# Code Transformation for Distributed Python ML Code

## 1 Python Abstract Syntax

```
stmt^* type\_ignore
                                                                                 (ModuleDef)
module
        ::=
              (@expr)^* def id (args) (-> expr)? : (#type:s)? stmt^*
                                                                                 (FunDef)
stmt
              (@expr)^* async def id (args) (\rightarrow expr)? : (\#type:s)? stmt^*
                                                                                (AsyncFunDef)
              (@expr)^* class id (expr^* keyword^*) : stmt^*
                                                                                 (ClassDef)
             return expr?
                                                                                 (Return)
             delete expr^*
                                                                                 (Delete)
             expr^* = expr (\#type:s)?
                                                                                 (Assign)
              expr binop = expr
                                                                                 (Augassign)
             expr : expr (= expr)?
                                                                                 (AnnAssign)
              (#type:s)? for expr in expr : stmt^* (else : stmt^*)?
                                                                                 (FORLOOP)
              (#type:s)? async for expr in expr : stmt^* (else : stmt^*)?
                                                                                (AsyncForLoop)
             while (expr) : stmt^* (else : stmt^*)?
                                                                                (WHILELOOP)
              if (expr) : stmt^* (else : stmt^*)?
                                                                                 (IF)
              (#type:s)? with with\_item^* : stmt^*
                                                                                 (WITH)
             (#type:s)? async with with_{-}item^* : stmt^*
                                                                                 (AsyncWith)
             match expr : match\_case^*
                                                                                 (Match)
             raise expr? (from expr)?
                                                                                (Raise)
             try : stmt^* exc\_handler^* (else : stmt^*)? (finally : stmt^*)?
                                                                                (TRY)
              assert expr expr?
                                                                                (Assert)
              import alias^*
                                                                                 (Import)
             from i\ id? import alias^*
                                                                                 (IMPORTFROM)
              global id^*
                                                                                 (GLOBAL)
             nonlocal id^*
                                                                                 (NonLocal)
                                                                                 (EXPRSTMT)
              expr
             pass
                                                                                 (Pass)
                                                                                 (Break)
             break
                                                                                 (CONTINUE)
              continue
```

```
(BOOLOP)
                     expr boolop expr
expr
                ::=
                              expr
                                                                                         (NAMEDEXPR)
                     expr :=
                     expr binop expr
                                                                                         (BINARYOP)
                                                                                         (UNARYOP)
                     unop expr
                     lambda args : expr
                                                                                         (LAMBDA)
                                                                                         (IFEXPR)
                     expr if expr else expr
                     \{(expr : expr)^* (**expr)^*\}
                                                                                         (DICTIONARY)
                     \{expr^*\}
                                                                                         (Set)
                      [expr^*]
                                                                                         (List)
                      (expr^*)
                                                                                         (Tuple)
                     \{expr: expr comprehension^*\}
                                                                                         (DICTIONARY COMP)
                      \{expr \ comprehension^*\}
                                                                                         (SetComp)
                      [expr comprehension^*]
                                                                                         (LISTCOMP)
                      (expr \ comprehension^*)
                                                                                         (GENERATORCOMP)
                     await expr
                                                                                         (AWAIT)
                     yield expr?
                                                                                         (Yield)
                                                                                         (YIELDFROM)
                     yield from expr
                     expr (compop \ expr)^*
                                                                                         (COMPOP)
                     expr (expr^* keyword^*)
                                                                                         (Call)
                     \{expr (!i)? (:expr)?\}
                                                                                         (FORMATTED VALUE
                     expr^*
                                                                                         (JoinedStr)
                     constant
                                                                                         (Constant)
                     expr.id
                                                                                         (Attribute)
                     expr[expr]
                                                                                         (Subscript)
                     *expr
                                                                                         (Starred)
                     **expr
                                                                                         (DoubleStarred)
                                                                                         (NAME)
                     expr? (:expr)? (:expr)?
                                                                                         (SLICE)
                     and | or
                                                                                         (BOOLOPERATOR)
boolop
                ::=
binop
                     + | - | * | @ | / | ** | << | >> | | | ^ | & | // | %
                                                                                         (BINOPERATOR)
                ::=
unop
                ::=
                     \sim | not | + | -
                                                                                         (Unoperator)
                     == | != | < | <= | > | >= | is | is not | in | not in
compop
                ::=
                                                                                         (COMPOPERATOR)
comprehension
                     for expr in expr (if expr)*
                                                                                         (Comprehension)
                ::=
                     async for expr in expr (if expr)*
                                                                                         (ASYNCCOMPREHEN
                     except expr? (as id)? : stmt*
exc\_handler
                                                                                         (EXCHANDLER)
                ::=
args
                     (arg (= expr)?)^*, (arg (= expr)?)^*, arg?, (arg (= expr)?)^*, arg?
