Abstract

In the 2022 world cup final against Argentina, Didier Deschamps made the controversial decision to substitute Giroud and Dembele just 40 minutes into the game. Although France later tied the game, was this due to luck or did these substitutions significantly increase the probability of a comeback for France?

To address this, one must be able to quantify the likelihood of scoring based on the players on the field, considering the impact of fatigue on the starters versus the fresh legs of the substitutes.To investigate such scenarios, we use historical event data from the English Premier League to analyze and predict goal-scoring dynamics. Using publicly available expected goal (xG) data, we estimate a teams xG rate per minute as a function of the game state G. This rate function $\lambda(G)$ is modeled to vary based on both player-specific contributions and the current game state.

We show that given $\lambda(G)$, a team's next goal timing closely follow a weibull distribution. To account for competition between teams, we model the joint distribution of goal scoring timings using a gaussian copula. We then show that simulating from this distribution can predict the game outcomes in an accurate and calibrated manner.

Using this model, we can evaluate the impact of substitutions on a team's win probability. This approach not only helps determine whether Didier Deschamps made a wise substitution but also provides a framework for assessing managerial decision-making as a whole. Overall this model can help assess the risk and reward of different tactical decisions on win probability, providing a tool for optimizing team performance and strategy throughout the match.

Introduction

Soccer, like all sports, is highly context-dependent, requiring decisions to be made with this context in mind. A game state G encompasses this context and is defined as the set of all relevant information—such as score differential, players on the field, and other situational factors—at any given time point. A natural question arises: how does G impact a team's probability of scoring? Unfortunately, attempts to model this directly often face challenges due to the scarcity of data, as there are, on average, only two goals scored per match.

While individual goal-scoring events are rare, shots are much more common. For any given shot s, let $p(s)$ denote the probability that the shot results in a goal. This probability, $p(s)$, is also known as a shot's expected goals ($xG(s)$). Estimates of expected goals, denoted $\hat{xG}$, are provided by sources like WorldfootballR. For the purposes of this report, we assume that $\hat{xG}(s) = xG(s)$ for all shots s. Given the prevalence of shots and their associated xG values, we can build a model to predict a team's goal-scoring rate λ as a function of the game state G.

Considerable work has been done on modeling goal-scoring dynamics across various sports. Holmes et al.[4] utilized player ratings from WhoScored for both starters and substitutes, estimating the distribution of goal differentials using a Skellam distribution. However, they assumed that goal-scoring rates are independent between teams, a premise that has been shown to be false [2]. They also assumed that, conditional on the players, goal-scoring rates remain constant.

Merritt et al.[6] discovered that scoring rates in professional sports tend to increase sharply near the end of time periods, such as quarters or halves. They demonstrated that goal-scoring events could be well approximated by a homogeneous Poisson process and that these events are largely memoryless across periods. They also noted that sports with fewer goals, such as hockey—similar to soccer—exhibit underdispersion relative to what is expected in a Poisson process.

Boshnakov et al.[1] modeled the time between goal-scoring events in soccer using a Weibull distribution, rather than the exponential distribution associated with a homogeneous Poisson process. The Weibull distribution is widely used in time-to-event analysis due to its flexibility in modeling both over- and underdispersion. To account for the dependence between teams in goal-scoring times, they used a copula. However, their model assumed a constant scoring rate for each team throughout the game.

A critical aspect that these papers overlook is the dynamic nature of goal-scoring rates throughout a game. Factors such as substitutions, strategic adjustments at halftime, and extra time cause the game state—and consequently, the goal-scoring rate—to change continuously. Therefore, it is essential to develop a model that can account for these adjustments when estimating win probability.

In this report, we present interpretable models for simulating soccer matches. Using English Premier League shot and event data from 2015 to 2022 as well as FIFA player ratings to proxy each team's offensive and defensive skill, we assess the impact of factors such as home-field advantage, extra time, score differential, and fatigue on a team's goal rate. Modeling the time between goals using a Weibull distribution and modelling the joint distribution of these times between teams using a gaussian copula,we demonstrate that game simulations based on these models align closely with actual game outcomes and are well-calibrated. The report concludes by assessing the effectiveness of EPL managers' substitutions during the 2017-2018 season as well as Didier Deschamps' controversial 40th minute substitution of Dembele and Giroud in the 2022 World Cup final.

