

Concurrent Algorithms

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Introduction

What we want: parallelism

- Definition of **parallelism: spatially coexisting**
- Parallel algorithms: using “spatially coexisting” computations
(e.g. “A || B”)
- Parallel architecture: utilizing “spatially coexisting” devices
(e.g. cores, memory, PCIe)
- Purpose: better time complexity & energy consumption
- Q: how to **communicate** among spatially coexisting things?
(e.g. job-stealing deque)

Communication channels

- FIFO queue
- Network (e.g. system bus, ethernet, ...)
- Database (e.g. key-value store, RDBMS)
- **Shared memory** (dominant for software)
- All channels are “**shared mutable state**”
- Summary: parallel things communicate among each other via shared mutable states

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Central
problem

What we need: concurrency

- Problem: how shared mutable states work
in the presence of temporally coexisting accesses?
- Definition of **concurrency: temporally coexisting**
- Parallelism requires concurrency
 - e.g. “(A || B); C” requires a channel from A & B to C
 - e.g. Parallel cores require shared-memory concurrency
 - e.g. job-stealing scheduler for parallel jobs require
job-stealing concurrent deque

Concurrent algorithms

- Algorithms for synchronizing (/coordinating/orchestrating) concurrent accesses to shared mutable states
 - Extremely nondeterministic & complex due to multiple threads of execution
- Usually in the form of **concurrent data structures**
 - e.g. job-stealing deque, concurrent hashmap, spin lock
- Goal: studying a few concurrent algorithms in shared-memory concurrency (dominant for SW)

Shared-memory concurrency

- Dominant concurrency abstraction for modern software
- Shared mutable state: memory
- Concurrent agents: multiple threads of execution
- Synchronization via memory location
 - e.g. message passing (passing value to another thread)

```
X=42;  
F=1      ||      while (F==0) { };  
                assert(X==42);
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Wrong

(store hoisting)

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Thread B: $Y=1$; $b=X$
(is $a=b=0$ possible?)

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Load hoisting

- Thread A: $a=Y$; $X=1$
Thread B: $b=X$; $Y=1$
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Store hoisting

- Thread A: $X++$
Thread B: $X++$
(is $X=1$ possible?)

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Non-atomicity

Common wisdom: locking

- **lock()**: disallowing hoisting of later instructions
 - Impossible: `lock(); A -> A; lock()`
 - Possible: `A; lock() -> lock(); A`
- **unlock()**: not hoisted across earlier instructions
 - Impossible: `A; unlock() -> unlock(); A`
 - Possible: `unlock(); A -> A; unlock()`

Common wisdom: locking

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Thread B: `do { lock(); f=F; unlock() } while(f==0);
assert(X==42)`

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Thread A: **X=42; lock(); F=1; unlock()**

Thread B: do { **lock(); f=F; unlock()** } while(f==0);
 assert(X==42)

- e.g. counter w/ proper locking (X=1 impossible):

Thread A: **lock(); X++; unlock()**

Thread B: **lock(); X++; unlock()**

Common wisdom: locking

- **Universal:** all sequential data structures turn into concurrent when protected with locks.
- **Deadlock-prone:** deadlock may happen if multiple threads try to acquire different locks
- **Insalable:** locking achieves safety basically by removing parallelism

Common wisdom: locking

- **Universal:** all sequential data structures turn into concurrent when protected with locks.
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- **Inscalable:** locking achieves safety basically by removing parallelism



Lock-free concurrent programming

Summary

- Parallelism: spatially coexisting
Concurrency: temporally coexisting
- Parallelism requires concurrent **shared mutable state**
- Shared memory is a dominant SMS for SW,
but it admits very strange relaxed behaviors
(trading simplicity for performance)
 - Locking works universally, but it is inscalable
 - Lock-free concurrency is hard due to relaxed behaviors

Key ideas of lock-free programming

Lock-free programming

Complex relaxed behaviors due to nondeterminism & reordering

- **Key challenge 1:** how to tame nondeterminism?
 - **Solution:** by using read-modify-write instructions
- **Key challenge 2:** how to tame reordering?
 - **Solution:** by using acquire load & release store

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Lock-free

- **Read-modify-write (RMW)** instructions:
Read and write a memory location in a single instruction
- **Taming nondeterminism by removing “critical” one**
(interleaving btw. read & write).

