

# Concurrent Algorithms

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# Introduction

# What we want: parallelism

- Definition of **parallelism: spatially coexisting**
- Parallel algorithms: using “spatially coexisting” computations  
(e.g. “A || B”)
- Parallel architecture: utilizing “spatially coexisting” devices  
(e.g. cores, memory, PCIe)
- Purpose: better time complexity & energy consumption
- Q: how to **communicate** among spatially coexisting things?  
(e.g. job-stealing deque)

# Communication channels

- FIFO queue
- Network (e.g. system bus, ethernet, ...)
- Database (e.g. key-value store, RDBMS)
- **Shared memory** (dominant for software)
- All channels are “**shared mutable state**”
- Summary: parallel things communicate among each other via shared mutable states

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Central  
problem

# What we need: concurrency

- Problem: how shared mutable states work  
in the presence of temporally coexisting accesses?
- Definition of **concurrency: temporally coexisting**
- Parallelism requires concurrency
  - e.g. “(A || B); C” requires a channel from A & B to C
  - e.g. Parallel cores require shared-memory concurrency
  - e.g. job-stealing scheduler for parallel jobs require  
job-stealing concurrent deque

# Concurrent algorithms

- Algorithms for synchronizing (/coordinating/orchestrating) concurrent accesses to shared mutable states
  - Extremely nondeterministic & complex due to multiple threads of execution
- Usually in the form of **concurrent data structures**
  - e.g. job-stealing deque, concurrent hashmap, spin lock
- Goal: studying a few concurrent algorithms in shared-memory concurrency (dominant for SW)

# Shared-memory concurrency

- Dominant concurrency abstraction for modern software
- Shared mutable state: memory
- Concurrent agents: multiple threads of execution
- Synchronization via memory location
  - e.g. message passing (passing value to another thread)

```
X=42;  
F=1      ||      while (F==0) { };  
                assert(X==42);
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**Wrong**

(store hoisting)

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Store hoisting

- Thread A:  $X++$   
Thread B:  $X++$   
(is  $X=1$  possible?)

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Non-atomicity



# Common wisdom: locking

- **lock()**: disallowing hoisting of later instructions
  - Impossible: `lock(); A -> A; lock()`
  - Possible: `A; lock() -> lock(); A`
- **unlock()**: not hoisted across earlier instructions
  - Impossible: `A; unlock() -> unlock(); A`
  - Possible: `unlock(); A -> A; unlock()`

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Thread B: do { **lock(); f=F; unlock()** } while(f==0);  
          **assert(X==42)**

- e.g. counter w/ proper locking (X=1 impossible):

Thread A: **lock(); X++; unlock()**

Thread B: **lock(); X++; unlock()**

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- **Inscalable:** locking achieves safety basically by removing parallelism

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Lock-free concurrent programming

# Summary

- Parallelism: spatially coexisting  
Concurrency: temporally coexisting
- Parallelism requires concurrent **shared mutable state**
- Shared memory is a dominant SMS for SW,  
but it admits very strange relaxed behaviors  
(trading simplicity for performance)
  - Locking works universally, but it is inscalable
  - Lock-free concurrency is hard due to relaxed behaviors

# Key ideas of lock-free programming



# Lock-free programming

**Complex relaxed behaviors due to nondeterminism & reordering**

- **Key challenge 1:** how to tame nondeterminism?
  - **Solution:** by using read-modify-write instructions
- **Key challenge 2:** how to tame reordering?
  - **Solution:** by using acquire load & release store

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Lock-free

- **Read-modify-write (RMW)** instructions:  
Read and write a memory location in a single instruction
- **Taming nondeterminism by removing “critical” one**  
(interleaving btw. read & write).