Methods

Datasets

We use four datasets to conduct all analyses in this report. The first two datasets provide information on match summaries and match events for all English Premier League (EPL) matches from 2015 to 2022. These datasets can be linked using a common game ID. The match summary (MS) dataset includes team rosters, jersey numbers, and game dates. The match events (ME) dataset contains a list of events for each game, which are parsed to identify the players and teams involved in substitutions, goals, and ejections. Both datasets are available at this Kaggle link.

We use WorldFootbalR's `load_understat_league_shots` function to obtain xG estimates and game information for all shots taken in the EPL between 2015 and 2022. This data is then joined with the MS dataset using team names and game dates to associate each shot with a specific game ID.

Additionally, we have access to FIFA ratings for all players across the five major soccer leagues from FIFA 15 to FIFA 22. This dataset includes player names and ratings for all positions and can be found here. We match each player name in the MS dataset to a corresponding player ID in the FIFA dataset.

In summary, we have 2600 games of EPL data from 2015 to 2022. For each game, we have access to all events, the xG of each shot, the players on the field at all times along with their FIFA ratings, and other relevant game state information. The process for merging datasets can be found in 'data_cleaning_and_wrangling.ipynb'.

XG Rate Estimation

We aim to estimate a team's goal-scoring rate as a function of the game state G. Understanding this relationship will allow us to quantify how various changes in the game state affect a team's ability to score. Generally, the primary factors influencing a team's scoring rate are its offensive capability and the defensive strength of the opponent. To estimate these abilities, we use player ratings from the FIFA video game as a proxy.

Specifically, we categorize positions as either offensive or defensive. Offensive positions include striker, left wing, right wing, center forward, left attacking midfielder, center attacking midfielder, and right attacking midfielder. Defensive positions include left back, right back, center back, left defensive midfielder, right defensive midfielder, central defensive midfielder, left wing back, and right wing back. A player's offensive rating is defined as the maximum rating across all offensive positions, while their defensive rating is defined similarly for defensive positions. For a team, the overall offensive and defensive ratings are calculated by summing the individual ratings of all players on the field, excluding the goalkeeper.

To measure fatigue, we calculate the average minutes played by all players on the field. For a given team, we denote this average as M. The fatigue difference between a team and its opponent is given by $M - M_o$, where M_o represents the opponent's fatigue. Positive values indicate that the team is less rested than its opponent.

Based on the plots below, we observe a linear relationship between the goal-scoring rate and each of the variables. When considering time-varying covariates, such as minutes left in the half, we rely on previous studies that found goal-scoring rates tend to remain constant throughout the game, except at the end of periods. To test this, we plotted the empirical CDF of goal-scoring times as seen in figure 1. If goal-scoring rates varied significantly with time, we would expect a non-linear CDF. However, the CDF appears linear, with a slope change at the 45-minute mark, indicating a change between halves. This suggests no significant time-dependent effect outside of the end of each half, consistent with previous findings [6].

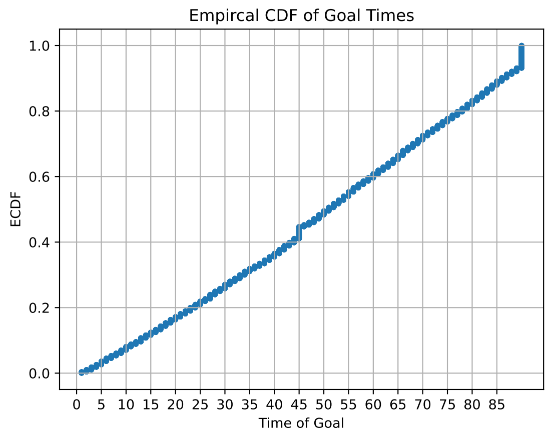

Figure 1: Empirical CDF of Goal Timings Across EPL Games Between 2015 and 2022

The following covariates are included in our model:

- Team offensive rating (Continuous)
- Opponent defensive rating (Continuous)
- Half (Binary)
- Extra time (Binary)
- Fatigue (Continuous)
- Home-away status (Binary)
- Score differential (Factor with levels "Down by 3+", "Down by 2", "Down by 1", "Tied", "Up 1", "Up 2", "Up 3+". Baseline level is "Down by 1".)