                                                                                         (Arguments)
                ::=
arg
                ::=
                     id expr? s?
                                                                                         (Argument)
keyword
                     id? = expr
                                                                                         (Keyword)
                ::=
                     id (.id)^* (as id)?
alias
                                                                                         (ALIAS)
                ::=
with\_item
                     expr (as expr)?
                                                                                         (WITHITEM)
                ::=
```

```
case pattern (if expr)? : stmt^*
                                                       (MATCHCASE)
match\_case
             ::=
pattern
             ::=
                  expr
                                                       (MATCHVALUE)
                  constant
                                                       (MATCHSINGLETON)
                                                       (MATCHSEQUENCE)
                  [pattern*]
                  *(id)?
                                                       (MATCHSTAR)
                  \{(expr: pattern)^* id?\}
                                                       (MATCHMAPPING)
                  expr (pattern^* (id = pattern)*)
                                                       (MATCHCLASS)
                  (pattern as)? id
                                                       (MatchAs)
                  pattern | pattern
                                                       (MATCHOR)
                                                       (MATCHWILDCARD)
                                                       (NoneLiteral)
constant
             ::=
                  None
                                                       (IntLiteral)
                  i
                                                       (FLOATLITERAL)
                  c
                                                       (ComplexLiteral)
                                                       (STRINGLITERAL)
                                                       (BOOLEANLITERAL)
                  (constant^*)
                                                       (TupleLiteral)
                                                       (Ellipsis)
                  i^*
                                                       (TypeIgnore)
type\_ignore
             ::=
                  Id
id
              \in
              \in
                  Str
s
                  {True, False}
              \in
i
              \in
                  \mathbb{Z}
f
              \in \mathbb{R}
                  \mathbb{C}
c
              \in
```

Note: there may be more constant terms such as an immutable container containing only constant elements. Please update the constant rule if you know such terms.

## 2 Transformation for TF2 Python Code

#### 2.1 Restrictions

- 1. All import statements must be placed at the top of a module.
- 2. The tensorflow module must be assigned to a variable only using an import statement.
- 3. Members of the tensorflow module must not be aliased.
- 4. print function call expressions must not introduce side-effects.
- 5. A dataset and an optimizer object must only be created once via an assignment statement with a function call expression and must not be aliased.
- 6. Variables storing a dataset or a optimizer object must not be reassigned to store another value that is not a dataset or an optimizer object.
- 7. A dataset and an optimizer object must not be created conditionally.
- 8. optimizer.apply\_gradients function call expressions must be expression statements or a direct RHS of an assignment statement.
- 9. Global variables storing an optimizer object and referred in functions must be defined before the functions' definitions and must not be changed after their initializations.
- 10. A checkpoint object must only be created once via an assignment statement with a function call expression and must not be aliased.

### 2.2 Rules

#### 2.2.1 Types and Auxiliary Functions

```
\tau ::= Module
                                    (Modules)
            Stmt
                                    (STATEMENTS)
                                    (Expressions)
            Expr
            Comprehension
                                   (Comprehensions)
            ExcHandler
                                   (EXCEPTION HANDLERS)
            Alias
                                   (ALIASES)
            With Item
                                   (WITHITEMS)
            Pattern
                                    (Patterns)
            Id
                                    (IDENTIFIERS)
            Str
                                   (STRINGS)
            \tau list
                                    (List of elements typed \tau)
\sigma \in \Sigma = Str^{\text{fin}} Id
                              ENVIRONMENT STORING MAPPINGS FROM STRINGS TO IDENTIFIERS
{\tt ClassNode} = Str \times {\tt ClassNode}?