# Spinlock (w/o reordering)

- The most basic lock implementation

- ```
struct spinlock {  
    lock: atomic<bool>;  
};
```
- ```
spinlock::lock(spinlock &l) {  
    while (!l.lock.swap(true)) {}  
}
```
- ```
spinlock::unlock(spinlock &l) {  
    l.lock.store(false);  
}
```

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**Taming reordering by  
acquire/release instructions**



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**Reordering  
possible**

# Spinlock (correct)

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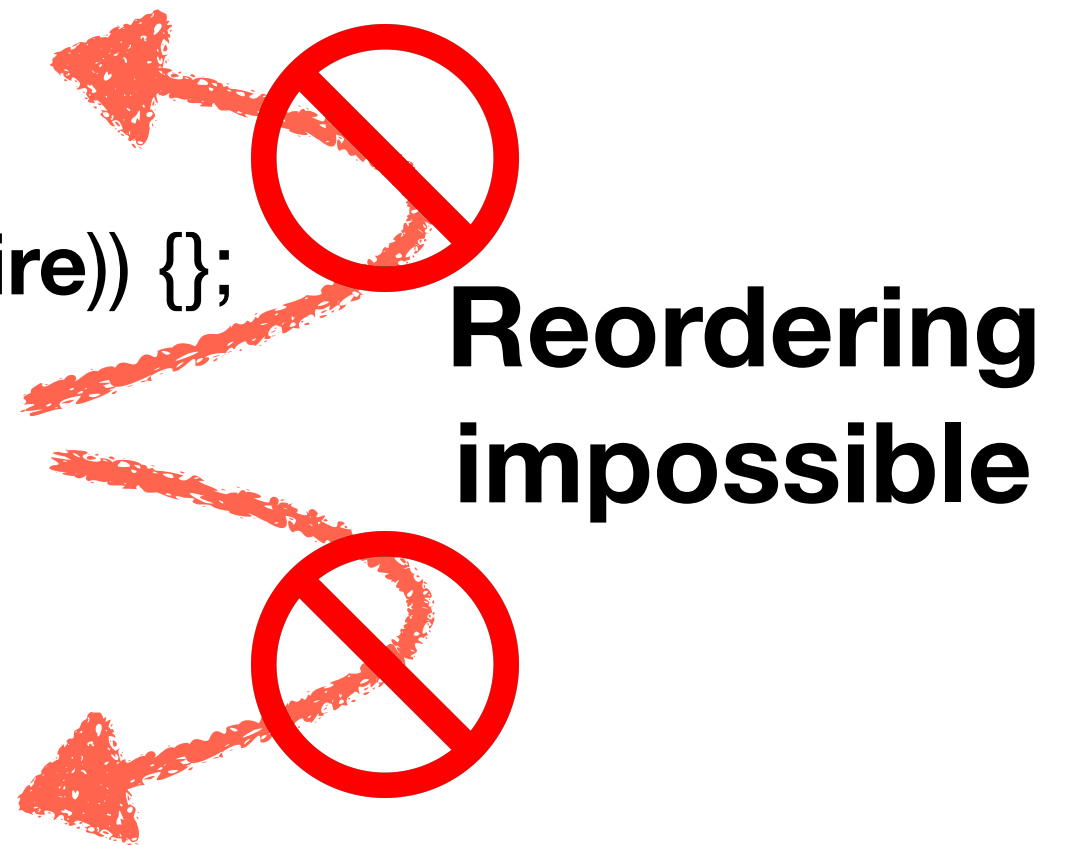
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# Versatility of RMW & RA

