

KDD 2025 – Research Track

# Oldie but Goodie: Re-illuminating Label Propagation on Graphs with Partially Observed Features

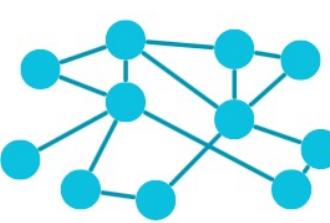
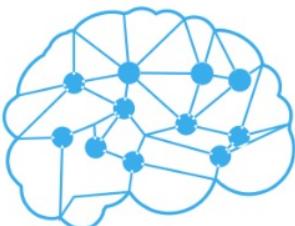
Sukwon Yun, Xin Liu, Yunhak Oh, Junseok Lee, Sungwon Kim,  
Tianlong Chen, Tsuyoshi Murata, Chanyoung Park



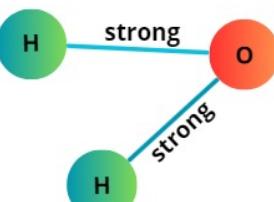
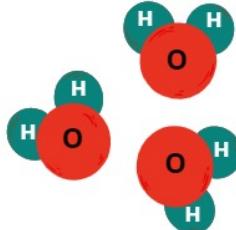
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# MOTIVATION: MISSING FEATURES IN GRAPH

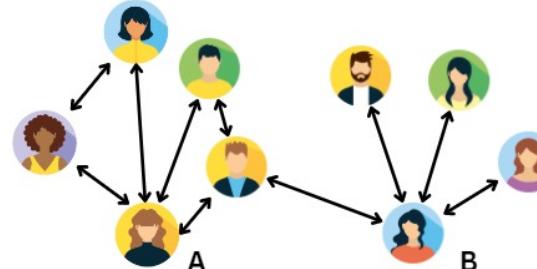
Various Types of Graph-Structured Data



Brain Networks

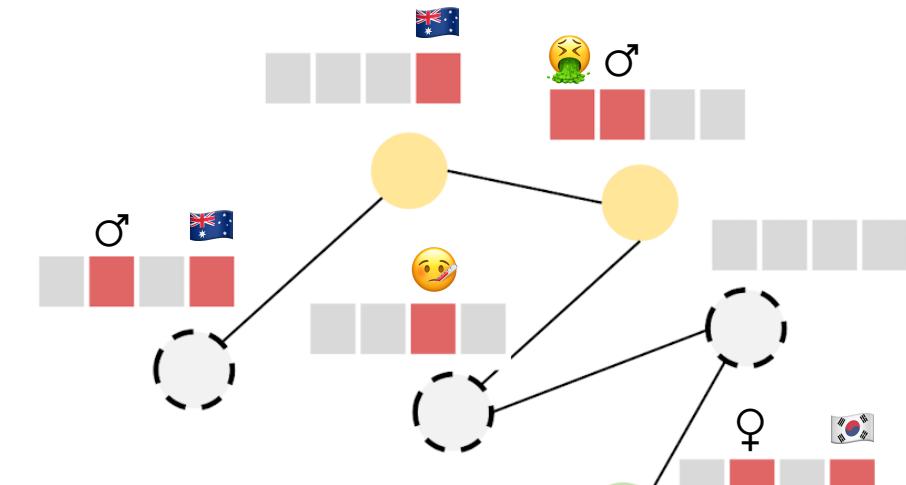


Chemical Compounds



Social Networks

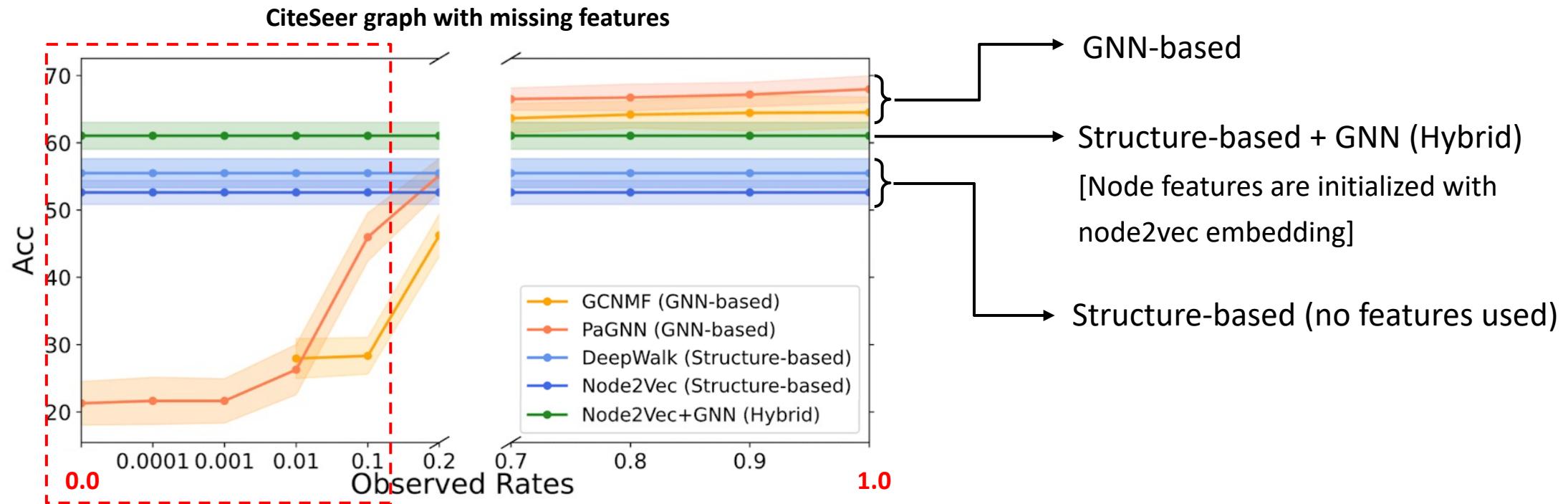
Real-world Scenarios



Missing Features

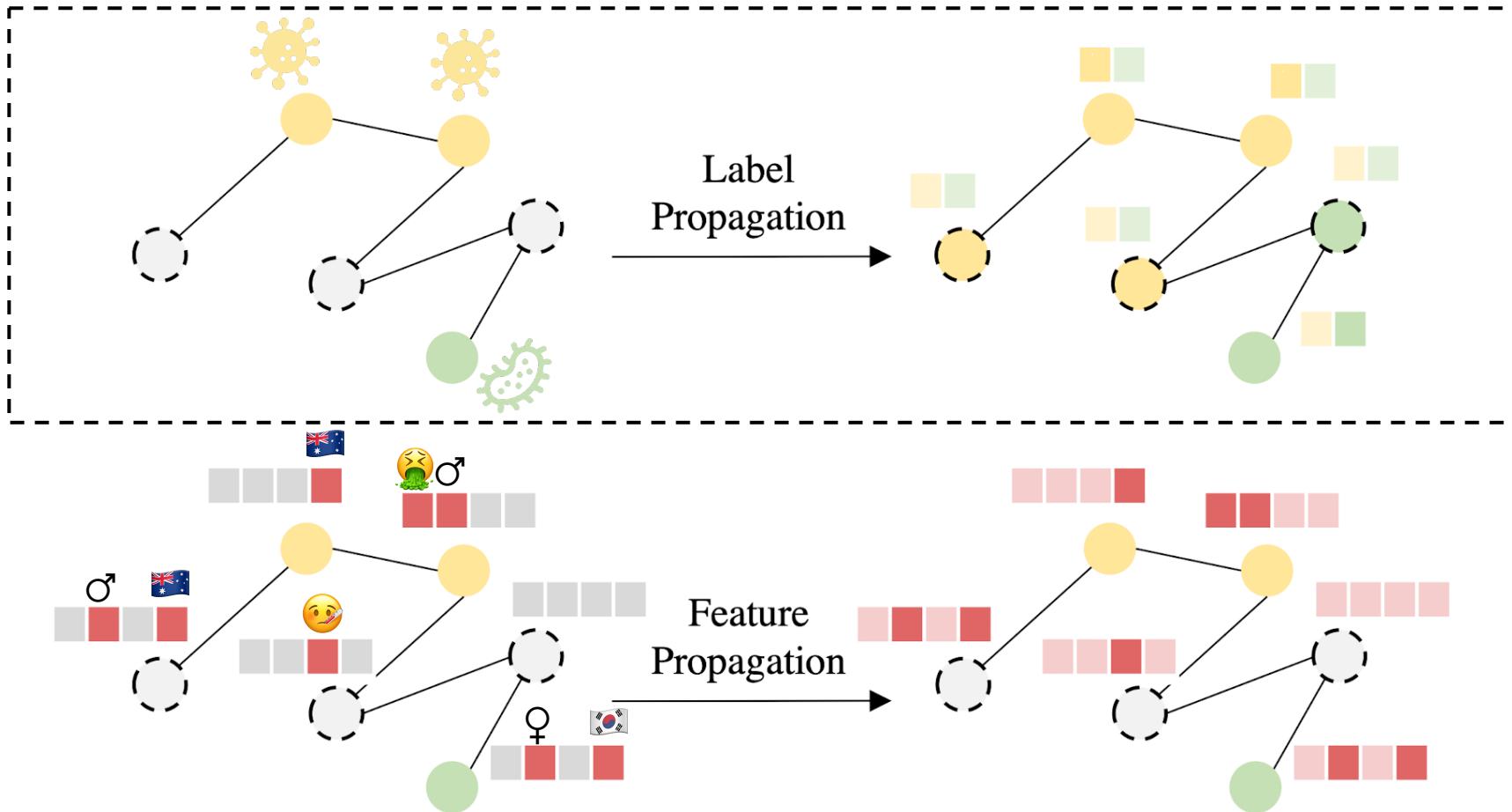
In real-world scenarios, **features are not always available** for graph data.