\mathtt{ns} \in \mathtt{Nodes} ::= \mathtt{ClassNode} \ \mathrm{list}
	extsf{vs} \in 	extsf{Vars} ::= Str \ 	extsf{list} \stackrel{	extsf{fin}}{	o} 	extsf{ClassNode}
\mathtt{cg} \in \mathtt{CG} = \mathtt{Nodes} 	imes \mathtt{Vars}
```

```
._1 : \tau \times \tau \to \tau Get the first element of the given pair
```

::  $\tau \to \tau$  list  $\to \tau$  list Append an element to a list (right-associative)

.nodes : ClassNode  $\rightarrow$  Nodes = .\_1 GET NODES OF CLASS GRAPH .vars : ClassNode  $\rightarrow$  Vars = .\_2 GET VARIABLES OF CLASS GRAPH

#### 2.2.2 Transformation Rules

#### A strict form of assignment statements

```
trans_{S}[id_{r} = expr_{1} \ (expr_{11} \dots expr_{1n} \ (id_{1} = )? \ expr_{21} \dots (id_{k} = )? \ expr_{2k}) \ (\#type:s)? \ ]](\sigma) = IF \ \sigma("tensor\_flow") = id_{t} \ AND \ expr_{1} = id_{t}.data.Dataset.expr_{3} \ THEN \\ ([id_{r} = expr_{1} \ (expr_{11} \dots expr_{1n} \ (id_{1} = )? \ expr_{21} \dots (id_{k} = )? \ expr_{2k}) \ (\#type:s)?], \\ \sigma["dataset" \mapsto id_{r}])
ELIF \ \sigma("tensor\_flow") = id_{t} \ AND \ expr_{1} = id_{t}.train.Checkpoint \ THEN \\ ([id_{r} = expr_{1} \ (expr_{11} \dots expr_{1n} \ (id_{1} = )? \ expr_{21} \dots (id_{k} = )? \ expr_{2k}) \ (\#type:s)?], \\ \sigma["checkpoint" \mapsto id_{r}])
ELIF \ \sigma("tensor\_flow") = id_{t} \ AND \ expr_{1} = id_{t}.optimizers.Adam \ THEN \\ IF \ id_{i} = learning\_rate \ WHEN \ 1 \leq i \leq k \ THEN \\ ([id_{r} = expr_{1} \ (expr_{11} \dots expr_{1n} \ (id_{1} = )? \ expr_{21} \dots id_{i} = expr_{2i} * hvd.size()
```

```
... (id_k = )? expr_{2k}) (#type:s)?], \sigma["optimizer" \mapsto id_r])
   ELSE
       ([id_r = expr_1 \ (expr_{11} * hvd.size() ... \ expr_{1n} \ (id_1 = )? \ expr_{21} ... \ (id_k = )? \ expr_{2k}) \ (\#type:s)?],
           \sigma["optimizer" \mapsto id_r])
ELIF \sigma ("optimizers") = id_t AND expr_1 = id_t. Adam THEN
   IF id_i = learning_rate WHEN 1 \le i \le k THEN
       ([id_r = expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... id_i = expr_{2i} * hvd.size())
           ... (id_k = )? expr_{2k}) (#type:s)?], \sigma["optimizer" \mapsto id_r])
   ELSE
       ([id_r = expr_1 \ (expr_{11} * hvd.size() ... \ expr_{1n} \ (id_1 = )? \ expr_{21} ... \ (id_k = )? \ expr_{2k}) \ (\#type:s)?],
           \sigma["optimizer" \mapsto id_r])
ELIF \sigma ("optimizer") = id_t AND expr_1 = id_t.apply_gradients THEN
   IF id_i = grads_and_vars WHEN 1 \le i \le k THEN
       LET id_z = \text{NewID}() IN
       ([id_z = expr_{2i},
       id_r = expr_1 (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots id_i = id_z \dots (id_k = )? expr_{2k}) (\#type:s)?,
       global hvd_broadcast_done,
       if not hvd_broadcast_done: [hvd.broadcast\_variables([x[1] for x in <math>id_z], root\_rank=0),
           hvd.broadcast_variables(id_t.variables(), root_rank=0),
           hvd\_broadcast\_done = True ]], \sigma)
   ELSE
       LET id_z = \text{NewID}() IN
       ([id_z = expr_{11},
       id_r = expr_1 \ (id_z \ expr_{12} \ ... \ expr_{1n} \ (id_1 = )? \ expr_{21} \ ... \ (id_k = )? \ expr_{2k}) \ (\#type:s)?