- We can implement spinlock w/ RMW & release/acquire.
- Actually, we can implement **most** concurrent data structures w/ RMW & release/acquire.
  - Concurrent stack, queue, hash table, trie, b-tree, balanced tree (AVL and red-black trees), ...
- Next class: implementing a concurrent linked list

# Concurrent linked list

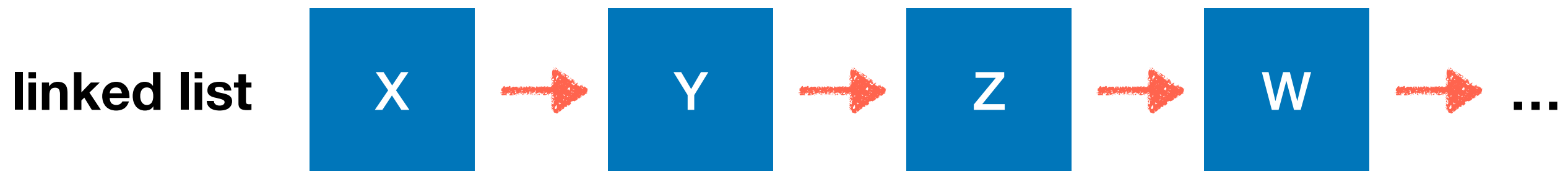
# (Singly) linked list





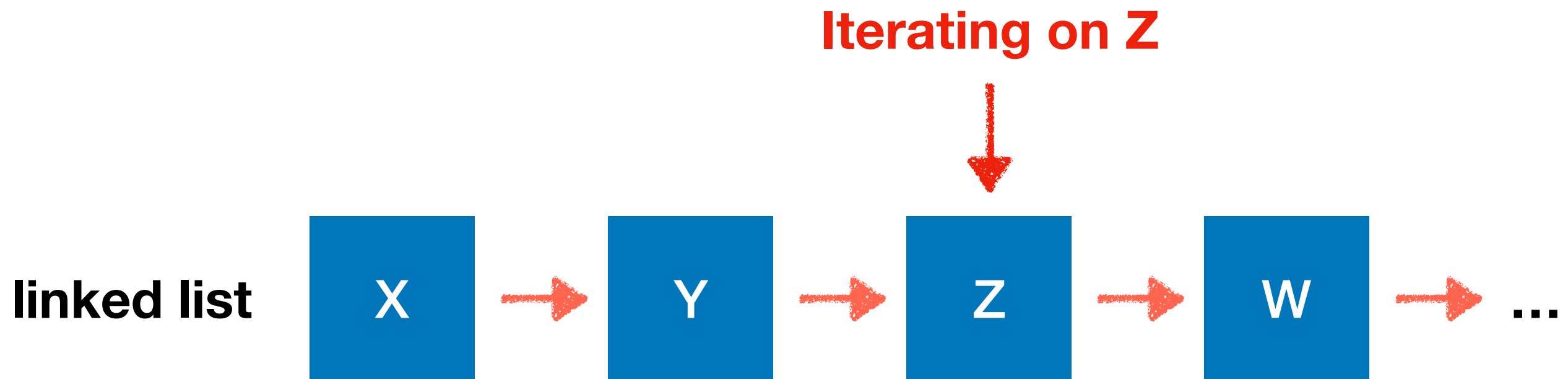
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Only one thread accesses  
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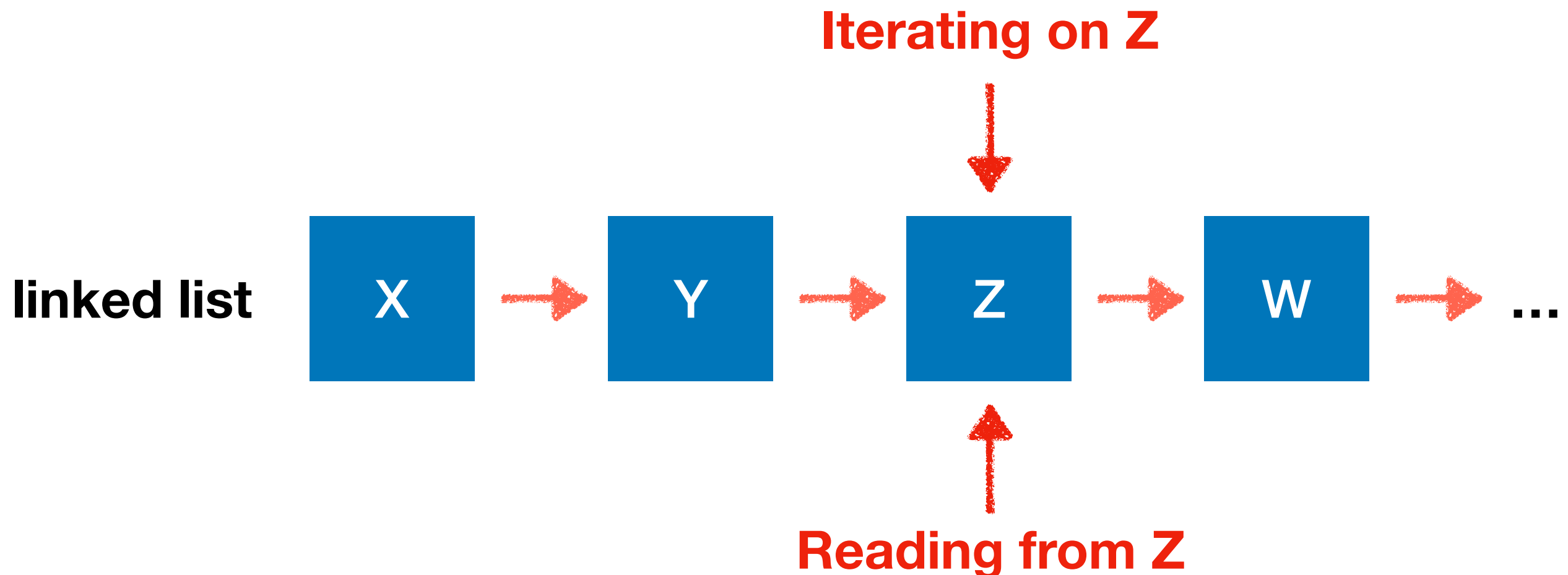