# EMPIRICAL MOTIVATION



For graphs with **severely missing features** (observed rate  $\leq 0.1$ ),  
**structure-based models outperform GNN-based models.**

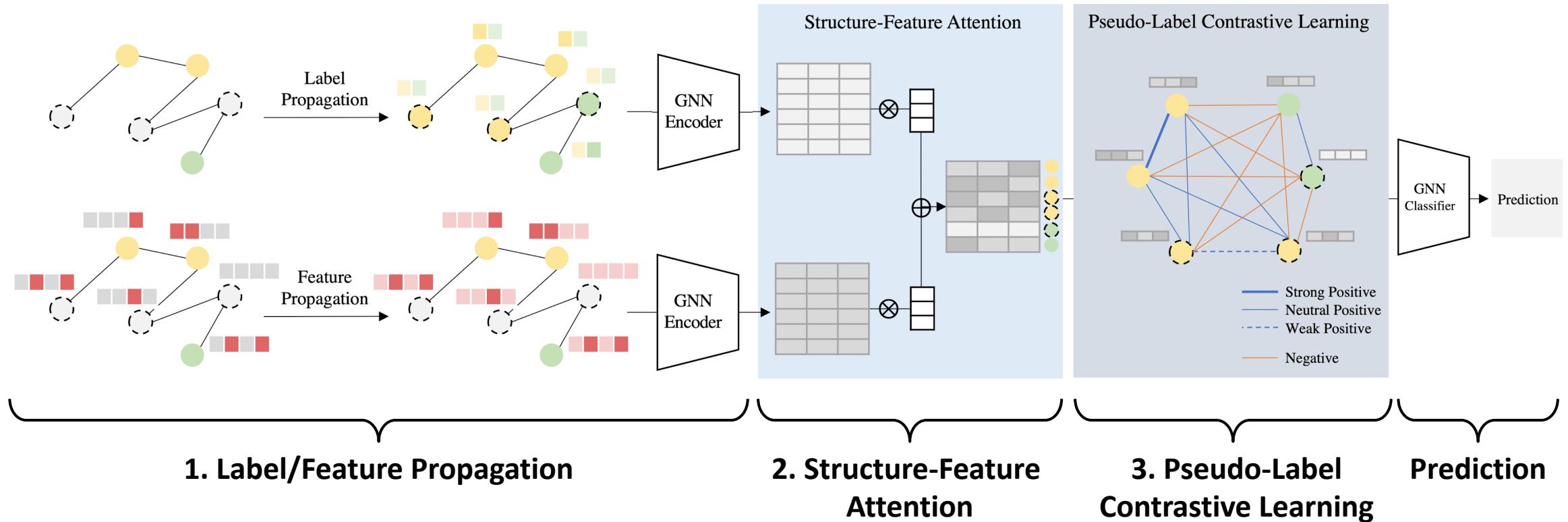
# LABEL PROPAGATION (OLDIE)



We revisit classical **Label Propagation (Oldie)** as an effective remedy for missing features, thanks to its ability to exploit both structure and label information.

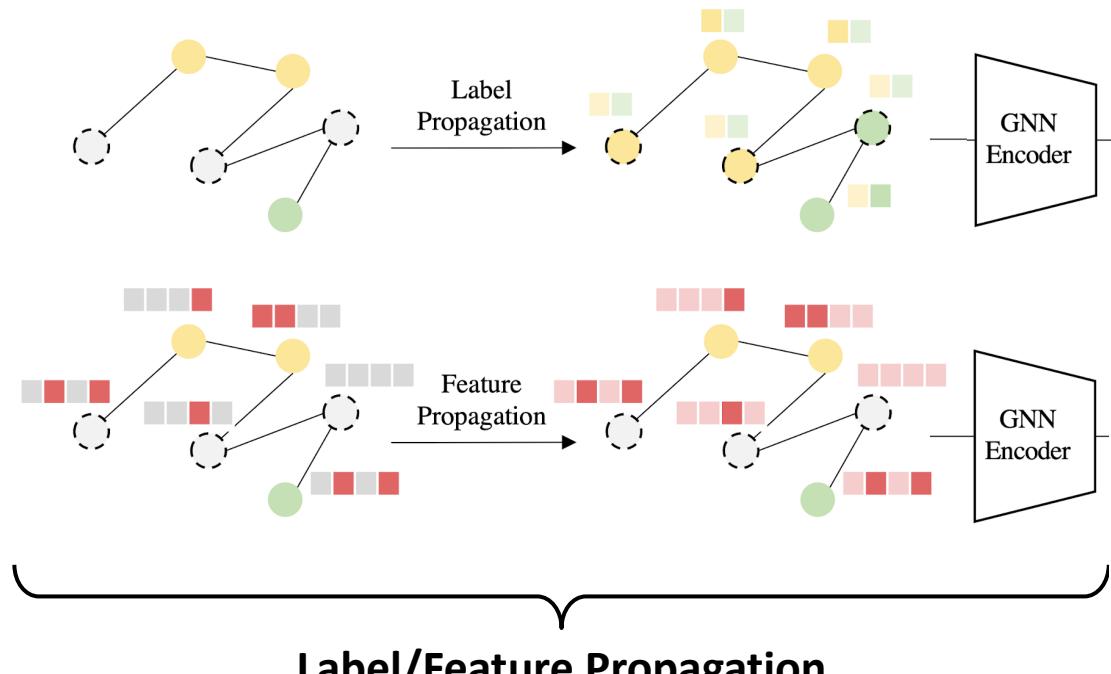
# METHODOLOGY (GOODIE)

We propose a hybrid approach that **bridges the gap between the traditional structure-based model and the recent GNN-based model, especially when only a few features are available.**



# METHODOLOGY

## ► Step 1. LP & FP branch



- Label Propagation

$$\mathbf{Y}^{(k+1)} = \hat{\mathbf{A}}_{sym} \mathbf{Y}^{(k)},$$

$$\mathbf{y}_i^{(k+1)} = \mathbf{y}_i^{(0)}, \forall i \leq m \quad \dots \text{replace with the known label if it exists}$$

$$\mathbf{H}^{LP} = \sigma(\hat{\mathbf{A}}_{sym} \hat{\mathbf{Y}} \mathbf{W}_{LP}), \quad \hat{\mathbf{Y}} = \mathbf{Y}^{(K)}$$

- Feature Propagation

$$\mathbf{X}^{(k+1)} = \hat{\mathbf{A}}_{sym} \mathbf{X}^{(k)},$$

$$\mathbf{x}_{i,d}^{(k+1)} = \mathbf{x}_{i,d}^{(0)}, \forall i \in \mathcal{V}_{known,d}, \forall d$$