       global hvd_broadcast_done,
       if not hvd_broadcast_done: [hvd.broadcast_variables([x[1] for x in id_z], root_rank=0),
           hvd.broadcast_variables(id_t.variables(), root_rank=0),
           hvd\_broadcast\_done = True ]], \sigma)
ELIF \sigma ("checkpoint") = id_t AND expr_1 = id_t.save THEN
   ([if hvd.rank() == 0: [id_r = expr_1 (expr_{11} ... expr_{1n} (id_1 =)? expr_{21} ... (id_k =)? expr_{2k}) (#type:s)?]],
       \sigma)
ELIF \sigma(\text{``keras''}) = id_k \text{ AND } expr_1 = id_k.\text{models.Sequential } \text{THEN}
   ([id_r = expr_1 (expr_{11} ... expr_{1n} (id_2 = )? expr_{21} ... (id_k = )? expr_{2k})], \sigma["model"\mapsto id_r])
ELIF \sigma("keras") = id_t AND expr_1 = id_t.optimizers.Adam THEN
   IF id_i = learning_rate WHEN 1 \le i \le k THEN
       (id_r = expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... id_i = expr_{2i} * hvd.size()
   ... (id_k = )? expr_{2k}) (#type:s)?
       id_r = \text{hvd.DistributedOptimizer}(id_r), \sigma[\text{"optimizer"} \mapsto id_r]
   ELSE
       ([id_r = expr_1 (expr_{11} * hvd.size() ... expr_{1n} (id_1 = )? expr_{21} ... (id_k = )? expr_{2k}) (\#type:s)?
       id_r = \text{hvd.DistributedOptimizer}(id_r), \sigma[\text{"optimizer"} \mapsto id_r])
ELSE ([id_r = trans_E \parallel expr_1 (expr_{11} \dots expr_{1n} (id_2 = )? expr_{21} \dots (id_k = )? expr_{2k}) \parallel (\sigma) (\#type:s)?],
       \sigma)
```

```
trans_S \llbracket expr_0 = expr_1 \text{ (#type:}s)? \rrbracket (\sigma) =
      IF \sigma(\text{"os"}) = id_{os} AND expr_0 = id_{os}.\text{environ ['CUDA_VISIBLE_DEVICES']}
      THEN ([], \sigma)
A general form of assignment statements except for the strict form
 trans_S[\![expr_1^* = expr_2 \text{ (#type:}s)?]\!](\sigma) = ([expr_1^* = trans_E[\![expr_2]\!](\sigma) \text{ (#type:}s)?], \sigma)
 trans_{S}[\![expr_{1}\ binop = expr_{2}\ ]\!](\sigma) = ([expr_{1}\ binop = trans_{E}[\![expr_{2}\ ]\!](\sigma)], \sigma)
 trans_S \llbracket expr_1 : expr_2 (= expr_3)? \rrbracket (\sigma) = ([expr_1 : expr_2 (= trans_E \llbracket expr_3 \rrbracket (\sigma))?], \sigma)
     IF expr_1 = id_1 AND \sigma("tensor_flow") = id_2
      AND expr_3 = id_2.data.Dataset.expr_4 (expr_5^* keyword^*) THEN
          ([expr_1: expr_2 = expr_3 (\#type:s)?], \sigma["dataset" \mapsto id_1])
     ELSE ([expr_1 : expr_2 (= trans_E [expr_3 ](\sigma))?], \sigma)
 trans_{S}[ (#type:s)? for expr_{1} in expr_{2} : stmt_{1}^{*} (else : stmt_{2}^{*})? ](\sigma) =
      ([(#type:s)? for expr_1 in trans_E[\![ expr_2 ]\!](\sigma) :
          trans_{\overline{S}}[stmt_1^*](\sigma)._1 (else : trans_{\overline{S}}[stmt_2^*](\sigma)._1)?], \sigma)
 trans_S[ (#type:s)? async for expr_1 in expr_2 : stmt_1^* (else : stmt_2^*)? ](\sigma) =
      ([(#type:s)? async for expr_1 in trans_E \llbracket expr_2 \rrbracket (\sigma) :
          trans_{\overline{S}} [\![\![ stmt_1^* ]\!] (\sigma)... 1  (else : trans_{\overline{S}} [\![\![ stmt_2^* ]\!] (\sigma)... 1)?], \sigma)
 trans_{S} while (expr) : stmt_{1}^{*} (else : stmt_{2}^{*})? (\sigma) =
      ([while (trans_E[\![expr]\!](\sigma)) : trans_{\overline{S}}[\![stmt_1^*]\!](\sigma).\_1 (else : trans_{\overline{S}}[\![stmt_2^*]\!](\sigma).\_1)?], \sigma)
 trans_{S}[\![ if (expr) : stmt^{*} (else : stmt^{*})? \![ \![ \![ \![ \![ \!]
