University of Illinois at Urbana-Champaign

ECE 311: Digital Signal Processing Lab

LAB 2: SOLUTIONS

Chandra Radhakrishnan *Peter Kairouz*

Problem 1

Refer to the MATLAB code for this problem

Problem 2

(a) The DTFT of the sequence is given by,

$$\begin{aligned} X_d(\omega) &= \sum_{-\infty}^{\infty} x[n]e^{-j\omega n} \\ &= \sum_{n=0}^{7} (0.7e^{-j\omega})^n \\ &= \frac{1-(0.7e^{-j\omega})^8}{1-e^{-j\omega})} \\ &= \frac{1-(0.7)^8 e^{-j8\omega}}{1-e^{-j\omega})} \end{aligned}$$

The magnitude and phase are shown in Fig. 1.

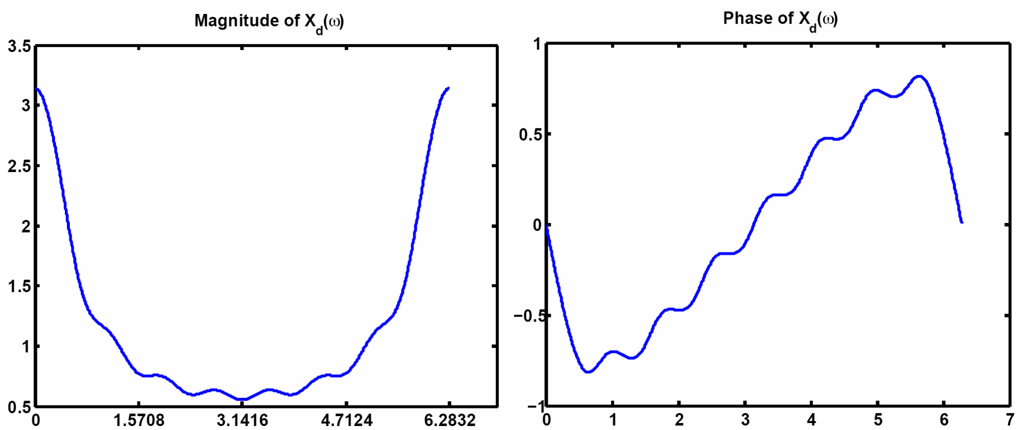

Figure 1: Problem 2 Part (a): Magnitude and Phase of $X_d(\omega)$

(b) The 8-point DFT of the sequence is shown in Fig. 2.

(c) The 16-point DFT of the sequence is shown in Fig. 3. Zero-padding sequences has the effect of sampling the DTFT at more number of points in the interval $(0, 2\pi)$. This improves the resolution of the spectrum and hence the DFT is now closer to the DTFT. However note that sampling at more number of points does not provide any new information compared to the DTFT.

(d) The 128-point DFT of the sequence is shown in Fig. 4.

(e) The plots from the DTFT of part (a) are equivalent to the 128-point DFT plots of part (d). For a length N DFT, the digital frequency $\omega = \frac{2\pi k}{N}$.