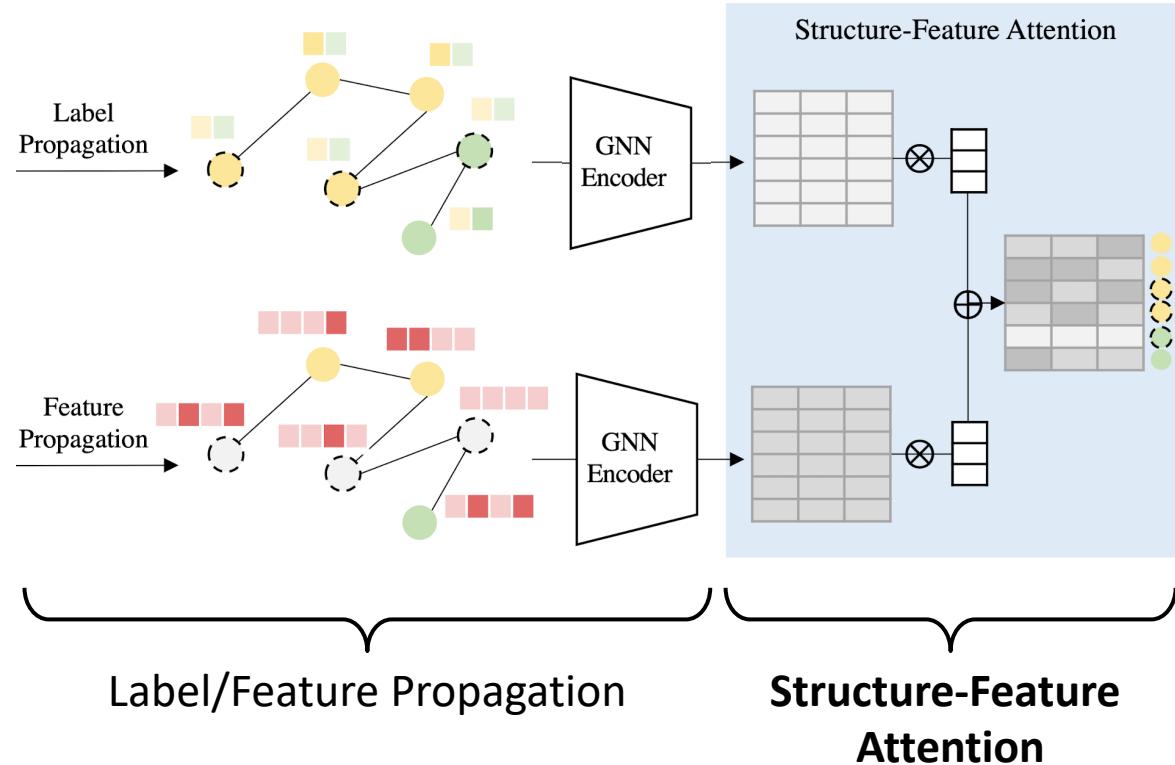
... replace with the known feature if available

$$\mathbf{H}^{FP} = \sigma(\hat{\mathbf{A}}_{sym} \hat{\mathbf{X}} \mathbf{W}_{FP}), \quad \hat{\mathbf{X}} = \mathbf{X}^{(K)}$$

We **compensate for missing label or feature values using a propagation** method until each converges at iteration  $K$ , and project both onto an embedding space.

# METHODOLOGY

## ► Step 2. Structure-Feature Attention



Low when features are sparse

High when features are abundant

$$\mathbf{z}_i = \alpha_{i,LP} \mathbf{h}_i^{LP} + \alpha_{i,FP} \mathbf{h}_i^{FP}$$

$$\alpha_{i,LP} = \frac{\exp(\text{LeakyReLU}(\mathbf{a}^\top \mathbf{h}_i^{LP}))}{\exp(\text{LeakyReLU}(\mathbf{a}^\top \mathbf{h}_i^{LP})) + \exp(\text{LeakyReLU}(\mathbf{a}^\top \mathbf{h}_i^{FP}))}$$

$$\alpha_{i,FP} = \frac{\exp(\text{LeakyReLU}(\mathbf{a}^\top \mathbf{h}_i^{FP}))}{\exp(\text{LeakyReLU}(\mathbf{a}^\top \mathbf{h}_i^{LP})) + \exp(\text{LeakyReLU}(\mathbf{a}^\top \mathbf{h}_i^{FP}))}$$

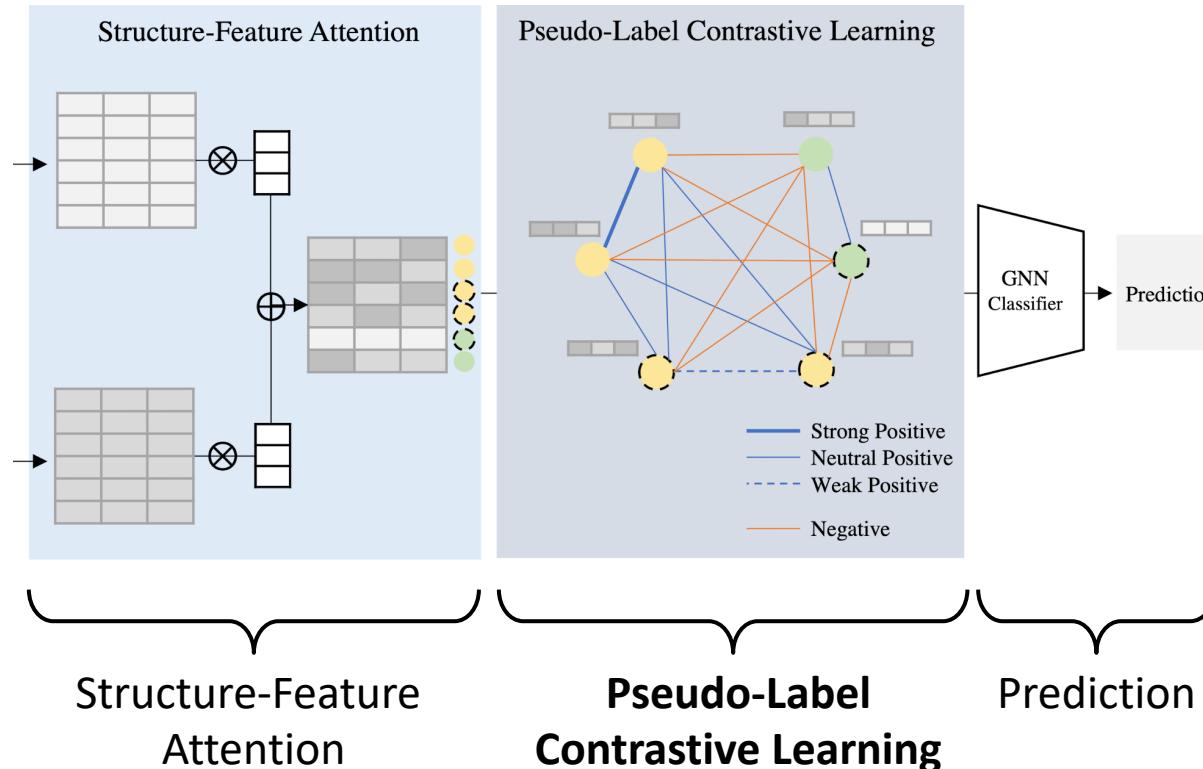
$$1 - \alpha_{i,FP}$$

$$\mathcal{L}_{ce} = \sum_{v \in \mathcal{V}_{tr}} \sum_{c \in C} CE(\mathbf{p}_v[c]), \mathbf{P} = \text{softmax}(\sigma(\hat{\mathbf{A}}_{sym} \mathbf{Z} \mathbf{W}_{cls}))$$

Due to the uncertainty in missing feature rates, we **adaptively capture attention between embeddings from each label and feature branch.**

# METHODOLOGY

## ► Step 3. Pseudo-Label Contrastive Learning (PseudoCon)



- Vanilla SupCon [1] loss

$$\mathcal{L}_{\text{sup}} = \sum_{i \in I} \frac{-1}{|P(i)|} \sum_{p \in P(i)} \log \frac{\exp(\mathbf{z}_i \cdot \mathbf{z}_p / \tau)}{\sum_{a \in A(i)} \exp(\mathbf{z}_i \cdot \mathbf{z}_a / \tau)}$$

- Pseudo-Label Contrastive loss (Ours)

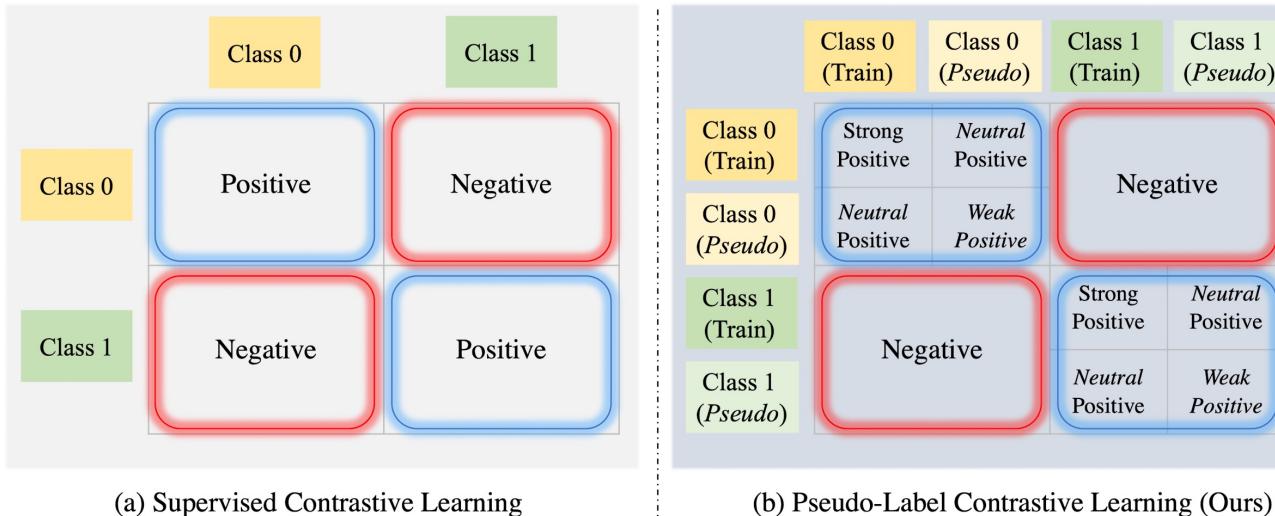
$$\mathcal{L}_{\text{pseudo}} = \sum_{i \in I} \frac{-1}{|P(i)|} \sum_{p \in P(i)} \boxed{w_{ip}} \cdot \log \frac{\exp(\mathbf{z}_i \cdot \mathbf{z}_p / \tau)}{\sum_{a \in A(i)} \exp(\mathbf{z}_i \cdot \mathbf{z}_a / \tau)}$$

[1] Supervised Contrastive Learning (NeurIPS 2020)

We **further leverage the potential of the LP branch by exploiting additional supervision** it provides. Specifically, we design a contrastive learning objective using the **pseudo-labels** generated by LP.