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**Concurrent  
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Multiple threads may access  
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**Concurrent  
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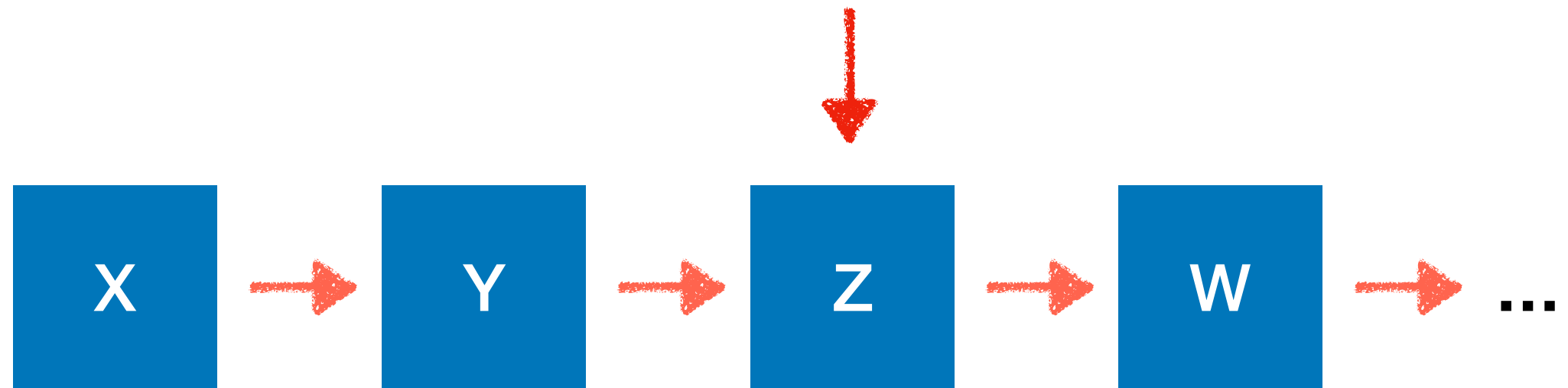


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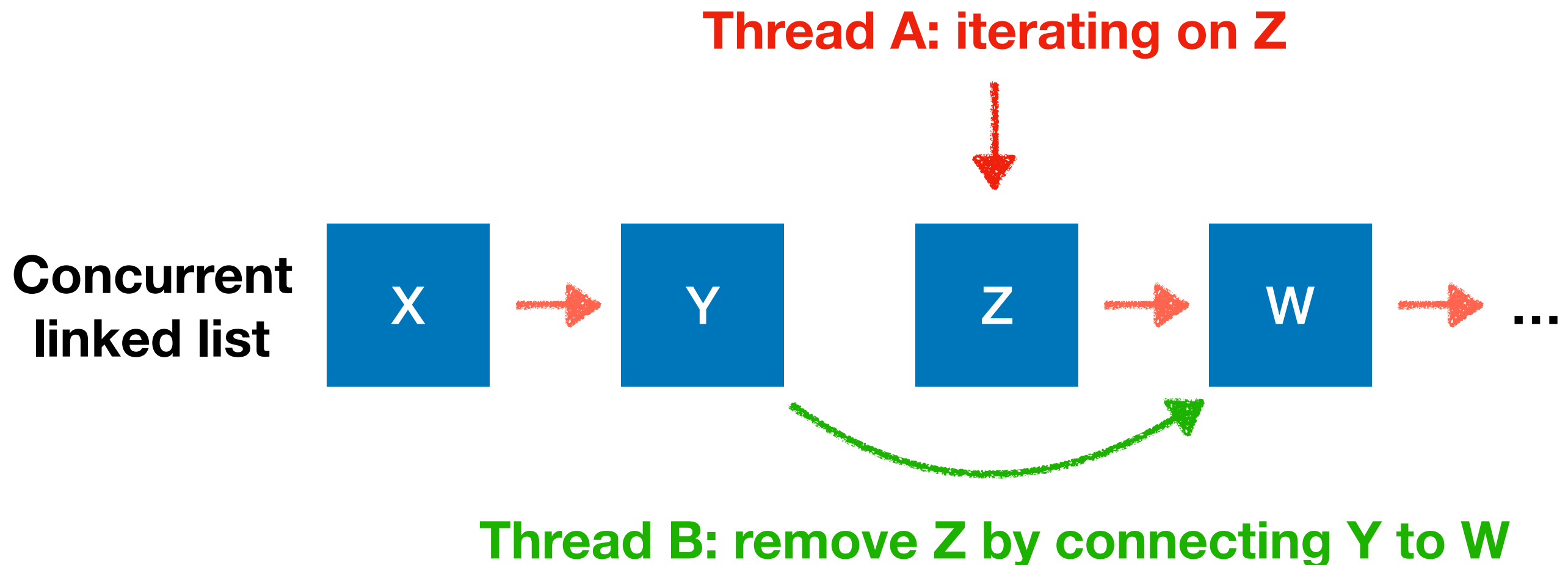
Thread A: iterating on Z

Concurrent  
linked list



# Concurrent linked list

Multiple threads may access a linked list



# Key Challenge

- **Challenge: how to synchronize linked list operations?**