For a period i of constant game state lasting $|t|$ minutes, let Y_i represent the amount of xG (expected goals) scored by a team. The expected goal rate is assumed to be linear in the coefficients representing the game state:

$$\lambda(G) = x_i^T \beta$$

where x_i is the vector representing the covariates listed above. Assuming that the expected goals scored in disjoint time periods are conditionally independent given the game state, we have:

$$\mathbb{E}[Y_i|x_i] = |t| x_i^T \beta$$
$$\mathbb{V}[Y_i|x_i] = |t| \sigma^2$$

where σ^2 is the variance for a period of length 1 minute. Therefore,

$$\frac{Y_i}{|t|} \sim \left[x_i^T \beta, \frac{\sigma^2}{|t|} \right]$$

This model can be fitted using a weighted least squares approach, with each observation's weight proportional to its length. Although the data is not normally distributed, the weighted least squares estimator is still the best linear unbiased estimator (BLUE), meaning its estimates have the lowest standard error among all unbiased estimators. Consequently, we fit β_G using weighted linear regression and perform bootstrap sampling to generate standard errors for the coefficients. Using a training set of 80% of games and aggregating the test set over each team per season, we find that the residuals between a team's xG and the predicted values based on the game state exhibit normal behavior with relative homoskedasticity as seen in figures 2-4. However, it is worth noting that teams expected to score very few or very many xG tend to underperform or overperform, respectively.

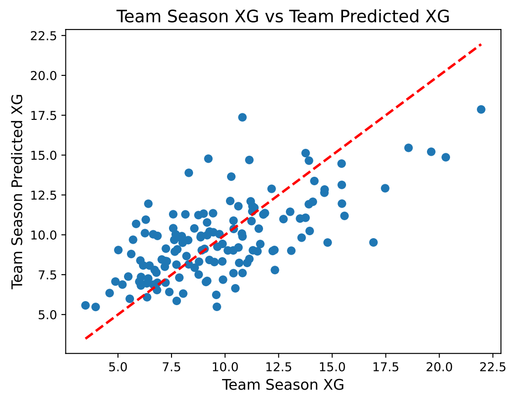

Figure 2: Model Predicted xG vs Observed xG on Test Set

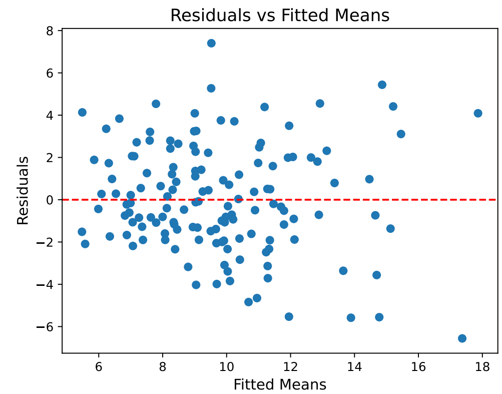

Figure 3: Model Residuals on Test Set

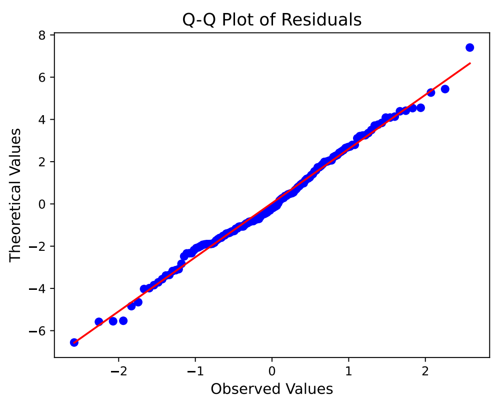

Figure 4: Model QQ plot

After refitting the model on the entire dataset, we obtain the following estimates and confidence intervals for the coefficients:

From table 1,we observe that the most significant factors influencing a team's scoring rate are the skill levels of the players on the field. As expected, home field advantage has a positive effect on scoring, equivalent to increasing the FIFA rating of each player on the home team by 3 points. The effects of extra time and the second half are also substantial, likely due to both teams increasing their aggression during these periods. Interestingly, a team's goal-scoring rate tends to decrease when they are behind, possibly due to the pressure to score leading to forced plays. However, this effect is only significant when a team is down by 2 or more goals. Finally, as expected, fatigue has a negative effect as players who are more tired tend to diminish in performance. Strictly using a team's own expected goals as a metric, this implies that a substitute who is one FIFA point worse on offense becomes preferable to the starter just 17.6 minutes into the game, further implying that a half's worth of fatigue is equal to a 2.5 point reduction in offensive performance.Similarly, using the opponent's goal rate as a metric, a half's worth of playing is approximately equal to a 3.5 point reduction in defensive performance. We more rigorously investigate the minimum time at which a substitution has a beneficial impact on win rate in the 'Results' section. Code for performing

Covariate	**Estimate (per 90 minutes)**	**2.5% Quantile**	**97.5% Quantile**
Offensive Rating	0.763 per 100	0.713 per 100	0.814 per 100
Opp. Defensive Rating	-0.581 per 100	-0.640 per 100	-0.523 per 100
Home Field Advantage	0.229	0.193	0.264
2nd Half	0.178	0.138	0.216
Extra Time	0.173	0.103	0.241
Fatigue	-0.00484	-0.00901	-0.00615
Score Differential (Up 3+)	0.0931	-0.103	0.292
Score Differential (Up 2)	0.161	0.0677	0.257
Score Differential (Up 1)	0.0347	-0.0262	0.0957
Score Differential (Tied)	-0.0148	-0.0634	0.0326
Score Differential (Down 2)	-0.0637	-0.139	0.0113
Score Differential (Down 3+)	-0.156	-0.258	-0.0472

Table 1: Estimates and Confidence Intervals for Each Coefficient

this analysis can be found in 'xg_prediction.ipynb' and 'goal_timing_eda.ipynb'.

Goal Timing Distribution

Marginal Distribution

Motivated by prior work [1], we hypothesize that the distribution of inter-arrival times follows a weibull distribution. Given a team's expected goal rate $\lambda(G)$, let T be the time until the next goal. We let

$$T \sim W(\frac{1}{\lambda(G)}, k)$$

Where $W(\mu, k)$ denotes a weibull distribution with mean μ and shape parameter k. The weibull is widely used in time to event analysis due it's ability to model processes where occurrence changes as a function of time. For $k > 1$, this means that the tails of the arrival time distribution are shorter than that of the exponential. For $k < 1$ the opposite is true. The exponential is a special case of the weibull when $k = 1$.

To estimate the shape parameter k, we focus on the first goal occurrence within the first 45 minutes of each game. This restriction is necessary because the goal-scoring rate typically changes in the second half and during extra time, which could skew the analysis. Additionally, most substitutions occur in the second half, so the first 45 minutes generally reflect a relatively constant game state and, therefore, a constant $\lambda(G)$. Via grid search, we find the maximum likelihood estimator of k to be 1.0853. From figures 5 and 6 we see from the hazard curve that the weibull distribution with $k = 1.0853$ is a very good fit whereas the exponential distribution significantly underestimates the survival proportion at all times.

Code for this analysis can be found in 'hazard_function.ipynb'.

Joint Distribution

It is intuitively obvious and widely accepted that there exists some form of dependence between teams that are competing against each other. Dixon et al[2] found empirical evidence of this by comparing the joint distribution of goals and that under an independence assumption. In order to model the dependence between competing teams we use a copula.

A copula is a useful way of modelling the joint distribution of random variables when given their marginal distributions. For uniformly distributed random variables U_1, U_2, a copula C specifies a joint cumulative distribution function over U_1 and U_2.