Spinlock (w/o reordering)

- The most basic lock implementation

- ```
struct spinlock {
 lock: atomic<bool>;
};
```
- ```
spinlock::lock(spinlock &l) {  
    while (l.lock.swap(true)) {}  
}
```
- ```
spinlock::unlock(spinlock &l) {
 l.lock.store(false);
}
```

# Lock-free programming

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- **X.load(acquire):** disallowing hoisting of later instructions
- **X.store(42, release):** not hoisted across earlier instructions

**Taming reordering by  
acquire/release instructions**



# Spinlock (incorrect)

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**Reordering  
possible**

# Spinlock (correct)

- The most basic lock implementation
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- ```
spinlock::lock(spinlock &l) {
 while (l.lock.swap(true, acquire)) {}
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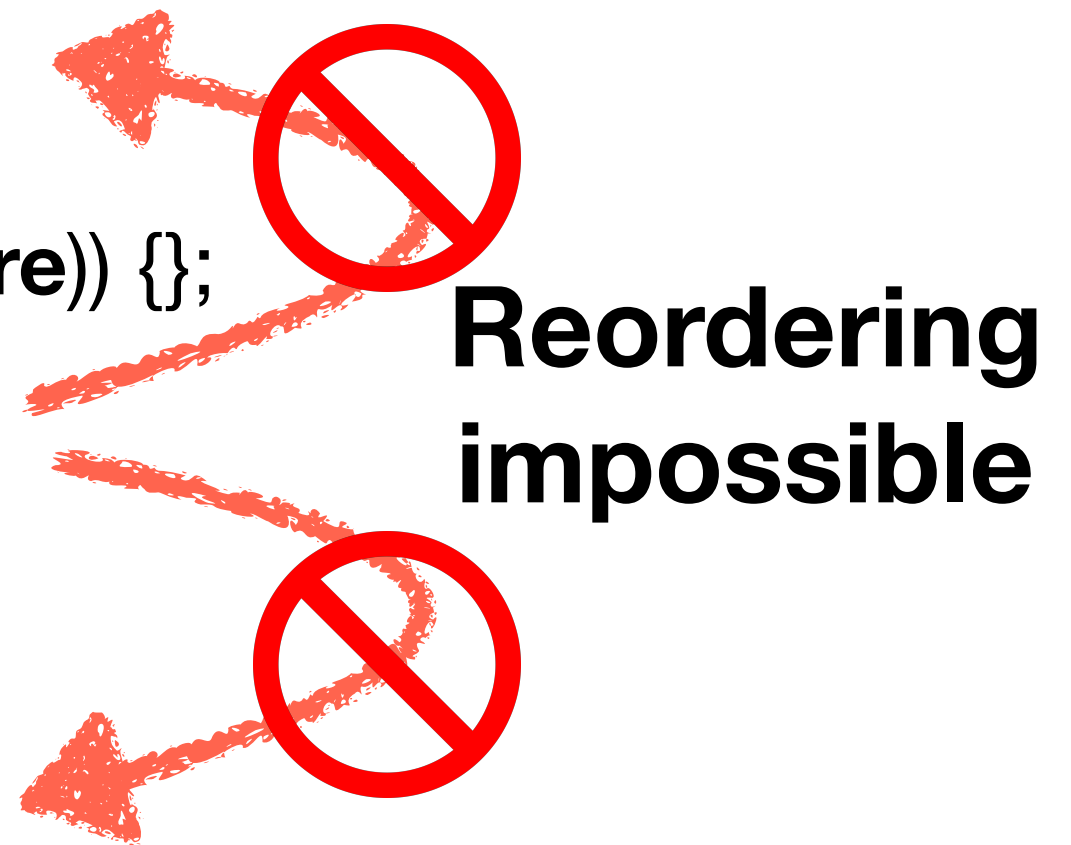
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# Versatility of RMW & RA

- We can implement spinlock w/ RMW & release/acquire.
- Actually, we can implement **most** concurrent data structures w/ RMW & release/acquire.
  - Concurrent stack, queue, hash table, trie, b-tree, balanced tree (AVL and red-black trees), ...