# METHODOLOGY

## ► Step 3. Pseudo-Label Contrastive Learning (PseudoCon)



- Pseudo-Label Contrastive loss (Ours)

$$\mathcal{L}_{\text{pseudo}} = \sum_{i \in I} \frac{-1}{|P(i)|} \sum_{p \in P(i)} w_{ip} \cdot \log \frac{\exp(\mathbf{z}_i \cdot \mathbf{z}_p / \tau)}{\sum_{a \in A(i)} \exp(\mathbf{z}_i \cdot \mathbf{z}_a / \tau)}$$

$$w_{ip} = \begin{cases} 1, & \text{if } i, p \in \hat{Y}_{\text{train}} \\ \tilde{y}_p, & \text{if } i \in \hat{Y}_{\text{train}} \text{ and } p \in \hat{Y}_{\text{pseudo}} \\ \tilde{y}_i, & \text{if } i \in \hat{Y}_{\text{pseudo}} \text{ and } p \in \hat{Y}_{\text{train}} \\ \tilde{y}_i * \tilde{y}_p, & \text{if } i, p \in \hat{Y}_{\text{pseudo}} \end{cases}$$

$$0 \leq \underbrace{\tilde{y}_i * \tilde{y}_p}_{\text{Weak Positive}} < \underbrace{\tilde{y}_i, \tilde{y}_p}_{\text{Neutral Positive}} \leq \underbrace{1}_{\text{Strong Positive}}$$

Final Loss of GOODIE:  $\mathcal{L}_{\text{final}} = \mathcal{L}_{\text{ce}} + \lambda \mathcal{L}_{\text{pseudo}}$

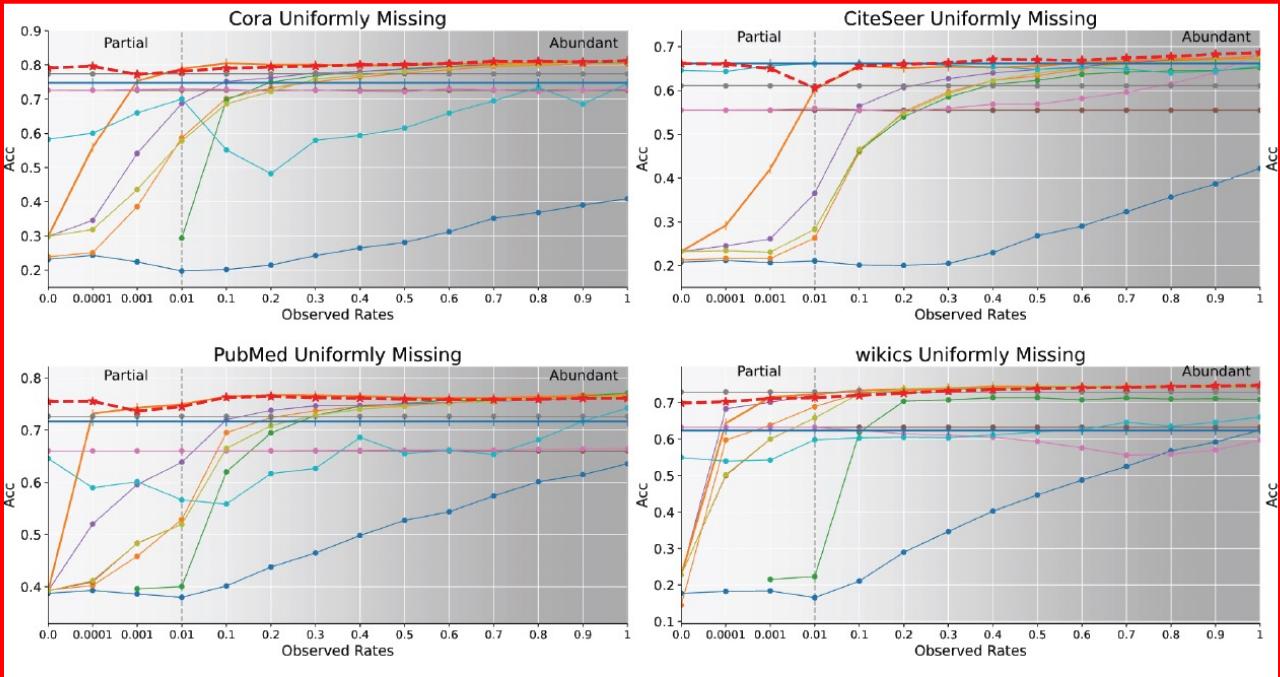
We adjust the **supervision weight based on the reliability of the labels.**

Cora
CiteSeer
PubMed
WikiCS
Coauthor CS
Coauthor Physics
OGBN-Arxiv

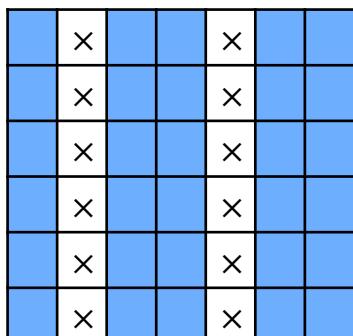
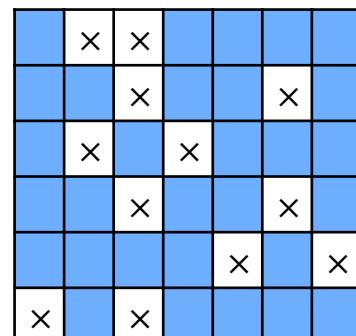
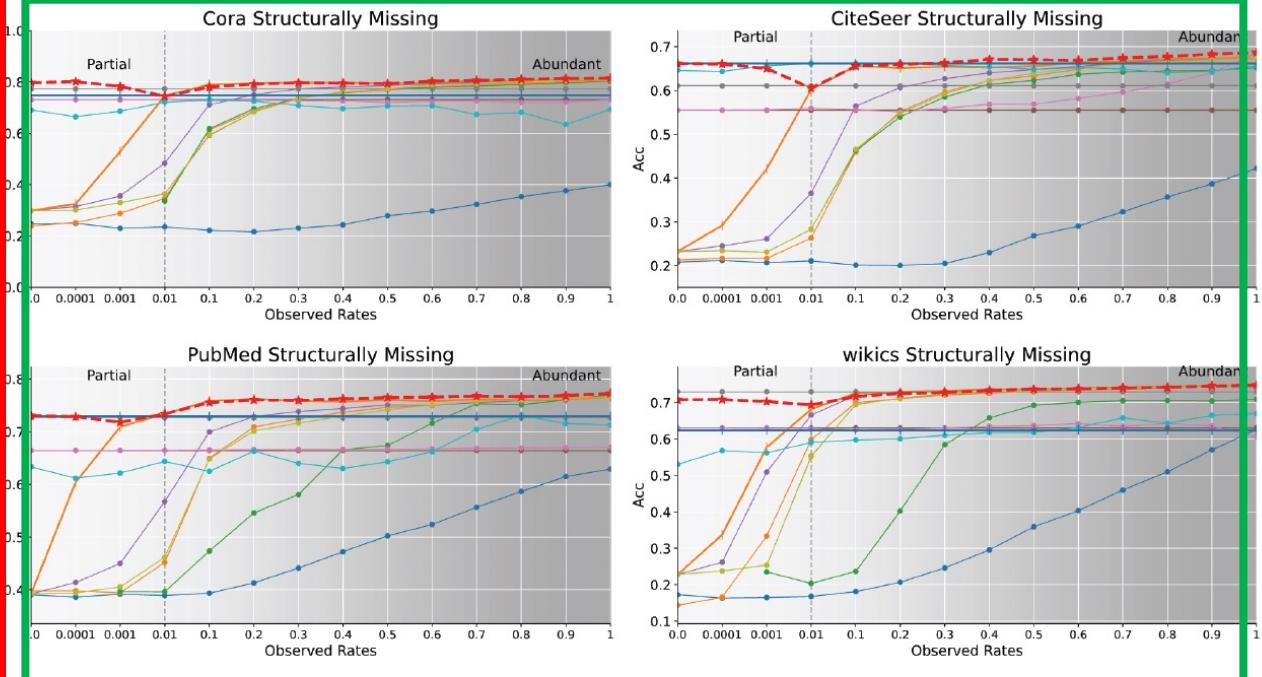
# EXPERIMENTS