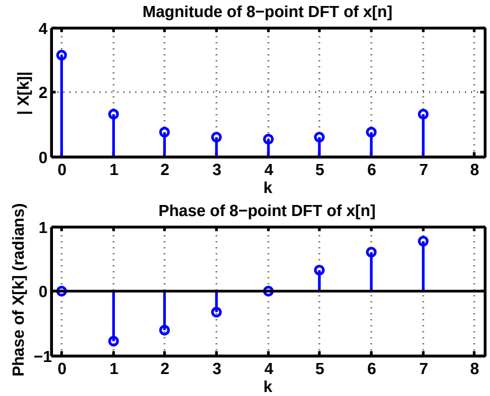

Figure 2: Problem 2 Part (b): 8-point DFT of the given sequence $x[n]$

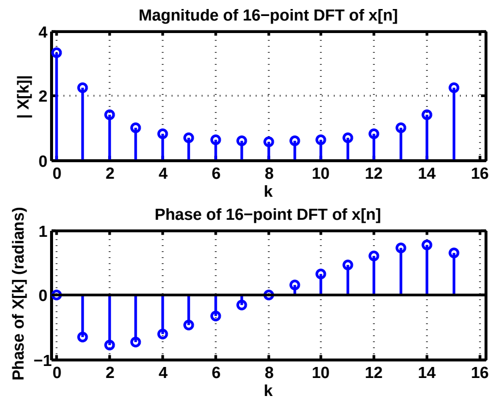

Figure 3: Problem 2 Part (c): 16-point DFT of the given sequence $x[n]$

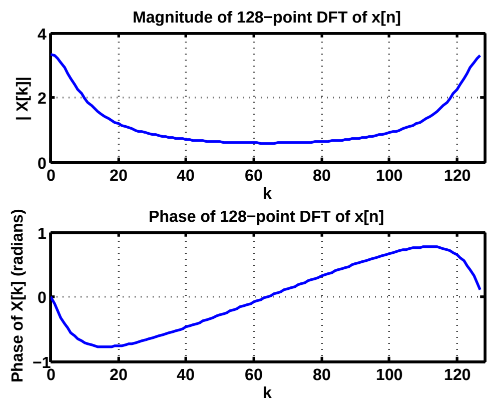

Figure 4: Problem 2 Part (d): 128-point DFT of the given sequence $x[n]$

Problem 3

(a) The result of convolving sequences $x[n], y[n]$ is $z[n] = x[n] * y[n] = [1, 2, 3, 4, 3, 2, 1]$. $z[n]$ is shown in Fig. 5.

(b) For two sequnces $x[n]$ and $y[n]$ for length M and N respectively, the circular convolution can be computed by "wrapping around" the samples that are obtained after time index $n = M - 1$ assuming $M \geq N$. Here $M = N = 4$. The linear

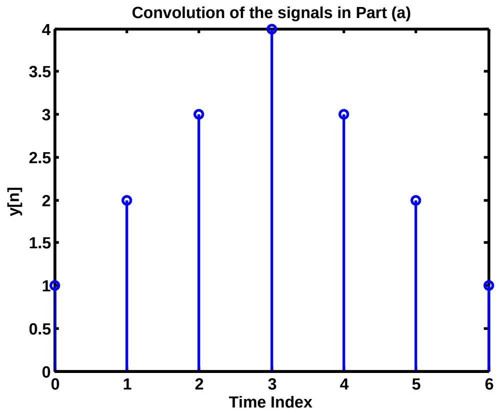

Figure 5: Problem 3 Part (a): Convolution of $x[n], y[n]$ in Part (a)

convolution of $x[n]$ and $y[n]$ in part (a) was computed to be,

$$z[n] = x[n] * y[n] = [1 \quad 2 \quad 3 \quad 4 \quad 3 \quad 2 \quad 1]$$

"Wrapping around" the samples after time index $M - 1 = 3$ we have,

$$z_c[n] = [4 \quad 4 \quad 4 \quad 4]$$

(c) Refer to the MATLAB code attached. Note carrying out the procedure in Part (b) implements the circular convolution of the two sequences.

(d) Zero- padding the input by one zero increases their length by 1. In this case the "wrapping" occurs after time index M.Zero-padding the inputs by 1 zero results in the following output,

$$z_c[n] = [3 \quad 3 \quad 3 \quad 4 \quad 3]$$

Now increase the length by two by adding two zeros. The "wrap" around occurs after time index $M+1$. The result is shown below,

$$z_c[n] = [2 \quad 2 \quad 3 \quad 4 \quad 3 \quad 2]$$

Finally adding one more zero yields an output which matches the linearly convolved output. Note that the result of linear convolution of two vectors $x[n]$ and $y[n]$ of lengths M and N respectively results in an output vector of length $M + N - 1$. Thus, we should zero-pad in a way that makes the length of each of the two vectors equal to $N + M - 1$. Hence $x[n]$ must be zero-padded by $N - 1$ and $y[n]$ by $M - 1$ respectively.