- Next class: implementing a concurrent linked list

# Concurrent linked list

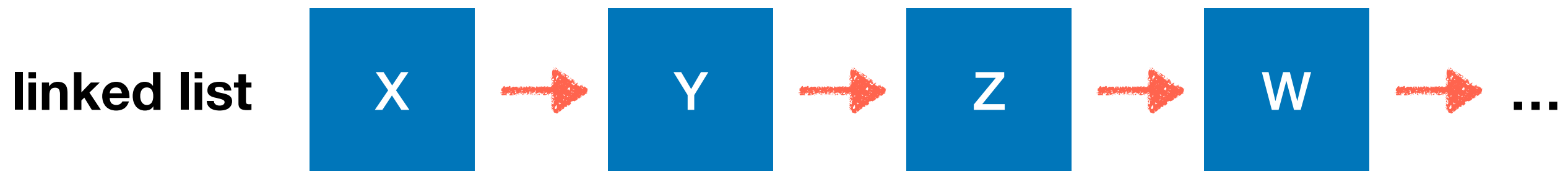
# (Singly) linked list





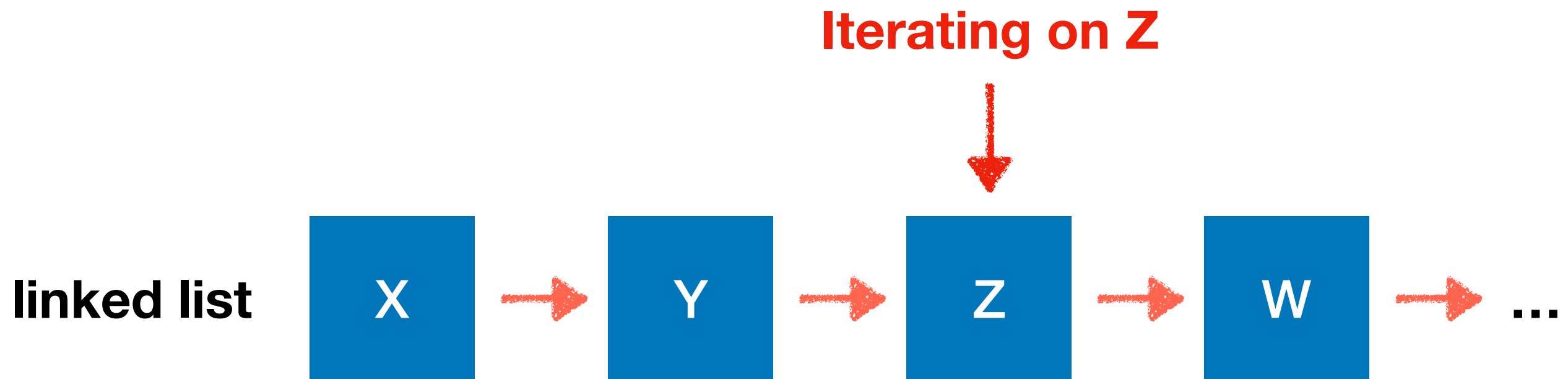
# (Singly) linked list

Only one thread accesses  
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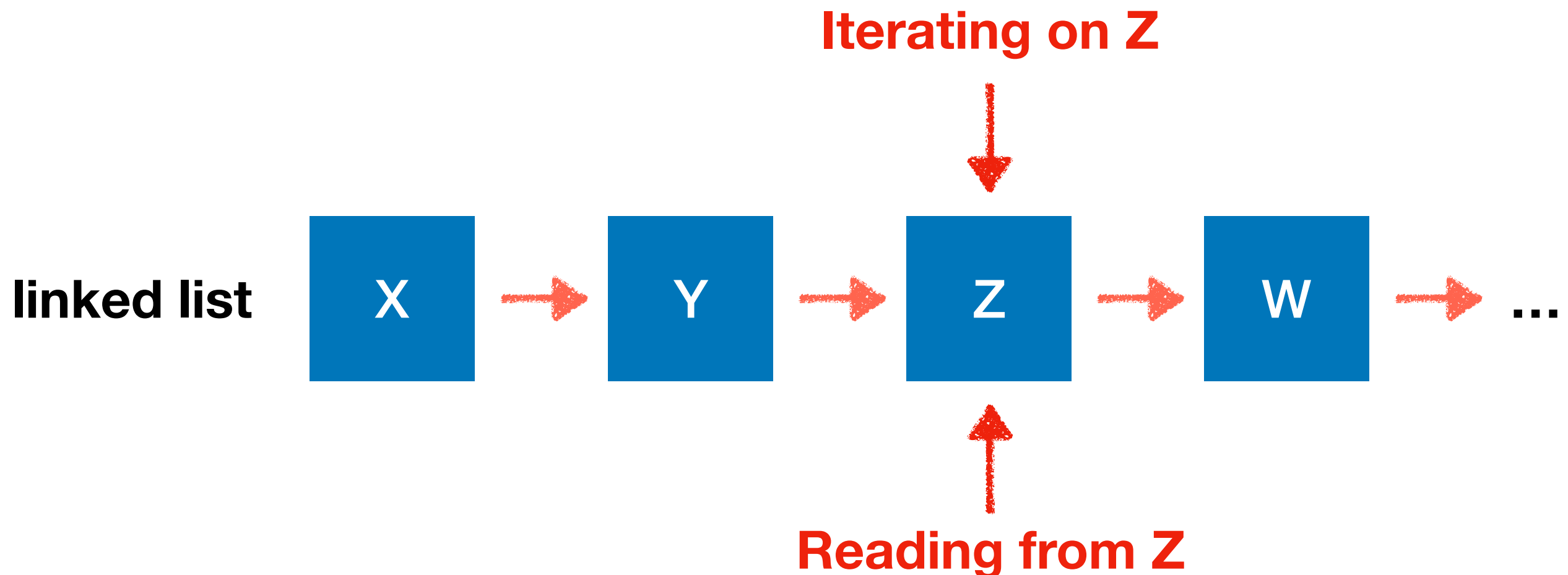
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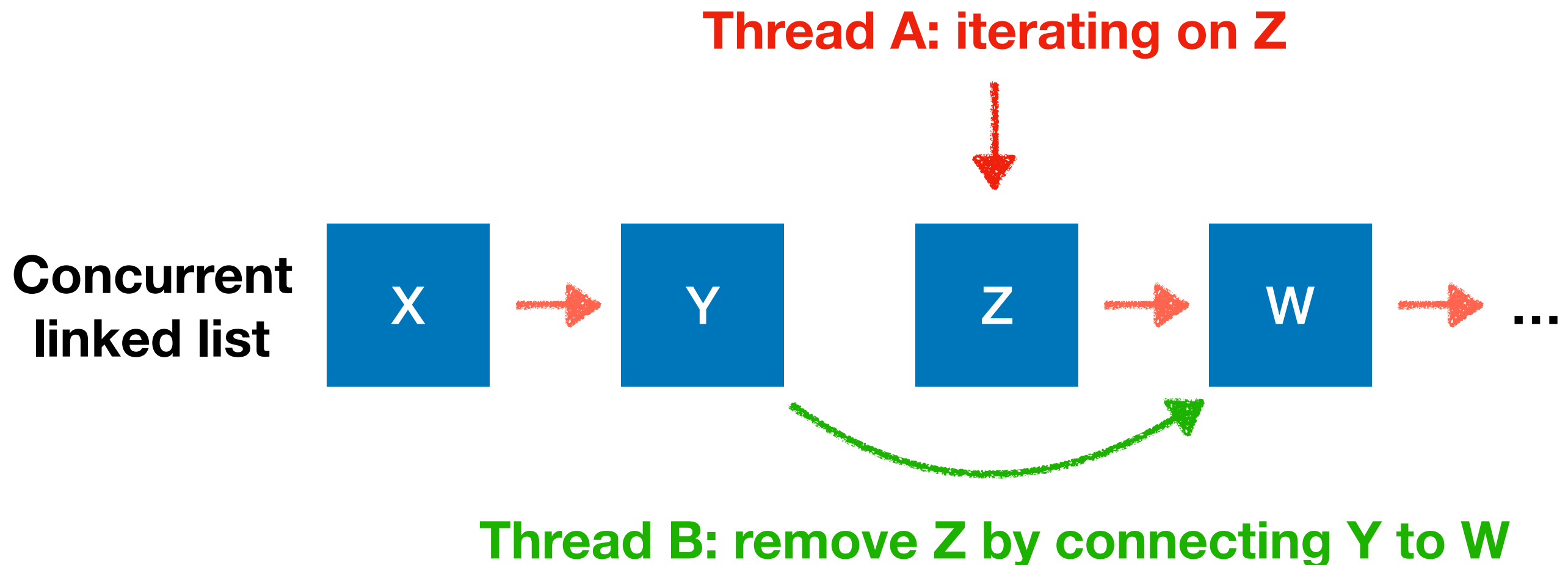
Thread A: iterating on Z

Concurrent  
linked list



# Concurrent linked list

Multiple threads may access a linked list