In the context of goal timings, let T_H and T_A be the time until the next goal for the home and away team respectively. From the section above,we assume that T_H and T_A follow some kind of weibull distribution. Applying the probability integral transform to T_H and T_A gives us U_H and U_A which are marginally uniform. Therefore if we can sample from the joint distribution of U_H and U_A, we can sample from the joint distribution

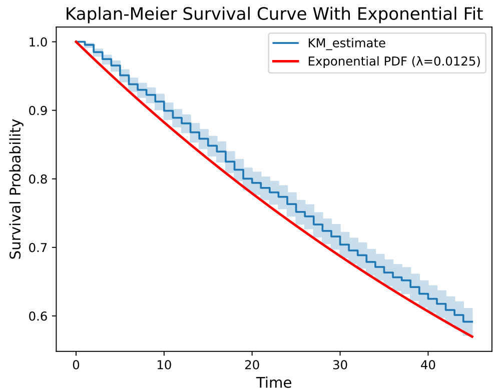

Figure 5: First Goal Kaplan-Meier Curve with Exponential Fit

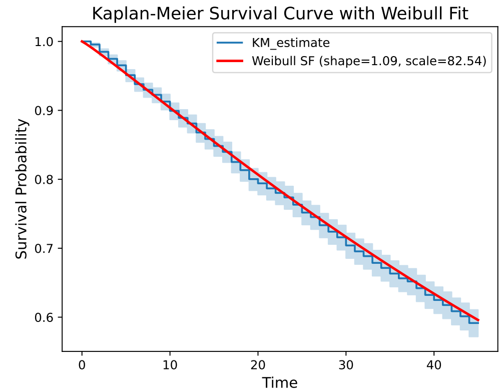

Figure 6: First Goal Kaplan-Meier Curve with Weibull Fit

of T_H and T_A.

Given a copula C, we can do exactly this as it specifies the joint distribution of U_H and U_A. Therefore finding the copula is equivalent to specifying the joint distribution between T_H and T_A. A formal proof of this fact exists in Sklar's theorem[7].

We model the joint distribution of T_H and T_A using a gaussian copula, that is that $(\Phi^{-1}(U_H), \Phi^{-1}(U_A))$ follows a bivariate normal distribution with correlation coefficient ρ. We expect ρ to be negative in general because teams are competitive rather than cooperative. Based on previous work detailing changes in game state dynamics based on whether a game is close or not[5], we fit a separate ρ for each half/extra time combination as well as if the game is tied or not. We find the maximum likelihood estimator of ρ in each case via binary search. A summary of our results are below.

Score Differential	Half	Extra Time	Correlation Estimate
Tied	1	0	-0.330
Tied	1	1	-0.436
Tied	2	0	-0.534
Tied	2	1	-0.780
Not Tied	1	0	-0.443
Not Tied	1	1	-0.795
Not Tied	2	0	-0.689
Not Tied	2	1	-0.865

Table 2: Correlation Estimates for Different Score Differentials, Halves, and Extra Time Scenarios

Table 2 shows a consistent pattern of correlations being negative which is to be expected in a competitive game. We also see that the magnitude of correlation increases in scenarios where teams grow more aggressive, that is during extra time and when the game is not tied as one team must push the pace due to needing a goal.

With the copula specified, we can now simulate from T_H and T_A at any point of any game. Code for this analysis can be found in 'copula.ipynb'.

Simulations

We use our copula and xG estimating function to simulate an entire match of soccer. We use the initial goal scoring rate estimates for both teams at the start of each game and draw samples from T_H and T_A until the game ends. We update the scoring rates if a team scores, extra time begins, or the half changes. For our simulations we will assume no substitutions occur. We partition the set of scoring rates between home teams and away team into 20 bins and simulate 2000 games between each combination of rates for a total of

400 simulations. For each game, we gather the initial scoring rates based on the xG rate model and match these rates to the closest simulation. Using this method we see that the actual results over 2600 games aligns closely with what is expected given the simulations as seen in figure 7. A Pearson goodness of fit test gives a pvalue of 0.498, suggesting that real game results follow the expected results closely. From figure 8 we also see that the simulations are well calibrated except for when a team is a large favorite or underdog. In this case, the favorites overperform.