## ► 1. Performance (%) of GOODIE on **Node Classification** across various observed feature rates

### Uniformly Missing



### Structurally Missing



- MLP
- Node2Vec+GNN
- Node2Vec
- GCN-LPA
- Node2Vec+Feats.
- C&S
- LP
- GCN
- GCN-NM
- GCNMF
- PaGNN
- FP
- GOODIE

# EXPERIMENTS

- 2. Performance (%) of GOODIE on **Node Classification** at missing rate (mr) of 99.99%

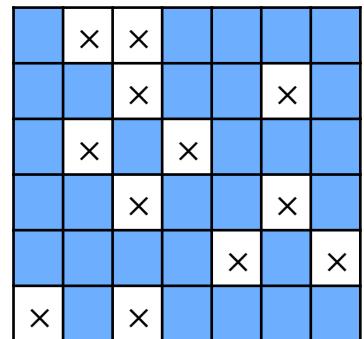
Dataset	GNN-based		Hybrid	Label/Feature Propagation		
	PaGNN	GCN-LPA	C&S	LP	FP	GOODIE
Cora	25.1±4.61	31.88±2.17	60.02±5.06	74.77±1.00	55.89±7.46	<b>79.58±1.01</b>
CiteSeer	22.6±1.77	24.24±1.07	65.55±1.61	66.15±1.67	51.06±4.17	<b>66.44±1.41</b>
PubMed	40.26±1.22	41.17±1.18	58.98±10.05	72.32±4.35	73.18±1.51	<b>75.54±0.65</b>
WikiCS	59.87±2.06	50.23±4.61	53.99±8.16	62.39±3.03	64.36±9.22	<b>70.28±2.57</b>
Co. CS	27.79±2.87	36.58±1.53	53.99±8.16	76.54±1.52	<b>78.54±0.63</b>	76.38±1.65
Co. Physics	44.16±5.89	53.34±1.44	45.03±23.68	85.86±1.91	<b>87.92±1.55</b>	87.86±1.61
OGBN-Arxiv	44.69±0.41	22.15±5.86	67.72±0.14	67.36±0.0	63.72±0.54	<b>69.04±0.14</b>
Average	38.78	37.08	57.90	72.20	67.81	<b>75.02</b>

# EXPERIMENTS

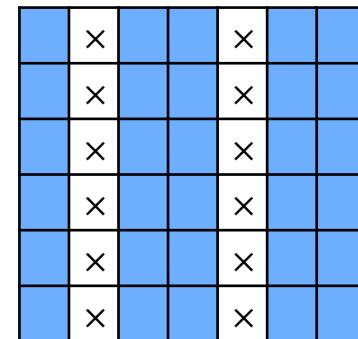
## ► 3. Performance (%) of GOODIE on **Link Prediction** at missing rate (mr) 0.9999

Dataset		Uniformly Missing				Structurally Missing			
		GCN	GCN-NM	FP	GOODIE	GCN	GCN-NM	FP	GOODIE
Cora	AUC	0.50±0.02	0.54±0.04	0.82±0.02	<b>0.84±0.02</b>	0.50±0.02	0.50±0.01	0.60±0.12	<b>0.84±0.02</b>
	AP	0.51±0.01	0.55±0.03	0.82±0.03	<b>0.85±0.01</b>	0.51±0.02	0.51±0.01	0.59±0.11	<b>0.84±0.02</b>
CiteSeer	AUC	0.50±0.02	0.50±0.04	0.75±0.09	<b>0.81±0.03</b>	0.48±0.02	0.50±0.02	0.60±0.12	<b>0.81±0.02</b>
	AP	0.50±0.01	0.53±0.04	0.76±0.10	<b>0.83±0.04</b>	0.50±0.01	0.50±0.02	0.59±0.12	<b>0.84±0.02</b>
PubMed	AUC	0.50±0.02	0.59±0.03	0.67±0.09	<b>0.75±0.04</b>	0.44±0.03	0.50±0.03	0.64±0.12	<b>0.70±0.04</b>
	AP	0.52±0.02	0.64±0.03	0.70±0.10	<b>0.79±0.04</b>	0.47±0.02	0.50±0.03	0.65±0.11	<b>0.74±0.04</b>
Coauthor CS	AUC	0.68±0.02	0.87±0.01	0.90±0.02	<b>0.93±0.01</b>	0.53±0.01	0.54±0.03	0.79±0.13	<b>0.87±0.04</b>
	AP	0.69±0.02	0.87±0.02	0.88±0.03	<b>0.93±0.01</b>	0.53±0.01	0.55±0.03	0.77±0.12	<b>0.86±0.05</b>

- Uniformly Missing

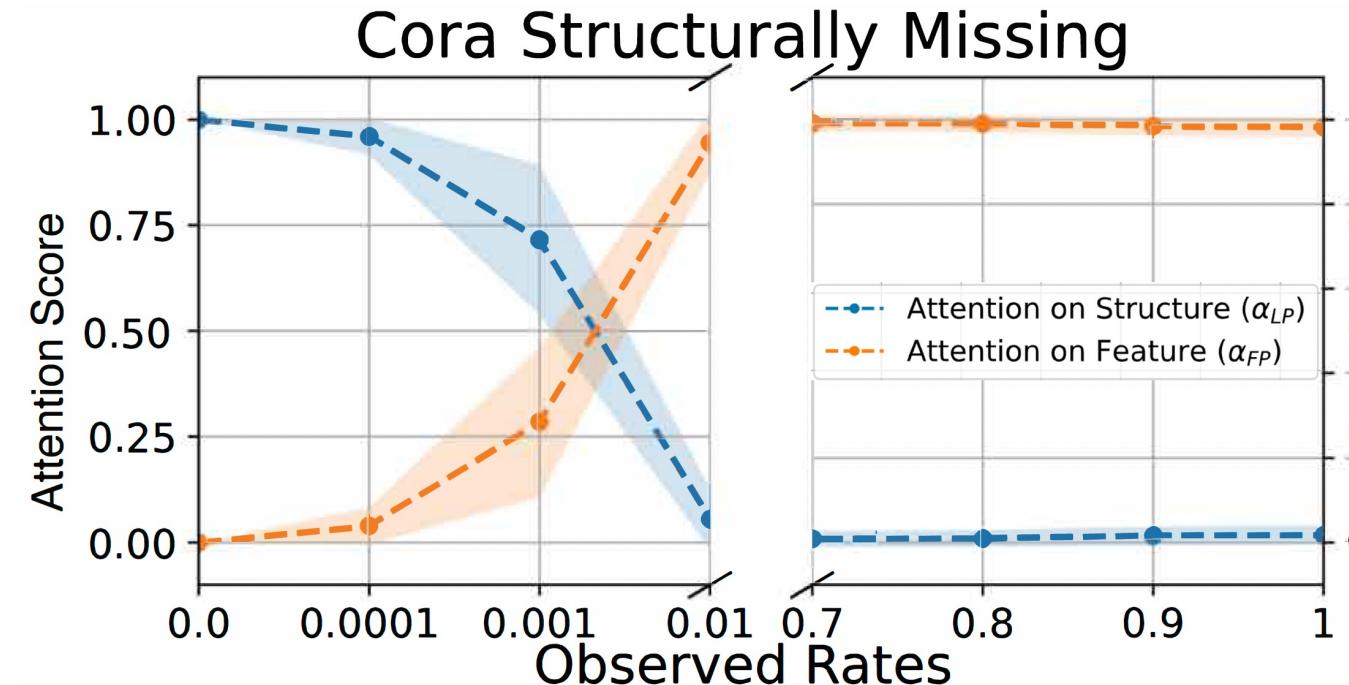
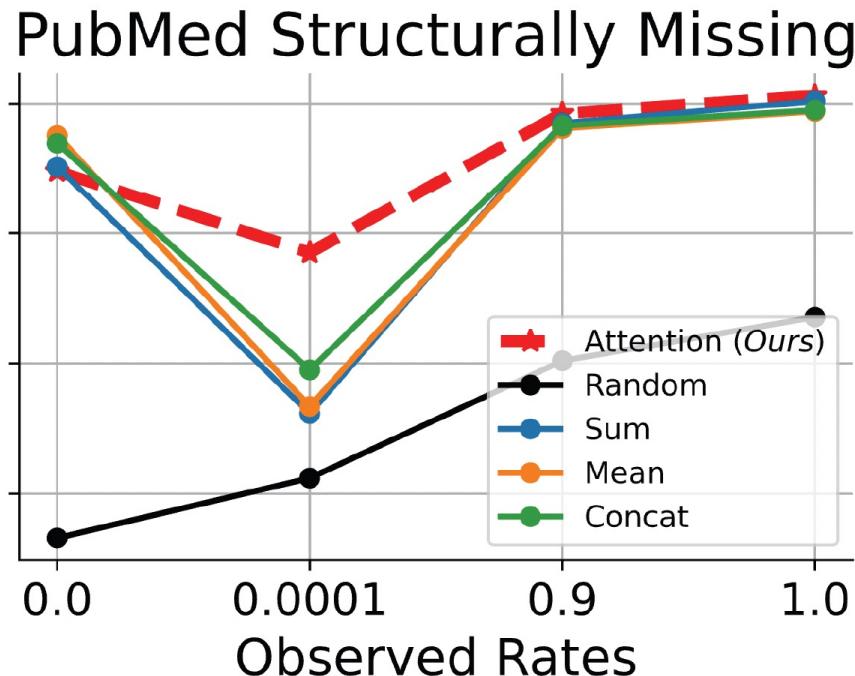