      ([if (trans_E \parallel expr \parallel (\sigma)) : trans_{\overline{S}} \parallel stmt^* \parallel (\sigma)._1 (else : trans_{\overline{S}} \parallel stmt^* \parallel (\sigma)._1)?], \sigma)
 trans_{S}[\![ (\#type:s)? with with\_item^* : stmt^* ]\!](\sigma) =
      LET with\_item^{*\prime}, \sigma_1 = trans_{\overline{W}} \llbracket with\_item^* \rrbracket (\sigma) IN
      LET stmt^{*\prime}, \sigma_2 = trans_{\overline{S}}[stmt^*](\sigma_1) IN
      IF \sigma_1 \setminus \sigma = ["gradient\_tape" \mapsto id] THEN
          ([(\#type:s)? with with\_item^{*\prime} : stmt^{*\prime},
          id = hvd.DistributedGradientTape(id), \sigma_2
      ELSE ([(#type:s)? with with\_item^{*\prime} : stmt^{*\prime}], \sigma_2)
 trans_{S}[ (\#type:s)?  async with with\_item^* : stmt^* ](\sigma) =
     LET with_item*', \sigma_1 = trans_{\overline{W}} [\![\![ with_item^* ]\!]](\sigma) IN
      LET stmt^{*}, \sigma_2 = trans_{\overline{S}} [stmt^*] (\sigma_1) IN
      IF \sigma_1 \setminus \sigma = ["gradient\_tape" \mapsto id] THEN
          ([(#type:s)? async with with_{-}item^{*\prime} : stmt^{*\prime},
          id = hvd.DistributedGradientTape(id), \sigma_2
      ELSE ([(#type:s)? async with with_item^{*\prime} : stmt^{*\prime}], \sigma_2)
```

```
trans_{S}[\![match\ expr\ :\ match\_case^*\ ]\!](\sigma) =
    ([match trans_E[\![expr]\!](\sigma) : (trans_C[\![match\_case]\!](\sigma))^*], \sigma)
trans_S raise expr_1? (from expr_2)? \|(\sigma) = ([raise \ expr_1? \ (from \ expr_2)?], \sigma)
trans_S \llbracket \text{try} : stmt_1^* = exc\_handler^* \text{ (else } : stmt_2^*)? \text{ (finally } : stmt_3^*)? \rrbracket (\sigma) =
    ([try : trans_{\overline{S}}[stmt_1^*]](\sigma)._1 (trans_H[exc\_handler](\sigma))^*
    (else : trans_{\overline{S}} [stmt_2^*](\sigma).1)? (finally : trans_{\overline{S}} [stmt_3^*](\sigma).1)?], \sigma)
trans_S[\![ assert expr_1 expr_2? [\![(\sigma) = ([assert trans_E[\![ expr_1 ]\![(\sigma) expr_2?], \sigma)
trans_S[\![\!] import alias^*[\![\!](\sigma)=
    LET \sigma_1 = trans_{\overline{A}} [\![ alias^* ]\!](\sigma) IN
   IF \sigma_1 \setminus \sigma = [\text{"tensor\_flow"} \mapsto id] THEN
       ([import alias*,
       import horovod.tensorflow as hvd,
       hvd_broadcast_done = False,
       hvd.init(),
       gpus = id.config.experimental.list_physical_devices('GPU'),
       for gpu in gpus: id.config.experimental.set_memory_growth(gpu, True),
       if gpus: id.config.experimental.set_visible_devices(gpus[hvd.local_rank()], 'GPU')], \sigma_1)
    ELSE ([import alias^*], \sigma_1)
trans_S[\![ import alias^* \![](\sigma) =
   LET \sigma_1 = trans_{\overline{A}} [\![ alias^* ]\!](\sigma) IN
   IF \sigma_1 \setminus \sigma = [\text{"tensor\_flow"} \mapsto id] THEN
       ([import alias^*,
       import horovod.tensorflow as hvd,
       hvd.init(),
       gpus = id.config.experimental.list_physical_devices('GPU'),
       for gpu in gpus: id.config.experimental.set_memory_growth(gpu, True),
       if gpus: id.config.experimental.set_visible_devices(gpus[hvd.local_rank()], 'GPU')], \sigma_1)