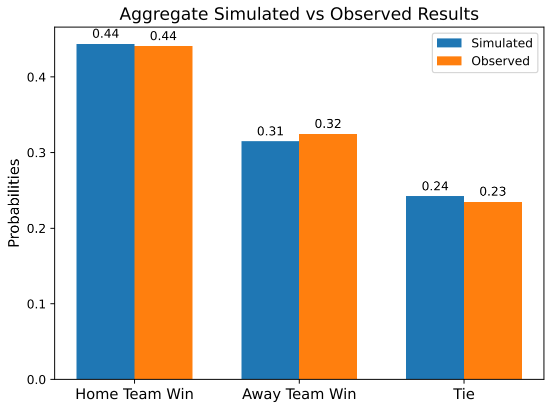

Figure 7: Simulation Win-Loss-Tie Proportions vs Observed

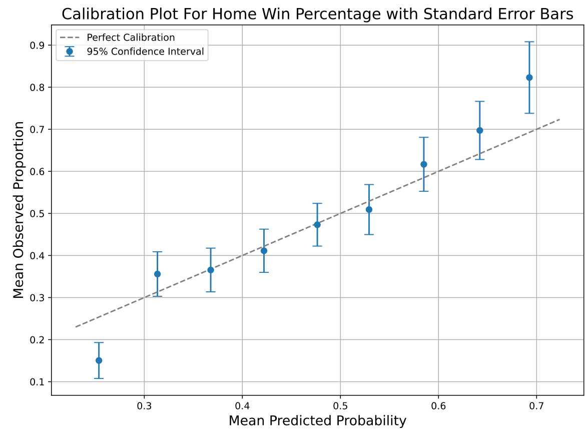

Figure 8: Calibration Plot for Home Teams

Code for this analysis can be found in 'copula.ipynb'.

Results

Evaluating EPL Manager Substitutions

While managers influence the game in many ways, one of the most direct ways they can influence the game is through substitutions. We analyse the substitutions made during the 2017-2018 EPL season for all clubs in order to evaluate which managers made the biggest differences through substitutions.

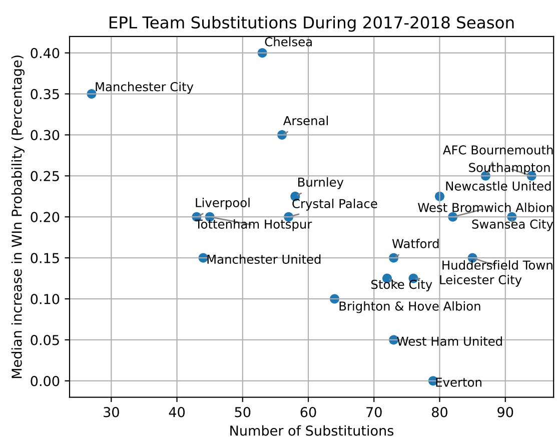

Figure 9: Median Effect of Substitutions on Win Probability Across Teams

As illustrated in figure 9, some of the top three clubs in terms of median value added are Arsenal, Chelsea, and Manchester City, led by Arsène Wenger, Antonio Conte, and Pep Guardiola respectively. Considering that these managers are widely recognized as three of the best in the world, these findings are to be expected. However, it is important to acknowledge that managers influence the game in various ways beyond making substitutions, so this chart should not be seen as a criticism of anyone. Code for this analysis can be found in 'substitution_analysis_function.ipynb'.

Optimal Substitution Time

With soccer being a game decided by which team scores the most goals, it is reasonable to deem substitutions that increase a team's xG rate or decreases the opponent's xG rate as "good" [3]. Substitution affects the game state by increasing/decreasing the total offensive and defensive ratings for the team as well as lowering the fatigue for the team overall. Let $\Delta O, \Delta D$ and ΔT be the change in offensive rating, defensive rating, and fatigue that the substitution causes. Then the overall effect of the substitution on the goal rate is