- Structurally Missing



# EXPERIMENTS

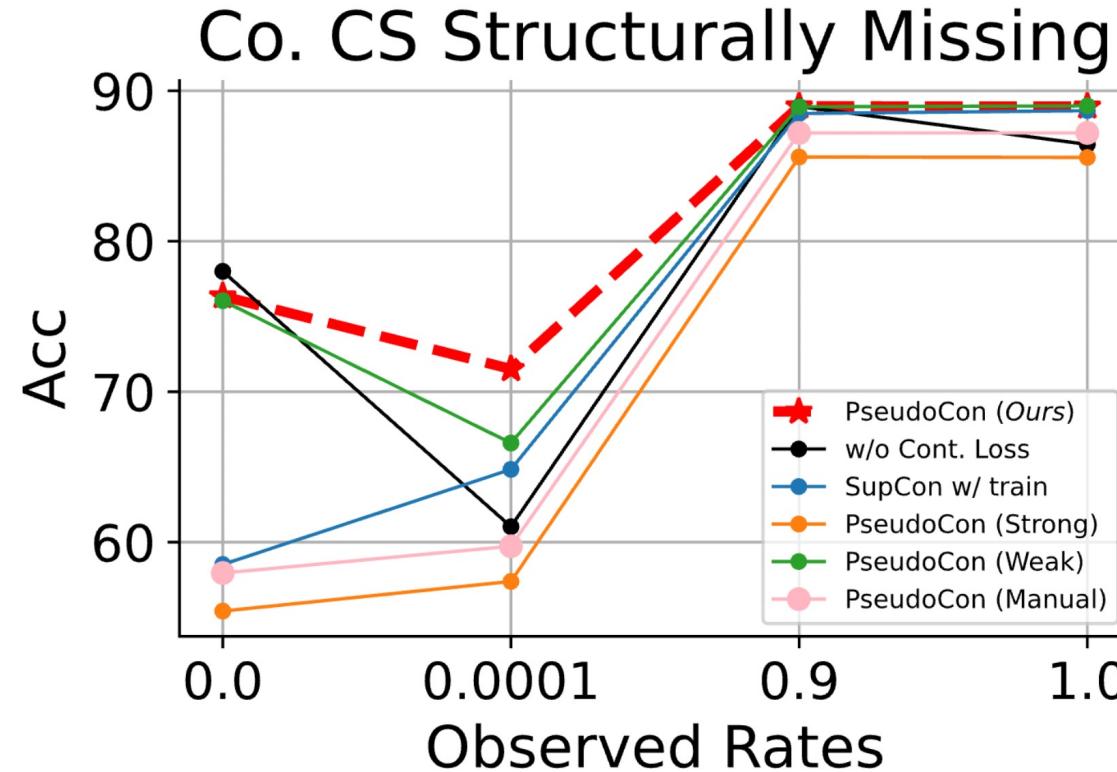
## ► 4. Effectiveness of Structure-Feature Attention and its Score at Different Observed Rates



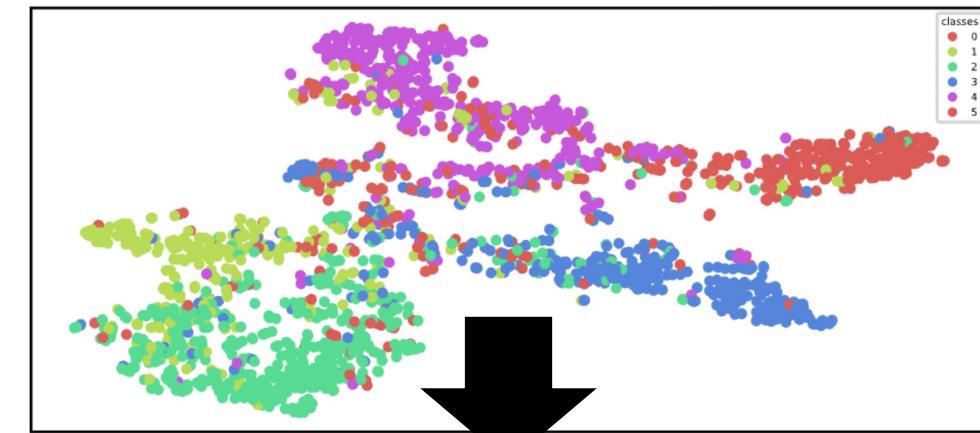
When handling embeddings from the FP and LP branches, **attention appears to be the most effective** strategy, as it naturally **places more weight on the FP branch as the observed rates increase**.

# EXPERIMENTS

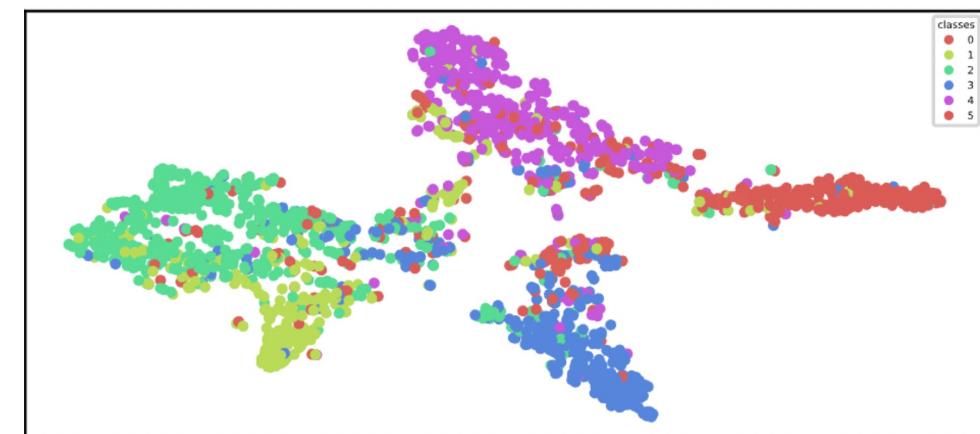
## ► 5. Effectiveness of PseudoCon Loss



GOODIE w/o PseudoCon ( $mr : 0.0$ )

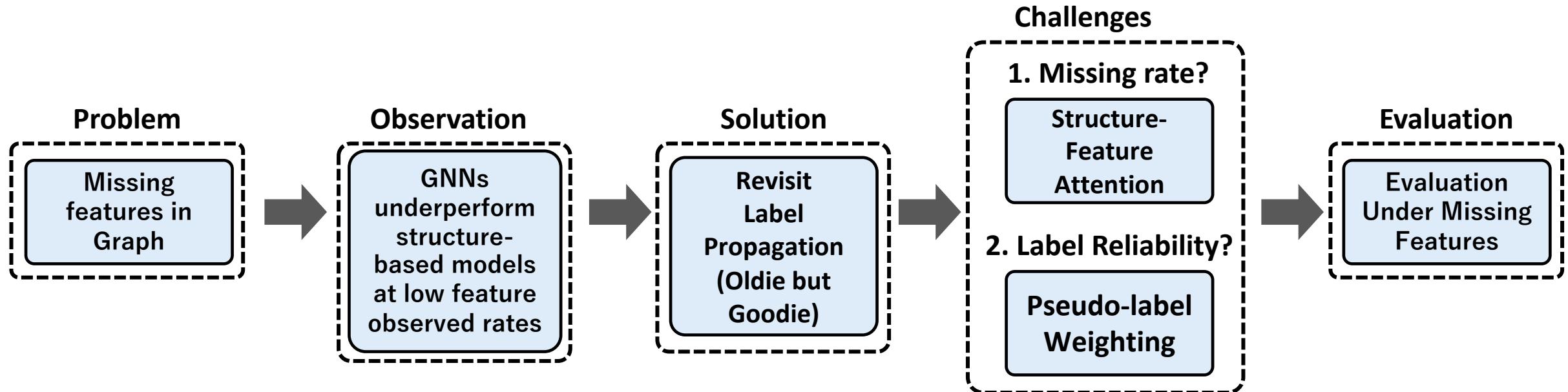


GOODIE ( $mr : 0.0$ )



PseudoCon appears **effective when both strong and weak positive pairs are involved**, as it **enhances intra-class embedding cohesion** while promoting inter-class separation.