  - Write operations: insertion, removal
  - Read operations: iteration



# Key Challenge

- **Challenge: how to synchronize linked list operations?**
  - Write operations: insertion, removal
  - Read operations: iteration
- **Key idea: using RMW & release/acquire instructions**
  - write operations are **atomic** thanks to RMW
  - All operations are tolerant to reordering thanks to RA
    - All accesses to a node' next pointer are release/acquire

# Insertion

**Concurrent  
linked list**



# Insertion

**Concurrent  
linked list**



**(1) Create a new node**

# Insertion

**Concurrent  
linked list**

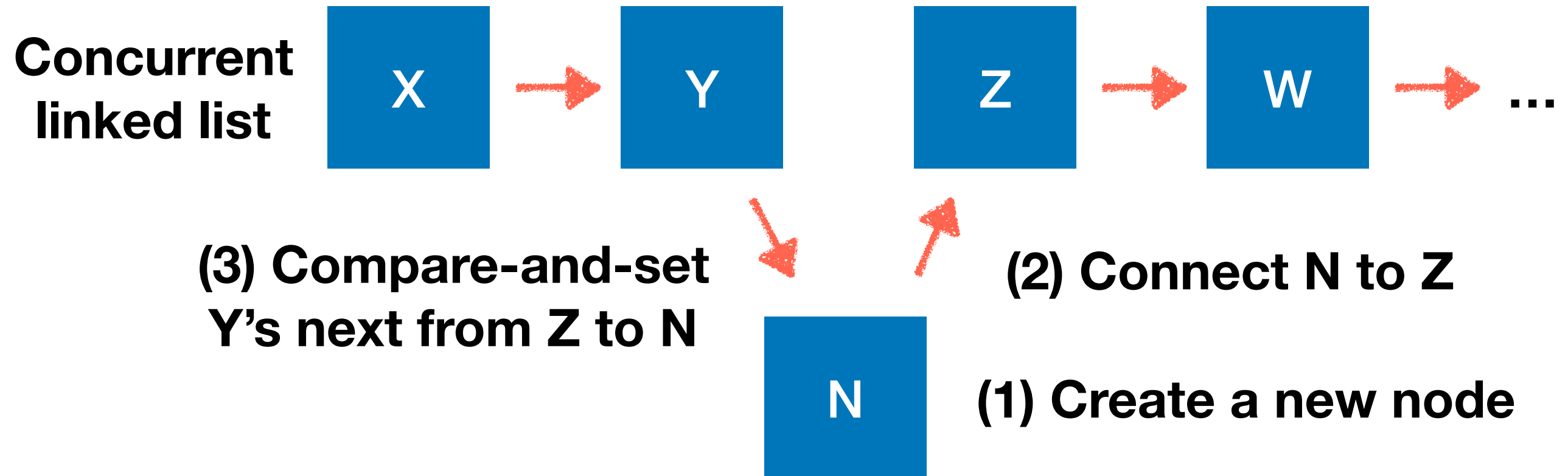