(e) Note that the vectors are of different length. The result of linear convolution is given below and is also shown in Fig. 6,

$$z_c[n] = [-3 \quad 2 \quad 1 \quad 28 \quad 54 \quad 48 \quad 38 \quad 20 \quad 4]$$

First we add zeros to $y[n]$ to make its length equal to $x[n]$. The circular convolution can now be done using the method similar to Part (b). In this case the wrap around occurs after time index $M - 1 = 6$. The result of circular convolution is shown below,

$$z_c[n] = [35 \quad 22 \quad 5 \quad 28 \quad 54 \quad 48]$$

Part (c)-(d) can be done similar to the earlier case. Note that you need to add 3 zeros to $x[n]$ and 5 zeros to $y[n]$ in order to get the result of linear convolution. The given MATLAB code can be used to implement transform domain convolution.

Problem 4 The file "tones.mat" has three frequencies in it. The signal is a sum of three tones shown below,

$$y1 = cos\left(\frac{\pi n}{4}\right) + 0.5cos\left(\frac{\pi n}{6}\right) + 0.5cos\left(\frac{\pi n}{8}\right)$$

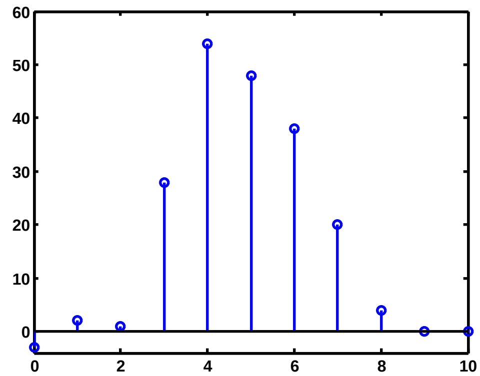

Figure 6: Problem 3 Part (e): Convolution of $x[n], y[n]$ in Part (e)

(a) First we use load the file and plot the spectrum with $N = 25$ DFT. The result is shown in Fig. 7. Taking $N = 25$ point DFT does not provide enough resolution to see the frequencies clearly.

(b) Figure 8 shows the result with $N = 35$ point DFT, we can now distinguish two frequencies. Finally we take $N = 100$ point DFT which clearly shows the three frequencies (Fig. 9).

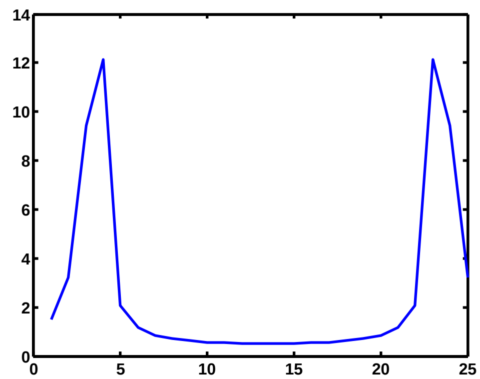

Figure 7: Problem 4: $N = 25$ point DFT of the signal.

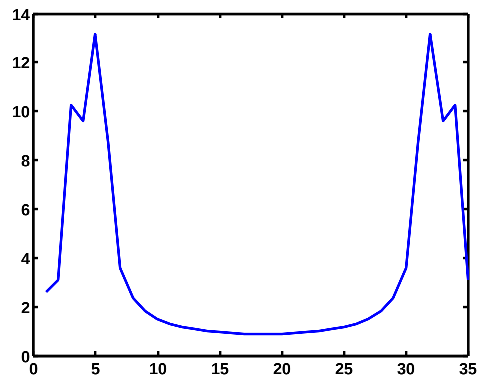

Figure 8: Problem 4: $N = 35$ point DFT of the signal.

Problem 5

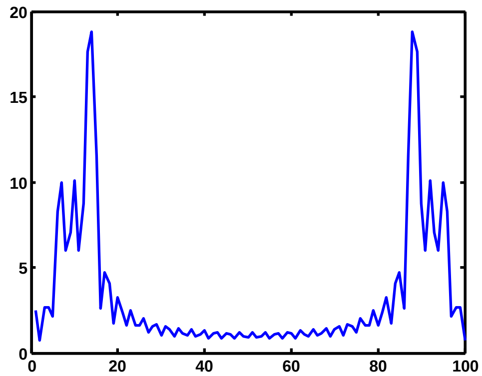

Figure 9: Problem 4: $N = 100$ point DFT of the signal.