# CONCLUSION



**Oldie but Goodie: Re-illuminating Label Propagation on Graphs with Partially Observed Features**

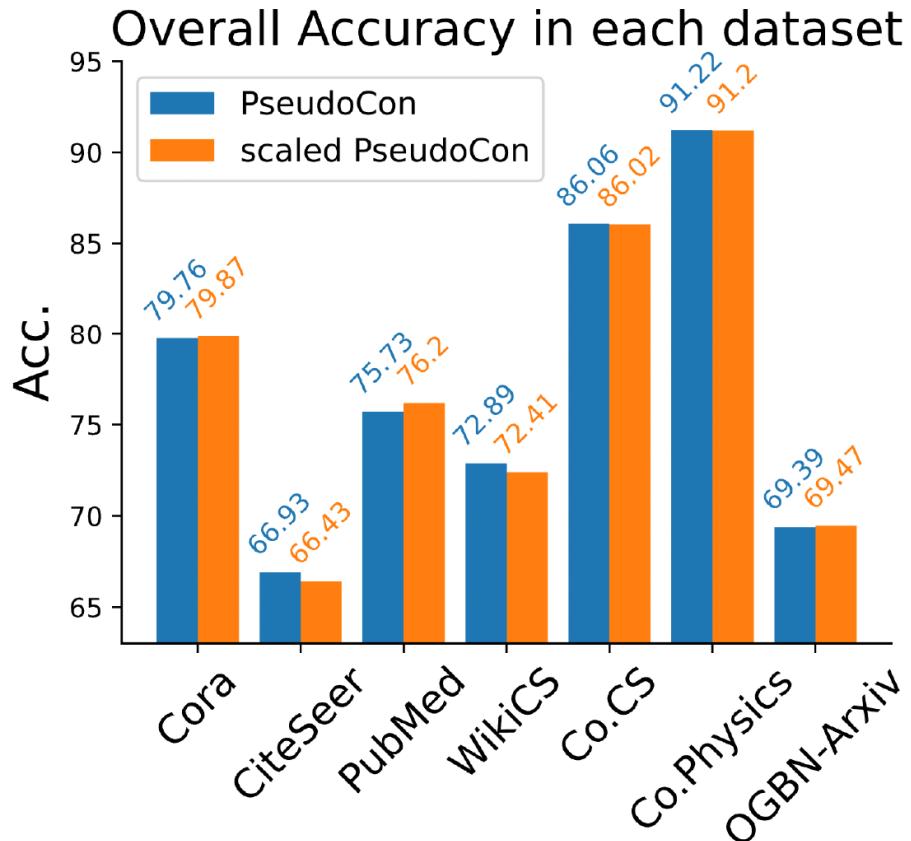
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Lab homepage: <https://dsail.kaist.ac.kr/>

# APPENDIX

# ENHANCING SCALABILITY OF PSEUDO-LABEL CONTRASTIVE LEARNING

## ► Comparison with Scaled PseudoCon Loss



(1) **PseudoCon Loss** →  $\mathcal{O}(N^2)$ ,  $N = \text{number of nodes}$

$$\mathcal{L}_{\text{pseudo}} = \sum_{i \in I} \frac{-1}{|P(i)|} \sum_{p \in P(i)} w_{ip} \cdot \log \frac{\exp(\mathbf{z}_i \cdot \mathbf{z}_p / \tau)}{\sum_{a \in A(i)} \exp(\mathbf{z}_i \cdot \mathbf{z}_a / \tau)}$$

(2) **Scaled PseudoCon Loss** →  $\mathcal{O}(C^2)$ ,  $C = \text{number of classes}$

$$\mathcal{L}_{\text{pseudo}} = - \sum_{c \in C} \log \frac{1}{\sum_{b \in B(c)} \exp(\mathbf{z}^c \cdot \mathbf{z}^b / \tau)}$$

where,  $\mathbf{z}^c = \frac{1}{|\hat{Y}^c|} \left( \underbrace{\sum_{i \in \hat{Y}_{\text{train}}^c} 1 \cdot \mathbf{z}_i}_{\text{Strong}} + \underbrace{\sum_{j \in \hat{Y}_{\text{pseudo}}^c} \tilde{y}_j \cdot \mathbf{z}_j}_{\text{Neutral \& Weak}} \right)$

**Class Prototypes**

**Strong**

**Neutral & Weak**

To reduce PseudoCon Loss complexity  $\mathcal{O}(N^2)$ , we introduce **scaled PseudoCon**, a **class prototype-based variant** that maintains strong performance with improved efficiency,  $\mathcal{O}(C^2)$ .

# METHODOLOGY

## Algorithm

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### Algorithm 1 Pseudocode for training GOODIE

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**Require:** A graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , Partially observed feature matrix  $\mathbf{X}^{(0)}$ , and train label matrix  $\mathbf{Y}^{(0)}$