    ELSE ([import alias^*], \sigma_1)
trans_{S}[\![\!] from i id? import alias^* [\![\!](\sigma) =
    LET \sigma_1 = trans_{\overline{A}} [\![ alias^* ]\!] (\sigma) IN
    IF id = \text{tensorflow AND } \sigma_1 \setminus \sigma = [\text{"keras"} \mapsto id] THEN
       ([from i id? import alias^*,
       import horovod.tensorflow.keras as hvd,
       hvd.init(),
       gpus = id.config.experimental.list_physical_devices('GPU'),
       for gpu in gpus: id.config.experimental.set_memory_growth(gpu, True),
       if gpus: id.config.experimental.set_visible_devices(gpus[hvd.local_rank()], 'GPU')], \sigma_1)
    ELSE ([from i id? import alias^*], \sigma_1)
```

```
trans_S[\![global\ id^*]\!](\sigma) = ([global\ id^*], \sigma)
 trans_S[\![ nonlocal id^*]\![(\sigma) = ([nonlocal <math>id^*], \sigma)
A strict form of expr statements
 trans_{S}[expr_{1} (expr_{11} ... expr_{1n} (id_{1} = )? expr_{21} ... (id_{k} = )? expr_{2k})](\sigma) =
    IF \sigma("optimizer") = id_t AND expr_1 = id_t.apply_gradients THEN
        IF id_i = grads\_and\_vars WHEN 1 \le i \le k THEN
           LET id_z = \text{NewID}() IN
           ([id_z = expr_{2i},
           expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... id_i = id_z ... (id_k = )? expr_{2k}),
           global hvd_broadcast_done,
           if not hvd_broadcast_done: [hvd.broadcast_variables([x[1] for x in id_z], root_rank=0),
              hvd.broadcast_variables(id_t.variables(), root_rank=0),
              hvd\_broadcast\_done = True ]], \sigma)
        ELSE
           LET id_z = \text{NewID}() IN
           ([id_z = expr_{11},
           expr_1 (id_z expr_{12} \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k}),
           global hvd_broadcast_done,
           if not hvd_broadcast_done: [hvd.broadcast_variables([x[1] for x in id_z], root_rank=0),
              hvd.broadcast_variables(id_t.variables(), root_rank=0),
              hvd\_broadcast\_done = True ]], \sigma)
    ELIF expr_1 = print OR expr_1 = expr.write OR expr_1 = expr.summary OR
        expr_1 = expr.save_weights OR expr_1 = expr.load_weights THEN
           ([if hvd.rank() == 0: expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... (id_k = )? expr_{2k})], \sigma)
    ELIF \sigma("checkpoint") = id_t AND expr_1 = id_t.save THEN
        ([if hvd.rank() == 0: [expr_1 (expr_{11} ... expr_{1n} (id_1 =)? expr_{21} ... (id_k =)? expr_{2k})]], \sigma)
    ELIF \sigma(\text{``model''}) = id_t \text{ AND } expr_1 = id_t.\text{fit THEN}
        IF id_i = verbose WHEN 1 \le i \le k THEN
           IF id_i = callbacks WHEN 1 \le j \le k THEN
              ([callback = [hvd.callbacks.BroadcastGlobalVariablesCallback(root_rank=0)]
               if hvd.rank() == 0: callbacks.append(expr_{2j})
              expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... id_i = 1 if hvd.rank() == 0 else 0
              ... id_i = \text{callbacks} \dots (id_k = )? | expr_{2k}) |, \sigma)
           ELSE
               ([expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ...