# Key Challenge

- **Challenge: how to synchronize linked list operations?**
  - Write operations: insertion, removal
  - Read operations: iteration



# Key Challenge

- **Challenge: how to synchronize linked list operations?**
  - Write operations: insertion, removal
  - Read operations: iteration
- **Key idea: using RMW & release/acquire instructions**
  - write operations are **atomic** thanks to RMW
  - All operations are tolerant to reordering thanks to RA
    - All accesses to a node' next pointer are release/acquire

# Insertion

**Concurrent  
linked list**



# Insertion

**Concurrent  
linked list**



**(1) Create a new node**

# Insertion

**Concurrent  
linked list**

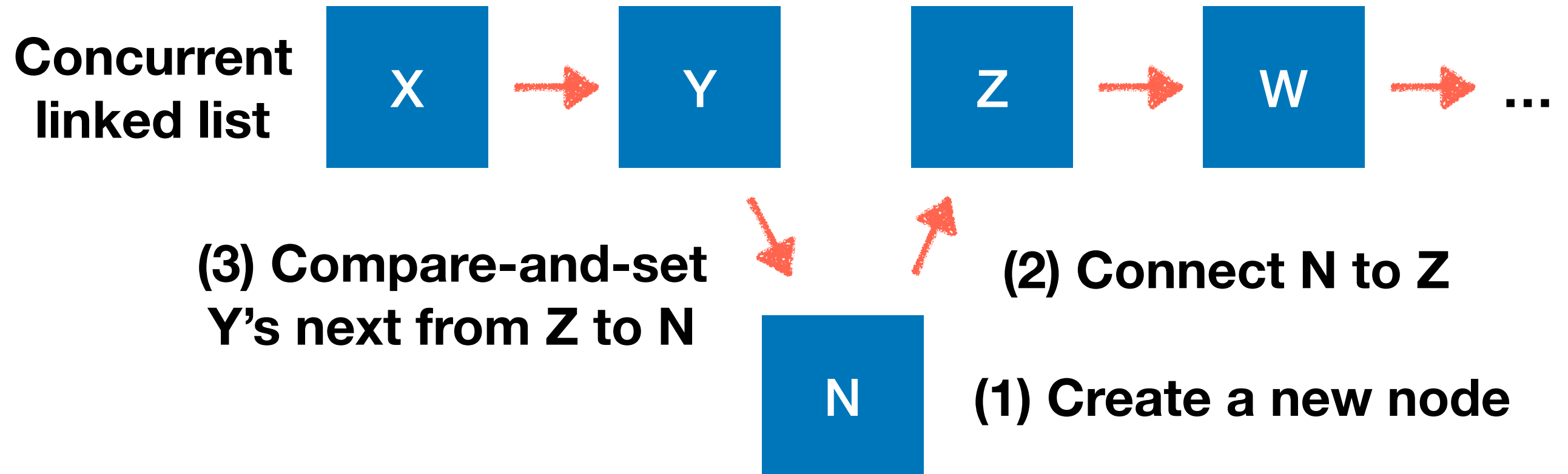


**(2) Connect N to Z**

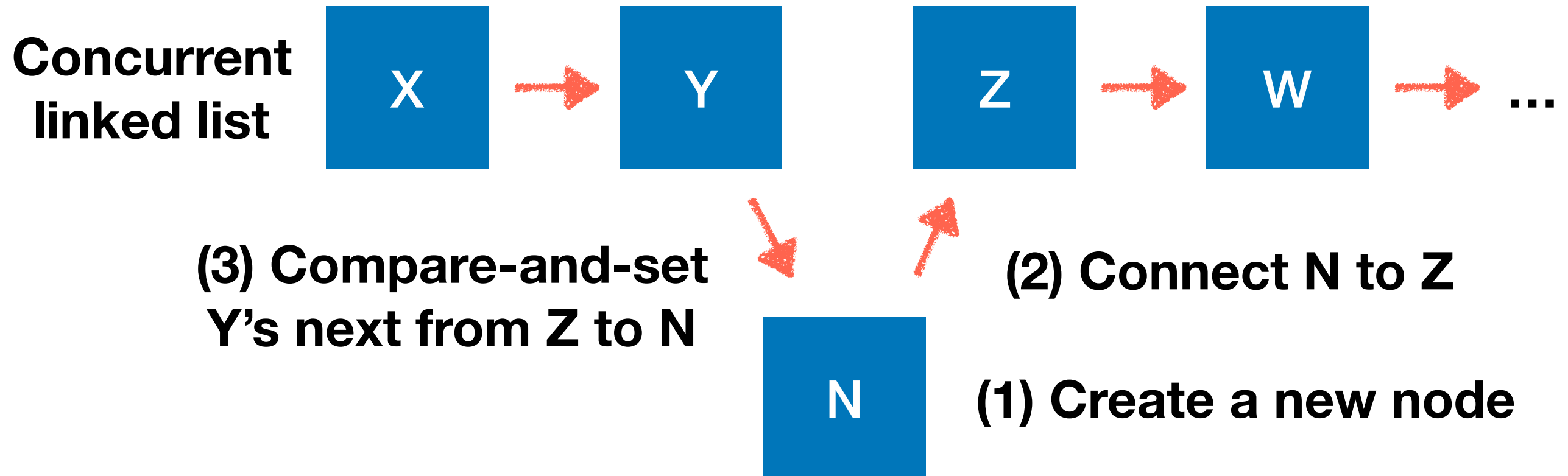


**(1) Create a new node**

# Insertion



# Insertion



**N should be btw. Y and Z:  
Atomicity from compare-and-set**

# Removal

**Concurrent  
linked list**



# Removal

(1) Mark Z's next  