**Ensure:** Final prediction matrix  $\mathbf{P} \in \mathbb{R}^{N \times |C|}$

- 1:  $\hat{\mathbf{A}}_{sym} = \tilde{\mathbf{D}}^{-1/2} \tilde{\mathbf{A}} \tilde{\mathbf{D}}^{-1/2}$
- 2:  $\hat{\mathbf{Y}}, \hat{\mathbf{X}}, \mathcal{W} = \text{PROPAGATION}(\mathbf{Y}^{(0)}, \mathbf{X}^{(0)})$
- 3: **while** Convergence **do**
- 4:    $\mathbf{P}, \mathbf{Z} = \text{TRAIN GOODIE}(\hat{\mathbf{Y}}, \hat{\mathbf{X}})$
- 5:    $\mathcal{L}_{ce} = \sum_{v \in \mathcal{V}_{tr}} \sum_{c \in C} CE(\mathbf{p}_v[c])$
- 6:    $\mathcal{L}_{pseudo} = \sum_{i \in I} \frac{-1}{|P(i)|} \sum_{p \in P(i)} w_{ip} \cdot \log \frac{\exp(\mathbf{z}_i \cdot \mathbf{z}_p / \tau)}{\sum_{a \in A(i)} \exp(\mathbf{z}_i \cdot \mathbf{z}_a / \tau)}$
- 7:    $\mathcal{L}_{final} = \mathcal{L}_{ce} + \lambda \mathcal{L}_{pseudo}$
- 8: **end while**

```

9: function TRAIN_GOODIE( $\hat{\mathbf{Y}}, \hat{\mathbf{X}}$ )
10:    $\mathbf{H}^{LP} = \sigma(\hat{\mathbf{A}}_{sym} \hat{\mathbf{Y}} \mathbf{W}_{LP})$ 
11:    $\mathbf{H}^{FP} = \sigma(\hat{\mathbf{A}}_{sym} \hat{\mathbf{X}} \mathbf{W}_{HP})$ 
12:   for  $i \in \mathcal{V}$  do
13:      $\alpha_{i,LP} = \frac{\exp(\text{LeakyReLU}(\mathbf{a}^\top \mathbf{h}_i^{LP}))}{\exp(\text{LeakyReLU}(\mathbf{a}^\top \mathbf{h}_i^{LP})) + \exp(\text{LeakyReLU}(\mathbf{a}^\top \mathbf{h}_i^{FP}))}$ 
14:      $\alpha_{i,FP} = \frac{\exp(\text{LeakyReLU}(\mathbf{a}^\top \mathbf{h}_i^{FP}))}{\exp(\text{LeakyReLU}(\mathbf{a}^\top \mathbf{h}_i^{LP})) + \exp(\text{LeakyReLU}(\mathbf{a}^\top \mathbf{h}_i^{FP}))}$ 
15:      $\mathbf{z}_i = \alpha_{i,LP} \mathbf{h}_i^{LP} + \alpha_{i,FP} \mathbf{h}_i^{FP}$ 
16:   end for
17:   return  $\text{softmax}(\sigma(\hat{\mathbf{A}}_{sym} \mathbf{Z} \mathbf{W}_{cls}))$ ,  $\mathbf{Z}$ 
18: end function
19: function PROPAGATION( $\mathbf{Y}^{(0)}, \mathbf{X}^{(0)}$ )
20:   for  $k = 0; k < K; k = k + 1$  do
21:      $\mathbf{Y}^{(k+1)} \leftarrow \hat{\mathbf{A}}_{sym} \mathbf{Y}^{(k)}$ 
22:      $\mathbf{y}_i^{(k+1)} \leftarrow \mathbf{y}_i^{(0)}, \forall i \leq m.$ 
23:   end for
24:    $\mathcal{W} = \text{WEIGHT CALCULATOR}(\mathbf{Y}^{(K)})$ 
25:   for  $k = 0; k < K; k = k + 1$  do
26:      $\mathbf{X}^{(k+1)} \leftarrow \hat{\mathbf{A}}_{sym} \mathbf{X}^{(k)}$ 
27:      $\mathbf{x}_{i,d}^{(k+1)} = \mathbf{x}_{i,d}^{(0)}, \forall i \in \mathcal{V}_{known,d}, \forall d.$ 
28:   end for
29:   return  $\mathbf{Y}^{(K)}, \mathbf{X}^{(K)}, \mathcal{W}$ 
30: end function
31: function WEIGHT_CALCULATOR( $\hat{\mathbf{Y}}$ )
32:    $\tilde{\mathbf{y}} = \max(\text{softmax}(\hat{\mathbf{Y}}))$ 
33:    $\mathcal{W} = [\mathbf{0}, \dots, \mathbf{0}]^T$ 
34:   for  $i = 0; i < N; i = i + 1$  do
35:     for  $j = i; j < N; j = j + 1$  do
36:       if  $\text{argmax}(\tilde{\mathbf{y}}_i) == \text{argmax}(\tilde{\mathbf{y}}_j)$  then
37:          $p \leftarrow j$ 
38:          $w_{ip} = \begin{cases} 1, & \text{if } i, p \in \hat{\mathcal{Y}}_{train} \\ \tilde{y}_p, & \text{if } i \in \hat{\mathcal{Y}}_{train} \text{ and } p \in \hat{\mathcal{Y}}_{pseudo} \\ \tilde{y}_i, & \text{if } i \in \hat{\mathcal{Y}}_{pseudo} \text{ and } p \in \hat{\mathcal{Y}}_{train} \\ \tilde{y}_i * \tilde{y}_p, & \text{if } i, p \in \hat{\mathcal{Y}}_{pseudo} \end{cases}$ 
39:         end if
40:         end for
41:     end for
42:     return  $\mathcal{W}$ 
43: end function

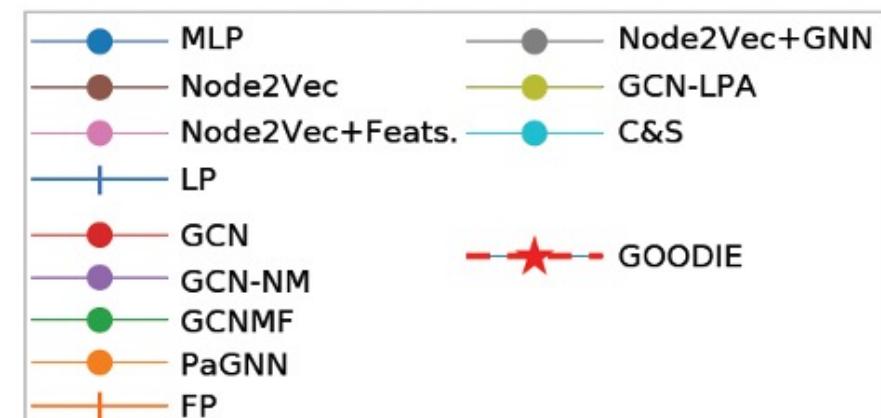
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# EXPERIMENTS

## Datasets and Baselines

Dataset	URL link to the dataset
Cora	<a href="https://github.com/shchur/gnn-benchmark">https://github.com/shchur/gnn-benchmark</a>
CiteSeer	<a href="https://github.com/shchur/gnn-benchmark">https://github.com/shchur/gnn-benchmark</a>
PubMed	<a href="https://github.com/shchur/gnn-benchmark">https://github.com/shchur/gnn-benchmark</a>
WikiCS	<a href="https://github.com/pmernyei/wiki-cs-dataset">https://github.com/pmernyei/wiki-cs-dataset</a>
Coauthor CS	<a href="https://github.com/shchur/gnn-benchmark">https://github.com/shchur/gnn-benchmark</a>
Coauthor Physics	<a href="https://github.com/shchur/gnn-benchmark">https://github.com/shchur/gnn-benchmark</a>
OGBN-Arxiv	<a href="https://ogb.stanford.edu/docs/nodeprop/#ogbn-arxiv">https://ogb.stanford.edu/docs/nodeprop/#ogbn-arxiv</a>

Baseline	URL link to the code
Node2Vec	<a href="https://github.com/aditya-grover/node2vec">https://github.com/aditya-grover/node2vec</a>
FP	<a href="https://github.com/twitter-research/feature-propagation">https://github.com/twitter-research/feature-propagation</a>
GCNMF	<a href="https://github.com/marblet/GCNmf">https://github.com/marblet/GCNmf</a>
GCN	<a href="https://github.com/tkipf/pygcn">https://github.com/tkipf/pygcn</a>
GCN-NM	<a href="https://github.com/twitter-research/feature-propagation">https://github.com/twitter-research/feature-propagation</a>
PaGNN	<a href="https://github.com/twitter-research/feature-propagation">https://github.com/twitter-research/feature-propagation</a>
GCN-LPA	<a href="https://github.com/hwwang55/GCN-LPA">https://github.com/hwwang55/GCN-LPA</a>
C&S	<a href="https://github.com/CUAI/CorrectAndSmooth">https://github.com/CUAI/CorrectAndSmooth</a>



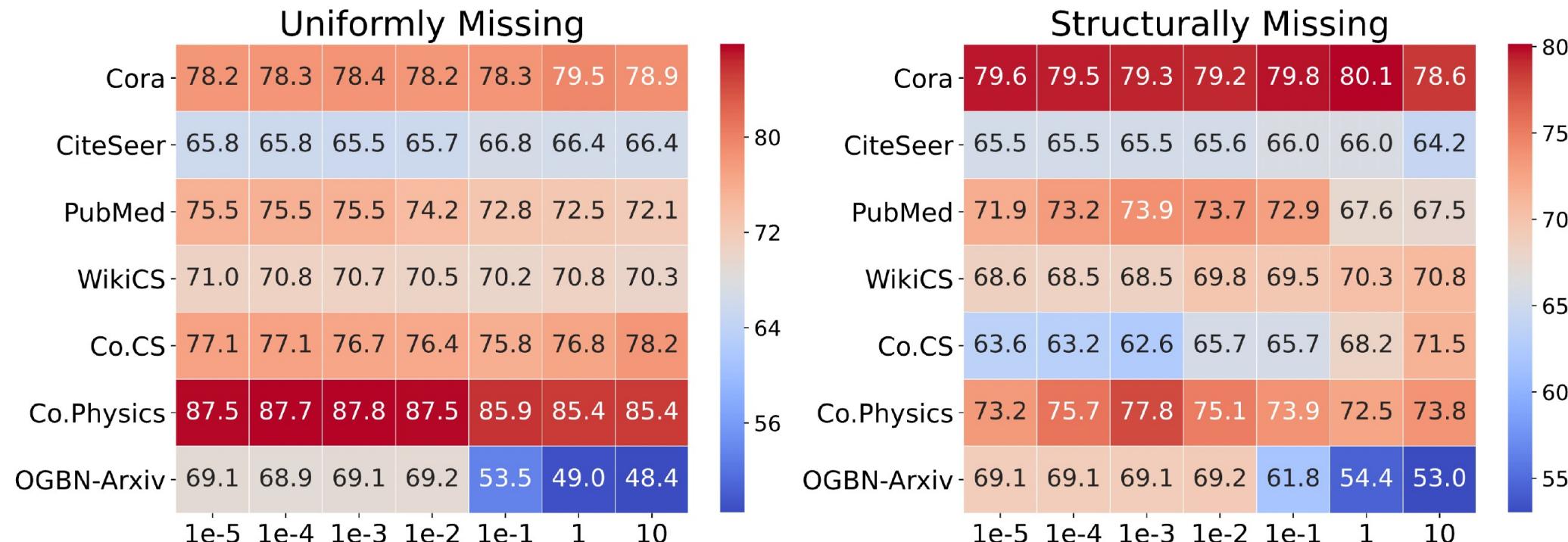
# EXPERIMENTS

## Hyperparameter Setting

Dataset	Common			Uniform	Structural
	$\alpha$	$\tau$	scaled	$\lambda$	$\lambda$
Cora	0.99	0.01	False	1	1
CiteSeer	0.999	0.01	False	1	1
PubMed	0.999	0.01	False	0.0001	0.1
WikiCS	0.9	0.01	False	10	10
Coauthor CS	0.99	0.01	False	0.01	10
Coauthor Physics	0.999	0.01	True	0.00001	0.001
OGBN-Arxiv	0.8	0.01	True	0.001	0.001

# EXPERIMENTS

- Sensitivity analysis on pseudo label controlling parameter,  $\lambda$  at missing rate (mr) 0.9999



Across diverse datasets,  $\lambda$  demonstrates robustness in the range from 1e-5 to 10.