                  id_i = 1 if hvd.rank() == 0 else 0 ... (id_k = )? expr_{2k}
                  callbacks = [hvd.callbacks.BroadcastGlobalVariablesCallback(root_rank=0)])], \sigma)
        ELSE
           IF id_i = callbacks WHEN 1 \le j \le k THEN
              ([callback = [hvd.callbacks.BroadcastGlobalVariablesCallback(root_rank=0)]
               if hvd.rank() == 0: callbacks.append(expr_{2i})
```

```
expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... id_j = callbacks ... (id_k = )? expr_{2k}
                        verbose = 1 if hvd.rank() == 0 else 0), \sigma
               ELSE
                   ([expr_1 \ (expr_{11} \ ... \ expr_{1n} \ (id_1 = )? \ expr_{21} \ ... \ (id_k = )? \ expr_{2k})
                        verbose = 1 if hvd.rank() == 0 else 0
                        \verb|callbacks = [hvd.callbacks.BroadcastGlobalVariablesCallback(0)]||, \sigma||
      ELIF \sigma(\text{``model''}) = id_t \text{ AND } expr_1 = id_t.\text{compile THEN}
          IF id_i = optimizer AND expr_{2i} = "adam" WHEN 1 \le i \le k THEN
               (id_z = tf.optimizers.Adam(learning_rate=0.001 * hvd.size())
               id_z = hvd.DistributedOptimizer(id_z)
               expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... id_i = id_z ... (id_k = )? expr_{2k}), \sigma)
          ELIF expr_{11} = "adam" THEN
               ([id_z = tf.optimizers.Adam(learning_rate=0.001 * hvd.size()))
               id_z = hvd.DistributedOptimizer(id_z)
               expr_1 (id_z ... expr_{1n} (id_1 = )? expr_{21} ... (id_k = )? expr_{2k}), \sigma)
          ELSE
               [trans_{\mathbb{E}} \| expr_1 (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k}) \| (\sigma) \|, \sigma)
      ELIF expr_1 = model.summary THEN
          ([if hvd.rank() == 0: expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... (id_k = )? expr_{2k})], \sigma)
      ELSE
          [trans_E \parallel expr_1 (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k}) \parallel (\sigma) \mid, \sigma)
A general form of expr statements except for the strict form
 trans_{S}[\![expr]\!](\sigma) = ([trans_{E}[\![expr]\!](\sigma)], \sigma)
 trans_S[\![ pass ]\!](\sigma) = ([pass], \sigma)
 trans_S[\![ break ]\!](\sigma) = ([break], \sigma)
 trans_S[\![ continue ]\!](\sigma) = ([continue], \sigma)
  trans_E : Expr \rightarrow \Sigma \rightarrow Expr
 trans_{E} \llbracket expr_{1} \ boolop \ expr_{2} \ \rrbracket(\sigma) = trans_{E} \llbracket expr_{1} \ \rrbracket(\sigma) \ boolop \ trans_{E} \llbracket expr_{2} \ \rrbracket(\sigma)
 trans_{E}[\![expr_{1} := expr_{2}]\!](\sigma) = expr_{1} := trans_{E}[\![expr_{2}]\!](\sigma)
 trans_{E} \llbracket expr_{1} \ binop \ expr_{2} \ \llbracket (\sigma) = trans_{E} \llbracket \ expr_{1} \ \rrbracket (\sigma) \ binop \ trans_{E} \llbracket \ expr_{2} \ \rrbracket (\sigma)
 trans_E \llbracket unop \ expr \ \rrbracket(\sigma) = unop \ trans_E \llbracket \ expr \ \rrbracket(\sigma)
 trans_{E} \llbracket \text{ lambda } args : expr \ \rrbracket(\sigma) = \text{ lambda } args : trans_{E} \llbracket expr \ \rrbracket(\sigma)
```

```
trans_{E} \llbracket expr_{1} \text{ if } expr_{2} \text{ else } expr_{3} \rrbracket (\sigma) =
           trans_{E} \llbracket expr_{1} \rrbracket (\sigma) if trans_{E} \llbracket expr_{2} \rrbracket (\sigma) else trans_{E} \llbracket expr_{3} \rrbracket (\sigma)
trans_{E} \llbracket expr_{1} \text{ if } expr_{2} \text{ else } expr_{3} \rrbracket (\sigma) =
           trans_{E} \llbracket expr_{1} \rrbracket (\sigma) if trans_{E} \llbracket expr_{2} \rrbracket (\sigma) else trans_{E} \llbracket expr_{3} \rrbracket (\sigma)