$$\Delta G = \Delta O\beta_O - \Delta D\beta_D - 2\Delta T\beta_F$$

where β_O, β_D, and β_F are the effects of team offense, team defense, and team fatigue. A decision rule based on ΔG being positive or not corresponds to the notion that a substitution is preferable when the net change in xg rate is to the substituting teams favor. Given that overall player ability is immutable within game, the only variable that a manager can affect is ΔT, that is the time of the substitution. Define the break even time of a substitution to be the minimum change in fatigue needed to make ΔG 0. Assume that the substituted player has played the entire game up until that point and that no ejections have occurred.Then the time at which which making a substitution is break even to the overall goal scoring rate is

$$T^\star = 11(\frac{\Delta O\beta_O - \Delta D\beta_D}{2\beta_F})$$

While the ΔG heuristic can be helpful for determining when to make a substitution, it does not take into account whether the substituting team is incentivized to score or to stop the opponent from scoring.

To investigate this, employ a simulation where both teams have the same scoring rate of 1.125 goals/90 which is the average over all teams. Consider making a substitution where the substitute is K_O FIFA points worse on offense and K_D FIFA points worse on defense. We ask the question "When is the earliest time $t^\star$ such that making the substitution at $t^\star$ increases the probability of winning?". We investigate this across 3 different score differentials,the substituting being down 1, tied, and up 1. We also restrict one of K_O and K_D to be 0 which allows us to answer the question "how many minutes are K FIFA points worth?". The results after simulating 2000 games under each scenario are below.

Offensive FIFA Points Lost	**10**	**9**	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**
Down 1	>90	>90	>90	>90	> 90	67	45	45	29	6
Tied	85	85	73	63	56	51	34	30	20	14
Up 1	47	45	45	44	28	27	19	18	12	9
ΔG **Heuristic**	88	80	71	62	53	44	36	27	18	9

Table 3: Minimum time at which making a substitution improves a team's win probability when substitute is worse on offense and equal on defense

As seen in Tables 3 and 4, when a team is leading by one goal, a "park the bus" strategy is most effective. Once a team takes the lead, it becomes immediately advantageous to substitute in defensive players to minimize the opponent's chances of scoring.This is evident as it becomes acceptable to substitute a player who loses 10 offensive points just 47 minutes into the game, whereas it is never advisable to substitute a player who loses 10 defensive points. When a team is trailing by one goal, the opposite is true: losing 10 defensive points is justifiable after only 32 minutes, whereas substituting a player who loses 10 offensive points is never advisable. When the game is tied, there is no clear consensus on which statistic is more

Defensive FIFA Points Lost	**10**	**9**	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**
Down 1	32	31	31	31	31	29	19	15	12	7
Tied	60	60	57	39	35	35	22	22	16	7
Up 1	>90	>90	82	81	72	51	37	30	17	9
ΔG **Heuristic**	67	61	54	47	41	34	27	21	14	7

Table 4: Minimum time at which making a substitution improves a team's win probability when substitute is worse on defense and equal on offense

valuable, as both offensive and defensive FIFA stats have significant impacts depending on the situation. In this case, the minimum time aligns closely with that offered by the ΔG heuristic indicating that when tied, a team should primarily focus on maximizing the difference in long run goals scored between the two teams. These simulations shed light into substitution strategies when both teams are equally skilled. One can imagine that these times may differ based on the opponent's offensive output. Because we are able to simulate game outcomes from any game state, to investigate these scenarios we can simply tune the relevant parameters and simulate, allowing for the assessment of whether a substitution would be beneficial at any stage of the game. These simulations highlight the scarcity of goals in soccer and the importance of adjusting strategies according to the game's current state. They also stress that substitution decisions should be dynamic and adaptive. Given that most substitutions in the EPL are made around the 80th minute, there is considerable potential for improving this aspect of strategy. Note that each scenario was simulated 3000 times and that for scenarios in which the win probability is small (e.g. down 1 at the 80th minute mark), 3000 simulations may not be enough to estimate the small probability that the team wins. However, in these scenarios, a substitution would not have made a large difference anyways. Additionally, it is crucial to recognize that substitution decisions should not rely solely on simulations but should also incorporate other strategic insights such as player matchups. Code for this analysis can be found in 'substitution_analysis_function.ipynb'.