(a) The DTFT of rectangular window spanning from 0 to $2L$ is:

$$\frac{sin\left(\frac{2L+1}{2}\right)\omega}{sin\frac{\omega}{2}}e^{-j\omega L}$$

The DFT is calculated by sampling the DTFT at $\omega = \frac{2\pi k}{M}$. Here $L = 150$ and the spectrum is sampled at $M = 900$ and $M = 1800$ points. First consider $M = 900$. The DFT of the windows $w_1[n]$ and $w_2[n]$ is given by,

$$\begin{aligned} W_1[k] &= \frac{sin\left(\frac{300}{2}\right)\frac{2\pi k}{900}}{sin\left(\frac{2\pi k}{2*900}\right)}.e^{-j2\pi k150/900} \\ &= \frac{sin\frac{\pi k}{3}}{sin\frac{\pi k}{2*900}}.e^{-j\pi k/3} \\ W_2[k] &= \frac{sin\frac{2\pi k}{3}}{sin\frac{2\pi k}{2*900}}.e^{-j2\pi k/3} \end{aligned}$$

Now $\frac{2\pi}{3} = \pi - \frac{\pi}{3}$, hence $W_1[k]$ and $W_2[k]$ are similar when $M = 900$ point DFT is computed since we are sampling two different spectrums at the same points. This makes the spectra look similar. We resolve this by sampling faster, i.e. choosing $M = 1800$. Figures 10 and 11 illustrate this. Note in Figure 11 the differences between the spectra can be clearly seen.

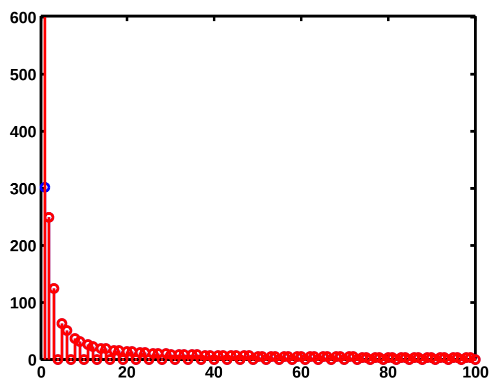

Figure 10: Problem 5 Part (a): DFT of the window functions $w_1[n]$ and $w_2[n]$ for $M = 900$.

(b) When $x[n]$ is multiplied by the window $w_1[n]$, we have two frequency components at 100 Hz and 102 Hz. Windowing in the time domain is equivalent to convolving with the sinc function computed in Part (a) and hence the frequency resolution corresponding to the window is $\left(\frac{2}{300}\right)F_s = 6.6$ Hz. As shown in Fig. 12, the frequencies could not be resolved using this

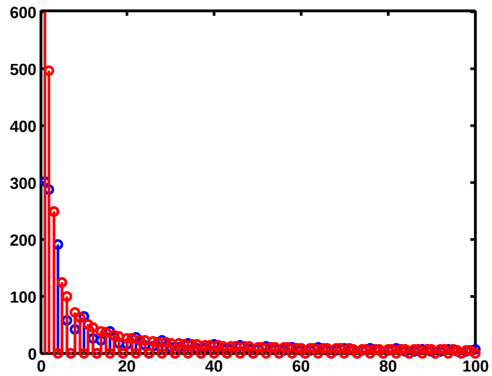

Figure 11: Problem 5 Part (a): DFT of the window functions $w_1[n]$ and $w_2[n]$ for $M = 1800$.

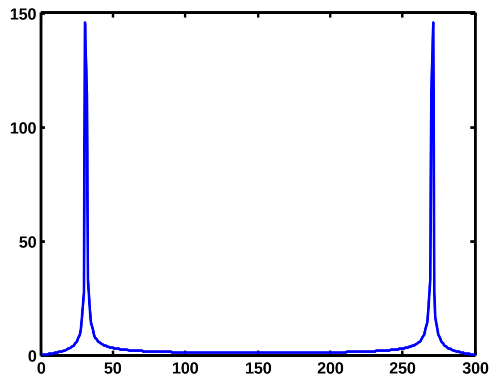

Figure 12: Problem 5 Part (b): DFT of $x_1[n] = x[n]w_1[n]$

window function and they are merged into one peak. (Note: Multiplying by window function $w_1[n]$ is equivalent to taking the first 300 samples of $x[n]$) Choosing a window of $w_1[n-300]$, we are capturing the signal with the frequency content 103 Hz and 105 Hz. As we saw earlier the frequency resolution is 6.6 Hz. Hence these frequencies also cannot be distinguished. The DFT of the signal $x[n]w_1[n-300]$ is shown in Fig. 13.

