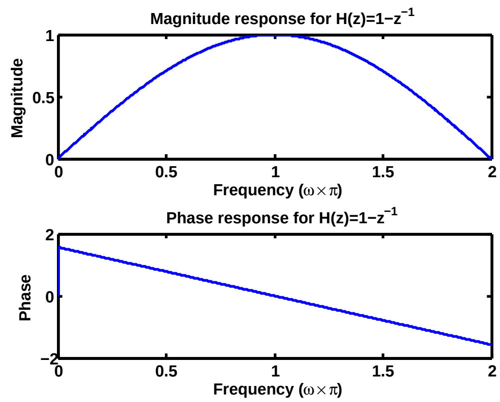

Figure 14: Magnitude and phase response for $H(z) = 1 - z^{-1}$