**(2) Connect N to Z**

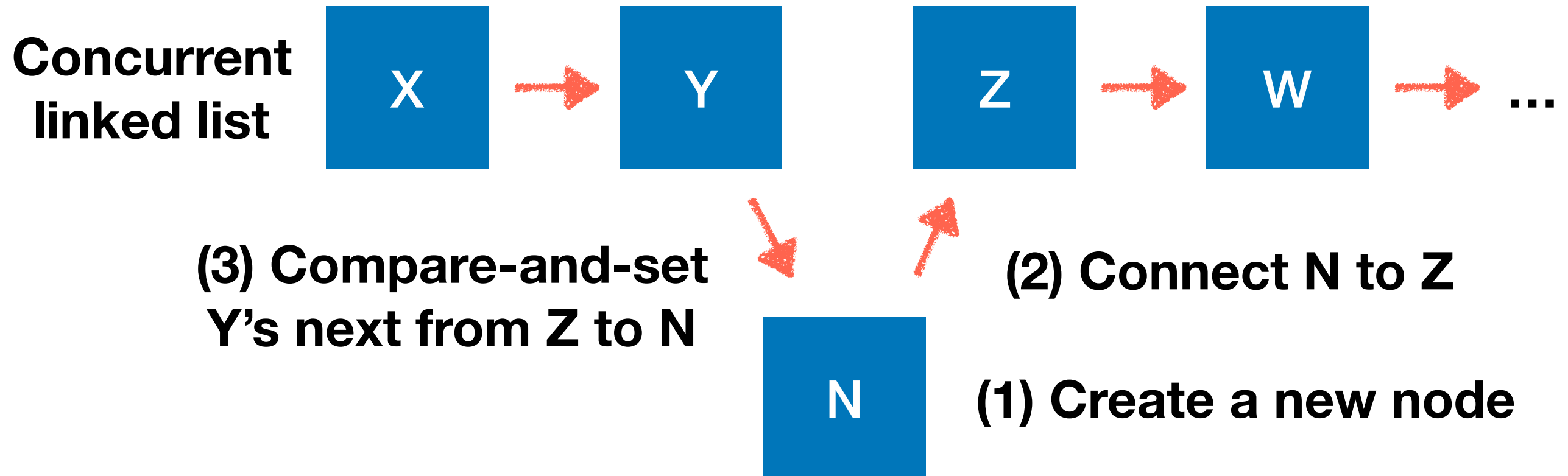


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# Insertion



# Insertion



**N should be btw. Y and Z:  
Atomicity from compare-and-set**

# Removal

**Concurrent  
linked list**



# Removal

(1) Mark Z's next  
as removed w/ RMW

Concurrent  
linked list

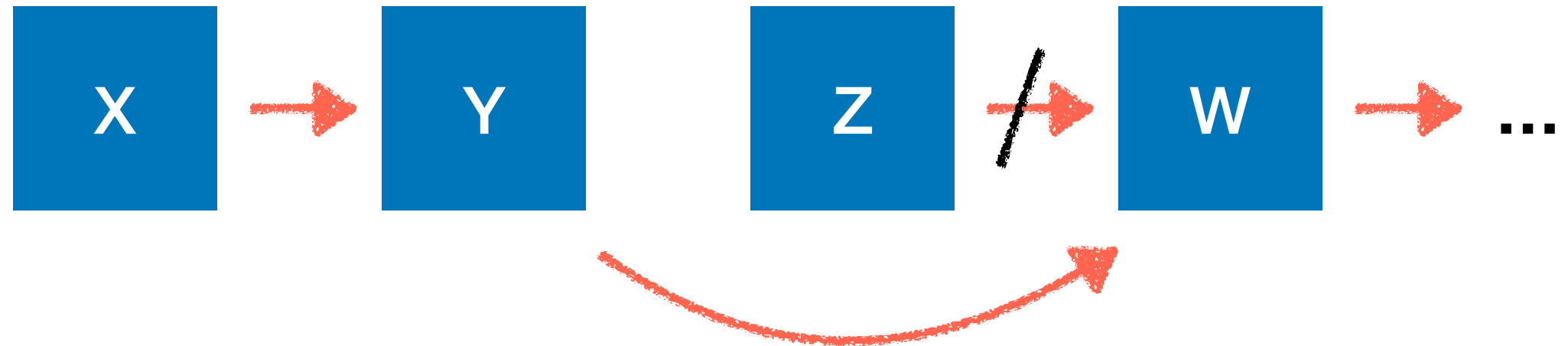




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Concurrent  
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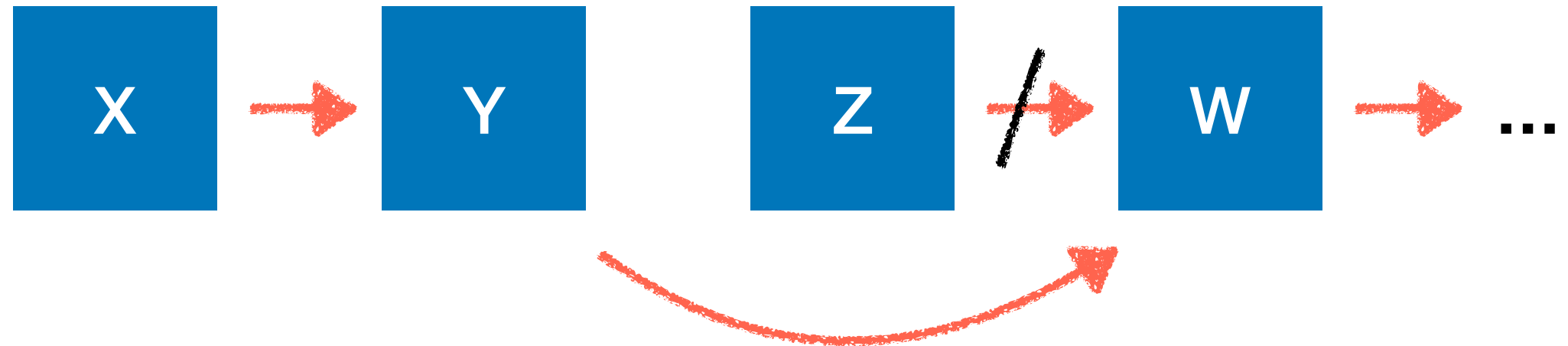


(2) Compare-and-set  
Y's next from Z to W