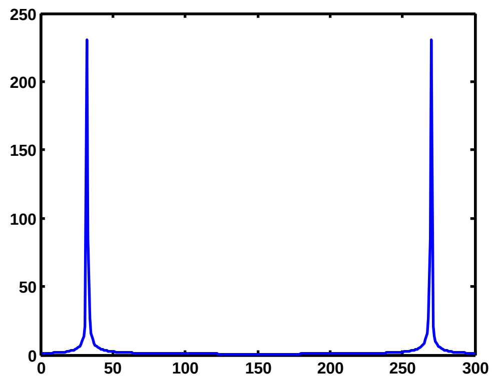

Figure 13: Problem 5 Part (b): DFT of $x_2[n] = x[n]w_1[n-300]$

(c) Now we use a window $w_2[n]$ to perform spectral analysis. Increasing the window size does not improve the resolution because the signal changes its frequency content during this period. The DFT is shown in Fig. 14

Problem 6 This problem makes use of the `spectrogram` function to get the spectrum of the speech signal in `mtlb` file. Note that

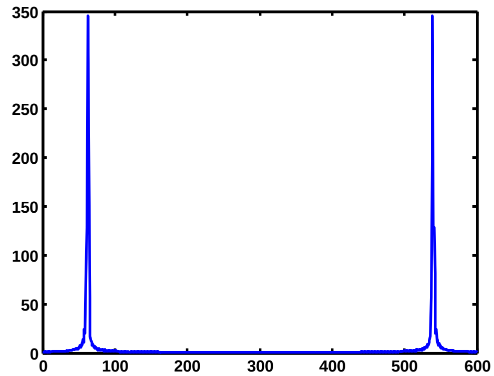

Figure 14: Problem 5 Part (c): DFT of $x_3[n] = x[n]w_3[n]$

all the parameters required for the function are given in the problem. Fig. 15 shows the spectrum of the speech signal. It can be seen from Fig. 15 that a speech signal could take one of two forms on short windows of time (30 ms): the fricative form, where the frequency content of the signal spans the entire frequency spectrum during that window of time (see the interval between sample number 2000 to sample number 2500 in the speech waveform) and the periodic form, where the frequency content looks very similar to that of a single tone signal during that window of time (see the interval from sample number 500 to sample number 2000 of the speech waveform). This is evident from the time and frequency spectrum of the signal where the single tone signal can be clearly seen in the interval corresponding to samples 500-2000 and the speech signal with the frequency content spanning the entire range can be seen in the interval corresponding to samples 2000-2500 in the spectrogram.

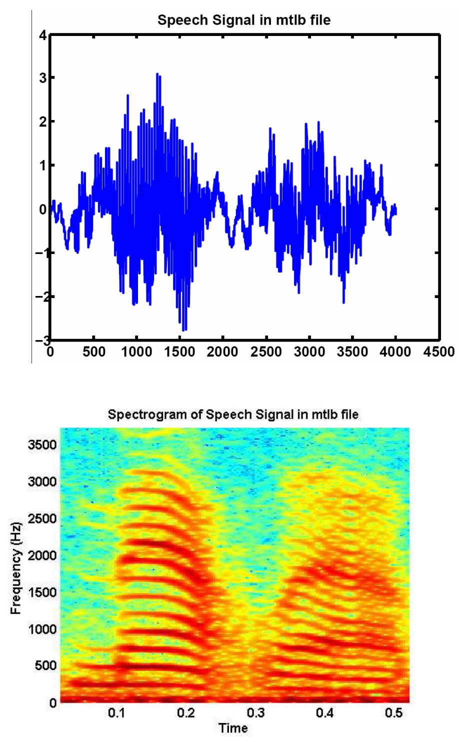

Figure 15: Problem 6 Spectrogram of the speech signal.