as removed w/ RMW

Concurrent  
linked list

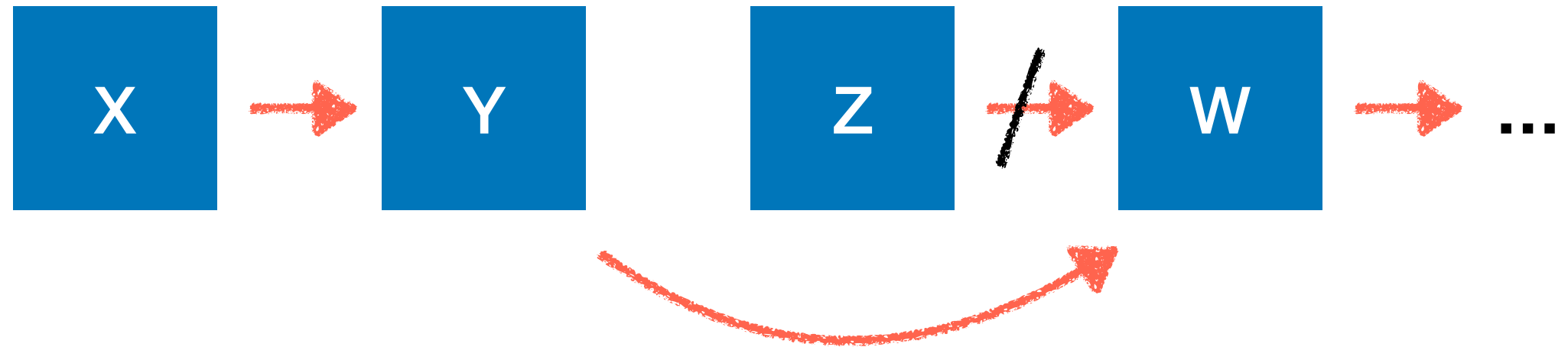




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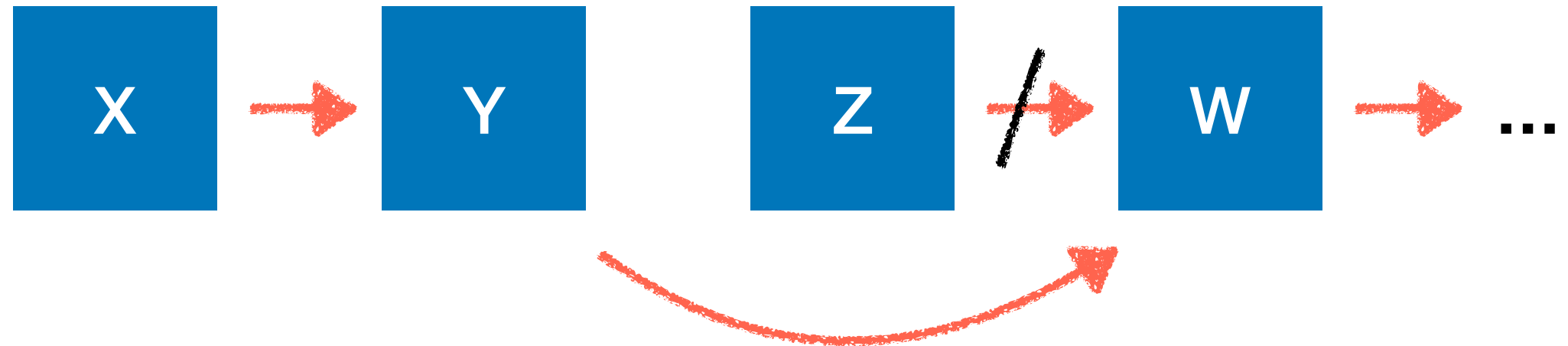


(2) Compare-and-set  
Y's next from Z to W

# Removal

(1) Mark Z's next  
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Concurrent  
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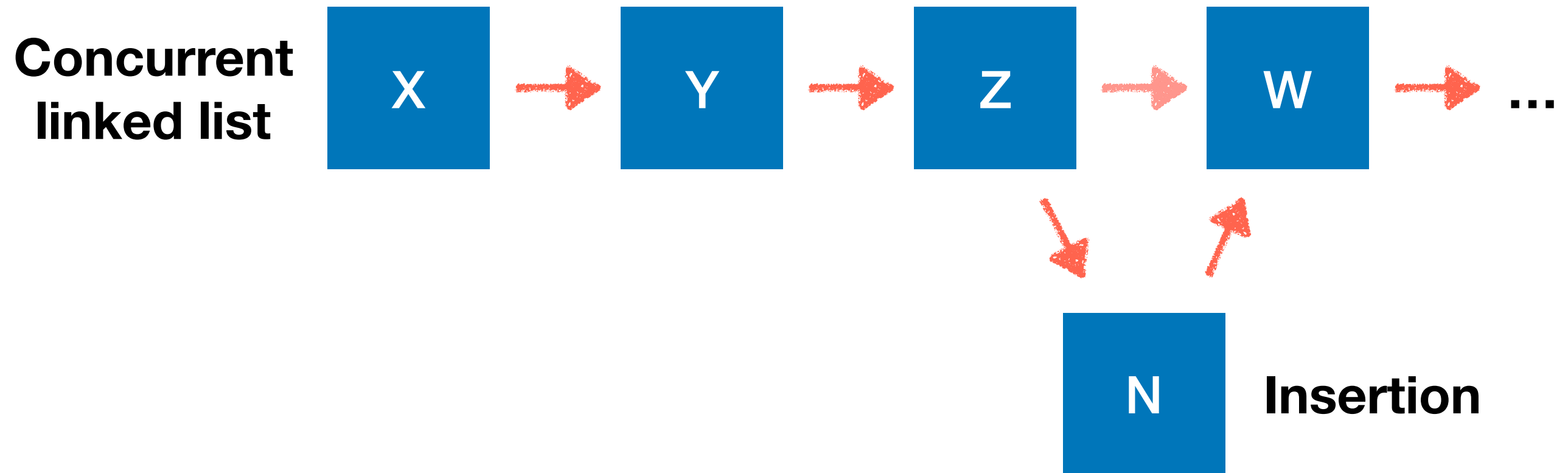
Only Z should be removed:  
Atomicity from compare-and-set

# Insertion & removal?

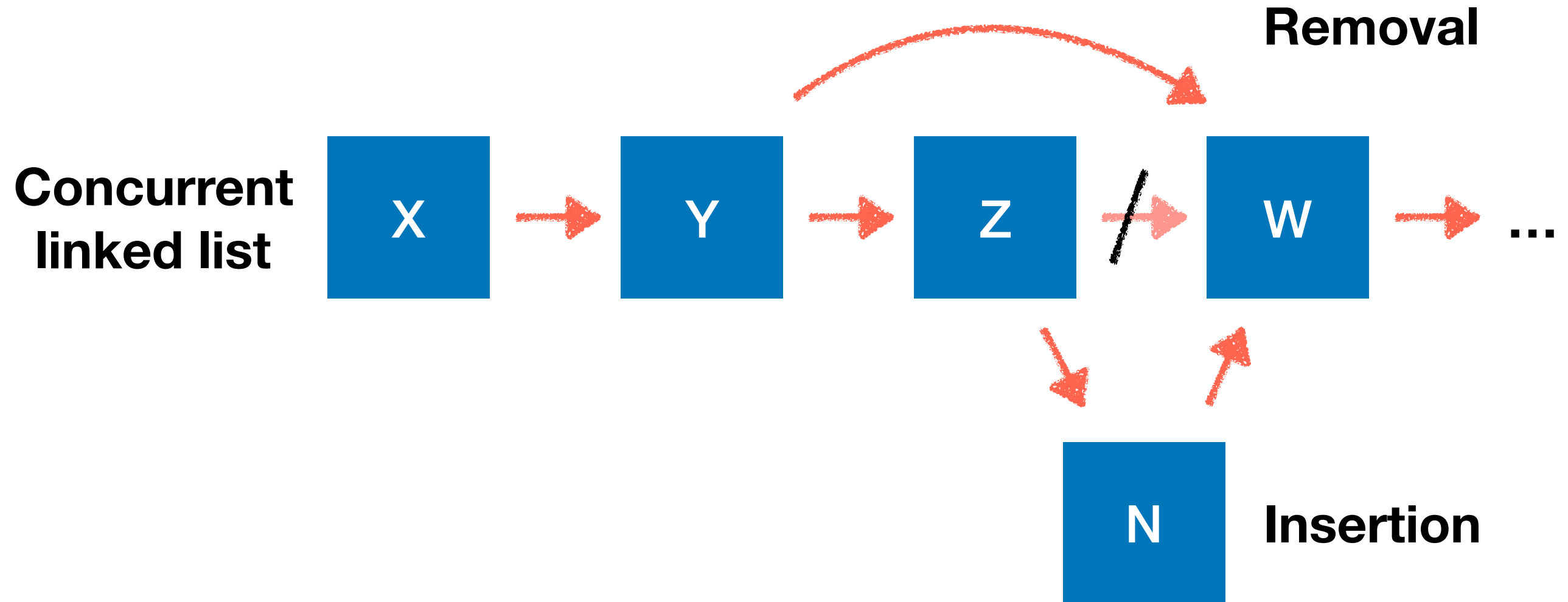