# Removal

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Concurrent  
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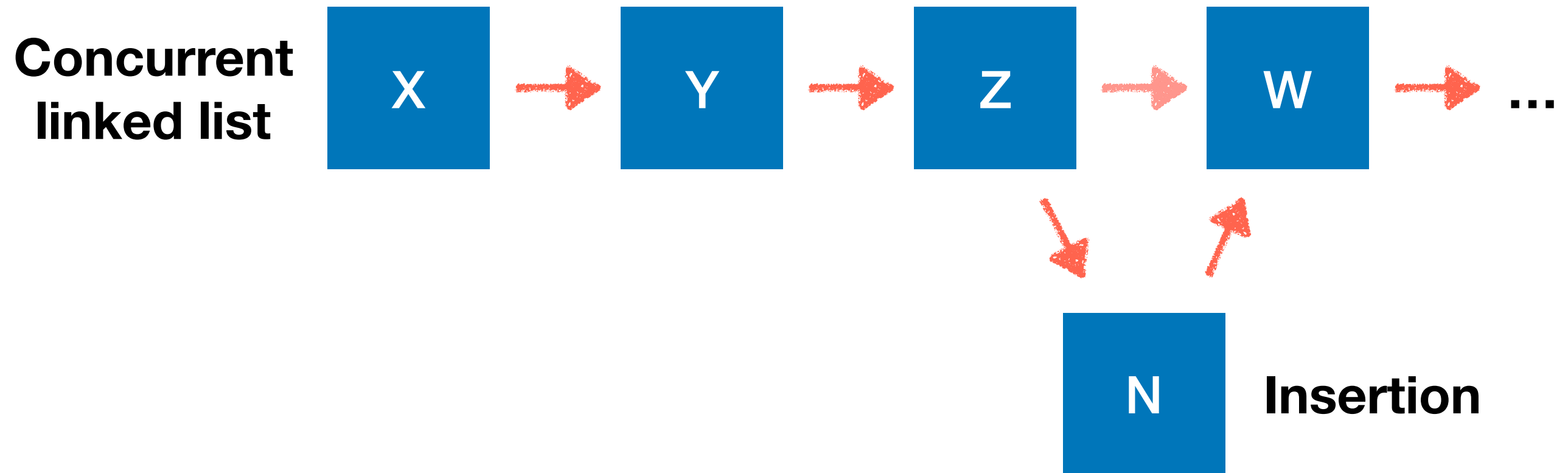
Only Z should be removed:  
Atomicity from compare-and-set

# Insertion & removal?

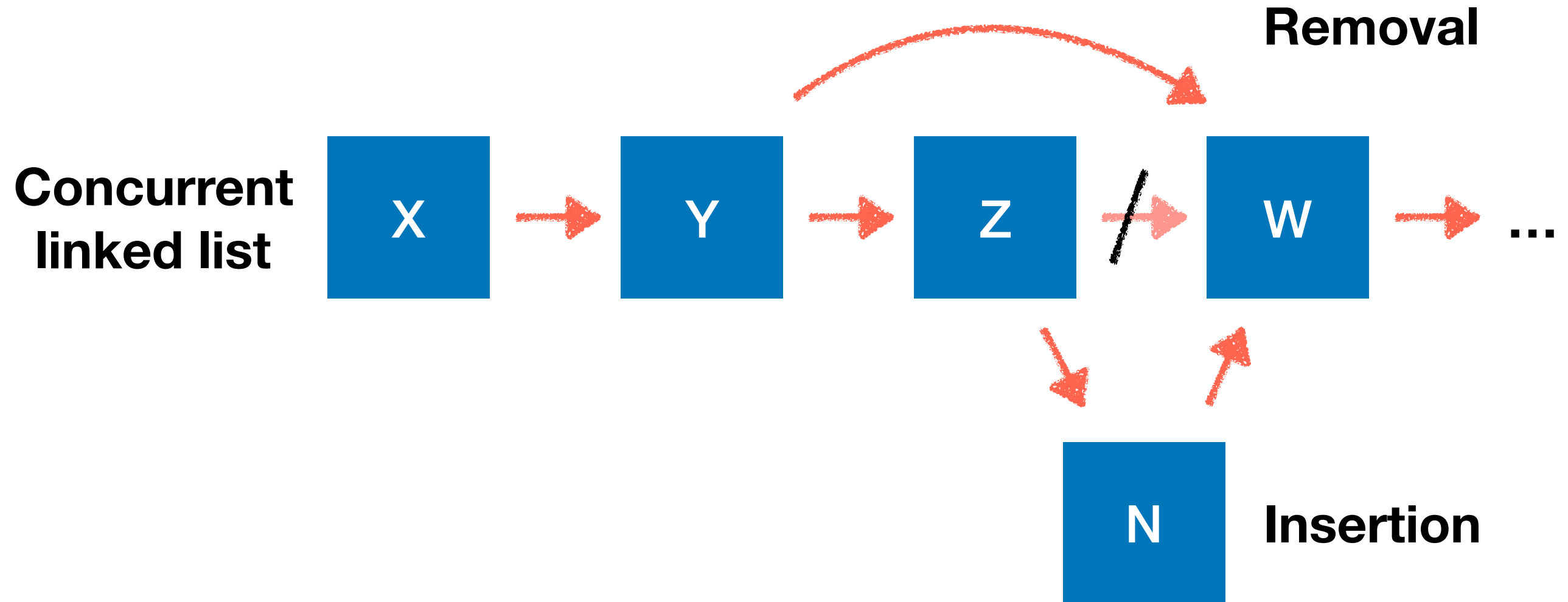
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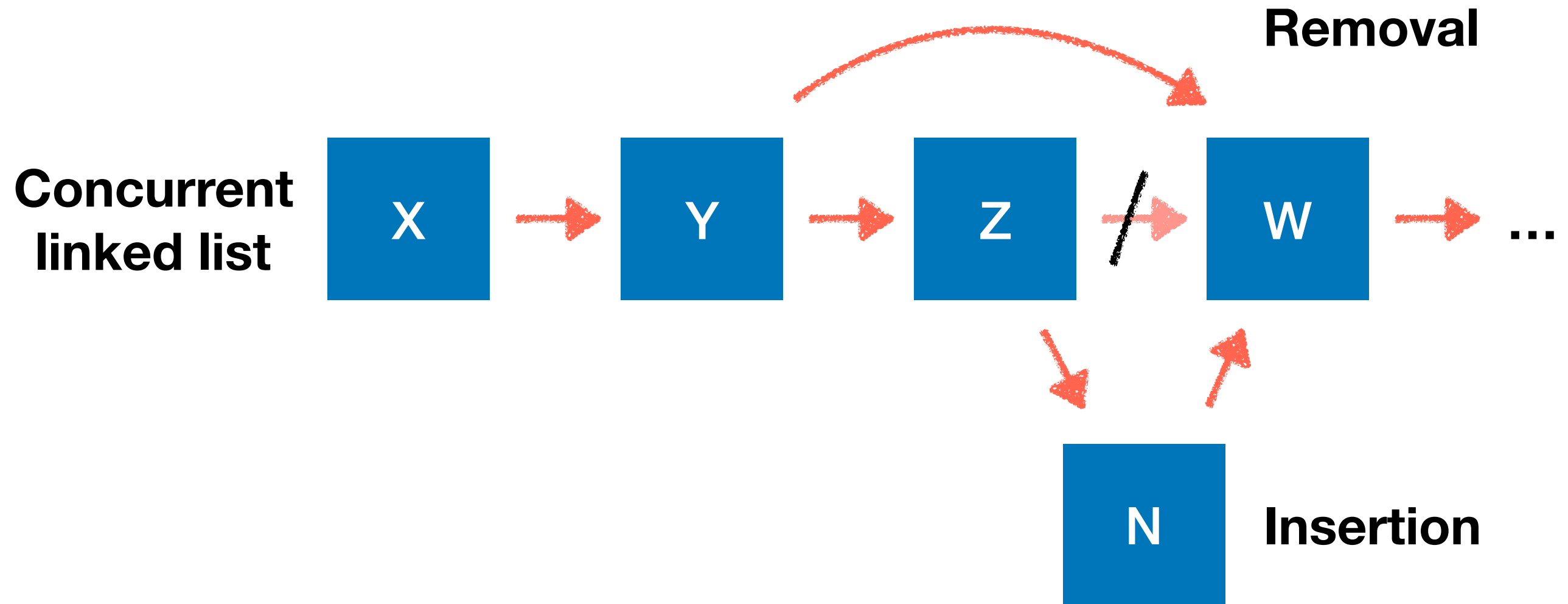
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Insertion and removal are synch.  
at Z's next

# Discussion

- **Theoretically optimal parallelism**
  - All read operations are executed in parallel
  - A write operation is executed in parallel w/ another op. if not accessing the same node
- **One of the earliest concurrent data structures**
- **Basis for hash tables**

# Why not stack or deque?

- Treiber's stack is not parallel: synchronizing all accesses
- More advanced, parallel stacks cannot be explained in 1.5h
- Job-stealing deque also cannot be explained in 1.5h
- **CS492: concurrent programming** (next semester)!



# End of the semester!

- **Q&A session:** 5th, June (Wednesday) 9:00am - 10:15am
- **Final exam:** 10th, June (Monday) 9:00am - 11:45am
- **Coverage:** Brent's theorem, systolic array, odd-even merge sort, bitonic merge sort, dynamic scheduling, concurrent algorithms