World Cup 2022 Revisited

In the 2022 World Cup final, Didier Deschamps, the manager of the French national team, made a bold and controversial decision to substitute Olivier Giroud and Ousmane Dembélé just 40 minutes into the match. At that point, France was trailing Argentina by two goals, and the early substitutions surprised many fans and analysts. Giroud, a key striker, and Dembélé, a dynamic winger, are both known for their attacking prowess, so taking them off the field so early in such a crucial game was a significant gamble. However, this gamble ultimately paid off, as France managed to tie the game and came close to winning before ultimately losing in penalty kicks.

France's comeback after the substitutions was impressive, but it's possible the team could have rallied similarly without the changes. To evaluate Deschamps' decision, we used simulations to assess how the substitutions affected France's win probability.In each scenario we simulated 5000 games to make sure that we assessed probabilities as accurately as possible. Without the substitutions, the likelihood of France winning was estimated at 3.2%, with an 8.44% chance of forcing a tie by the end of regulation. After the substitutions, the probability of winning increased slightly to 3.82%, and the chance of tying rose to 8.98%. Although France's overall chances of winning only improved from 7.42% to 8.31%, the decision to make the substitutions was still optimal.

This becomes even clearer when considering additional factors: Giroud and Dembele had already logged significant minutes throughout the tournament and so the effect of fatigue becomes even more pronounced. Combined with Deschamps' belief that the changes would better counter Argentina's pressure—something our model doesn't account for—the decision to substitute is more than justified.

Ultimately, this decision was about making the best of a difficult situation, as France's chances of winning were already slim. But in a game decided by the finest margins, every percentage point counts.Code for this analysis can be found in 'substitution_analysis_function.ipynb'.

Conclusions and Further Directions

In this report, we have explored the impact of various factors on a team's goal-scoring dynamics and win probability in soccer, with a particular focus on the timing and effectiveness of substitutions. By leveraging historical event data from the English Premier League, we developed a framework for estimating a team's probability of winning a match given the current game state. This model also allow us to evaluate how in-game decisions, such as substitutions, can influence the outcome of a match. Our simulations demonstrated that strategic adjustments, including the timing of substitutions, play a critical role in maximizing a team's chances of success.

While the simulation procedure discussed in this report provides valuable insights into the dynamics of soccer matches and the impact of managerial decisions, there are several areas for further exploration and refinement that could enhance the model's accuracy and applicability:

- Uncertainty Estimation in Simulations: The simulations are based on point estimates of parameters used in the xG model and time to score model.This makes it difficult to develop prediction intervals for win probabilities. Fitting those models using a Bayesian modelling strategy would solve this problem as we would could simulate from the joint posterior distribution of all of our parameters prior to each simulation.

- Player-Specific Fatigue: Incorporating player-specific fatigue levels, rather than assuming uniform fatigue across a team, could improve the precision of the model. This would account for variations in individual stamina and the differing physical demands of various positions.

- Player/Position-Specific Importance: Further analysis is needed to quantify the specific contributions of different players and positions to both offensive and defensive efforts. This could involve weighting the impact of each player based on their role and skill level, leading to more accurate assessment of substitution strategies.

- Random Intercept/Effects Model: Introducing a random intercepts or random effects model could help account for team-specific characteristics, such as the influence of the manager, the unique abilities of the goalkeeper, or the overall team strategy. This would allow the model to better capture the variability between different teams and contexts.

- Exploration of Different Copula Models: While the Gaussian copula was used in this analysis, exploring alternative copula models could provide a better fit for the joint distribution of goal timings. Different copulas might capture the dependencies between team performances more effectively, especially under varying game conditions.

- Integration of Tracking Data: Tracking data would enable a more detailed analysis of how player movements and positioning contribute to game outcomes.

- Utility as a Betting Tool: Given the model's ability to simulate match outcomes, there is potential to develop it further as a betting tool. By predicting win probabilities and likely scorelines with greater accuracy, the model could offer valuable insights for betting markets.

- Team-Specific Parameters: Finally, refining the model to include team-specific parameters, such as historical performance trends, tactical preferences, and player cohesion, could improve the precision of predictions. This would allow for more accurate assessments of how individual teams are likely to perform under different game states.

References

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