**Concurrent  
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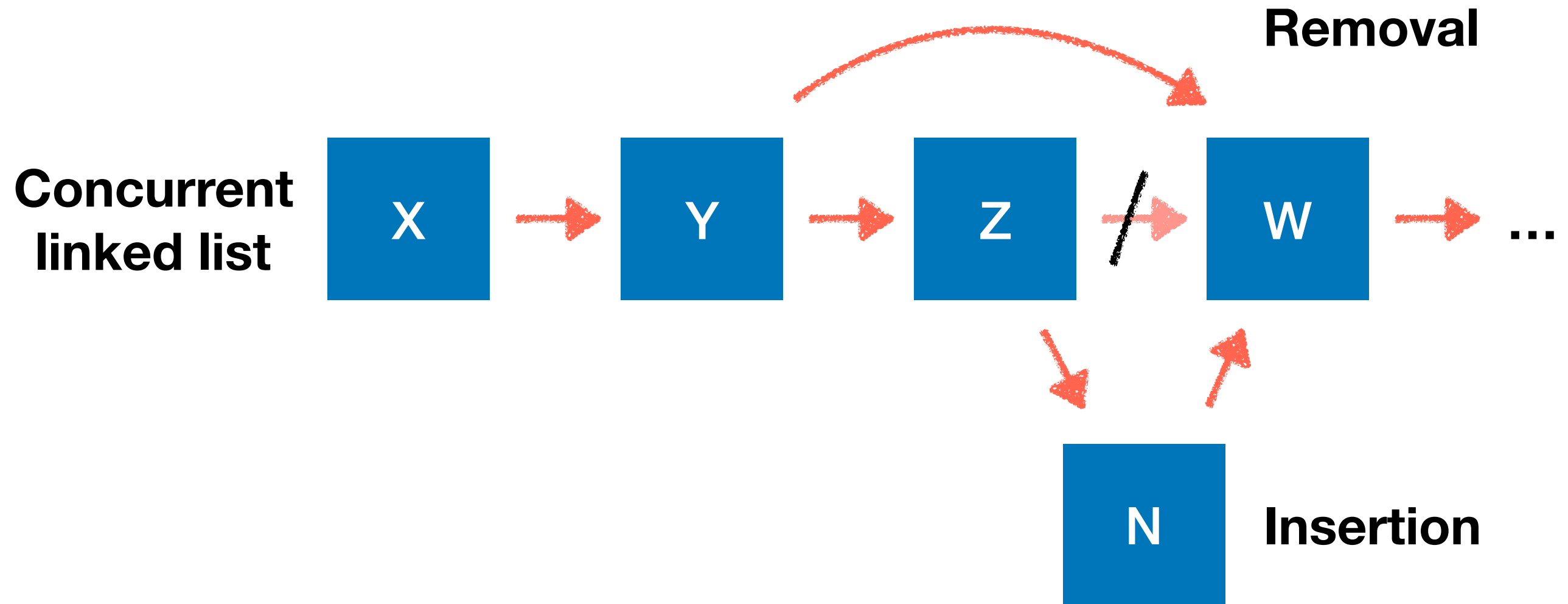
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Insertion and removal are synch.  
at Z's next

# Discussion

- **Theoretically optimal parallelism**
  - All read operations are executed in parallel
  - A write operation is executed in parallel w/ another op. if not accessing the same node
- **One of the earliest concurrent data structures**
- **Basis for hash tables**

# Why not stack or deque?

- Treiber's stack is not parallel: synchronizing all accesses
- More advanced, parallel stacks cannot be explained in 1.5h
- Job-stealing deque also cannot be explained in 1.5h
- **CS492: concurrent programming** (next semester)!



# End of the semester!

- **Q&A session:** 5th, June (Wednesday) 9:00am - 10:15am
- **Final exam:** 10th, June (Monday) 9:00am - 11:45am
- **Coverage:** Brent's theorem, systolic array, odd-even merge sort, bitonic merge sort, dynamic scheduling, concurrent algorithms