trans_{E} \llbracket \{(expr_{1} : expr_{2})^{*} \ (**expr_{3})^{*} \} \rrbracket (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \rrbracket (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \rrbracket (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \rrbracket (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \rrbracket (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \rrbracket (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \rrbracket (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \rrbracket (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \rrbracket (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \rrbracket (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \rrbracket (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \rrbracket (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \Vert (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \Vert (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \Vert (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \Vert (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \Vert (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \Vert (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \ (**expr_{3})^{*} \} \Vert (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \} \Vert (\sigma) = \{(expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma) \} \| (expr_{1} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \| (expr_{2} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \| (expr_{2} : trans_{E} \rrbracket (\sigma))^{*} \| (expr_{2} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \| (expr_{2} : trans_{E} \llbracket expr_{2} \rrbracket (\sigma))^{*} \| (expr_{2} : trans_{E} \rrbracket (\sigma))^{*} \| (expr_{2} :
trans_{E}[\![ \{expr^*\} ]\!](\sigma) = \{(trans_{E}[\![expr]\!](\sigma))^*\}
trans_E[ [expr^*] ](\sigma) = [(trans_E[expr](\sigma))^*]
trans_E[\![ (expr^*) \ ]\!](\sigma) = ((trans_E[\![ expr \ ]\!](\sigma))^*)
trans_{\mathbb{E}}[\![expr_1:expr_2:comprehension^*]\!](\sigma) = \{expr_1:trans_{\mathbb{E}}[\![expr_2:]\!](\sigma) \ (trans_{\mathbb{E}}[\![comprehension]\!](\sigma))^*\}
trans_E[\![\{expr\ comprehension^*\}\]](\sigma) = \{trans_E[\![\ expr\]](\sigma)\ (trans_O[\![\ comprehension]\!](\sigma))^*\}
trans_E[\![expr\ comprehension^*]\!](\sigma) = [trans_E[\![expr\ ]\!](\sigma)\ (trans_O[\![comprehension]\!](\sigma))^*]
trans_{E} \llbracket (expr \ comprehension^*) \ \rrbracket (\sigma) = (trans_{E} \llbracket \ expr \ \rrbracket (\sigma) \ (trans_{O} \llbracket comprehension \rrbracket (\sigma))^*)
trans_E \llbracket await expr \rrbracket(\sigma) = await <math>trans_E \llbracket expr \rrbracket(\sigma)
trans_E yield expr? \ \|(\sigma) = \text{yield } (trans_E \ expr \ \|(\sigma))?
trans_{E}[\![\  \, \text{yield from } expr\ ]\!](\sigma) = \text{yield from } trans_{E}[\![\  \, expr\ ]\!](\sigma)
trans_{E}[\![expr_{1}\ (compop\ expr_{2})^{*}\ ]\!](\sigma) = trans_{E}[\![expr_{1}\ ]\!](\sigma)\ (compop\ trans_{E}[\![expr_{2}\ ]\!](\sigma))^{*}
trans_E[\![expr_1 (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k})]\!](\sigma) =
           IF \sigma(\text{"dataset"}) = id_t AND expr_1 = id_t.take THEN
                      IF id_i = \text{count} WHEN 1 \le i \le k THEN
                                 expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... id_i = expr_{2i} // hvd.size() ... (id_k = )? expr_{2k})
                      ELSE
                                 expr_1 (expr_{11} // hvd.size() ... expr_{1n} (id_1 = )? expr_{21} ... (id_k = )? expr_{2k})
           ELSE
                      trans_{E} \llbracket expr_{1} \rrbracket (\sigma) \ (trans_{E} \llbracket expr_{11} \rrbracket (\sigma) \dots \ trans_{E} \llbracket expr_{1n} \rrbracket (\sigma)
                                 (id_1 = )? trans_E[\![expr_{21}]\!](\sigma) \dots (id_k = )? trans_E[\![expr_{2k}]\!](\sigma))
trans_{E}[\![\{expr_{1} \quad (!i)? \quad (:expr_{2})?\}]\!](\sigma) = \{expr_{1} \quad (!i)? \quad (:expr_{2})?\}
```

```
trans_{E} \llbracket expr^* \rrbracket (\sigma) = expr^*
trans_E \llbracket constant \rrbracket (\sigma) = constant
trans_E[\![expr.id\ ]\!](\sigma) = (trans_E[\![expr\ ]\!](\sigma)).id
trans_{E}[\![expr_{1}[expr_{2}]]\!](\sigma) = trans_{E}[\![expr_{1}]\!](\sigma)[trans_{E}[\![expr_{2}]]\!](\sigma)]
trans_E \llbracket *expr \rrbracket (\sigma) = *expr
trans_E \llbracket **expr \rrbracket (\sigma) = **expr
trans_E \llbracket id \rrbracket (\sigma) = id
trans_{E}[\![expr_{1}?\ (:expr_{2})?\ (:expr_{3})?\ ]\!](\sigma) = trans_{E}[\![expr_{1}\ ]\!](\sigma)?\ (:trans_{E}[\![expr_{2}\ ]\!](\sigma))?\ (:trans_{E}[\![expr_{3}\ ]\!](\sigma))?
 trans_O: Comprehension \rightarrow \Sigma \rightarrow Comprehension
trans_O\llbracket for expr_1 in expr_2 (if expr_3)* \rrbracket(\sigma)= for expr_1 in trans_E\llbracket expr_2 \rrbracket(\sigma) (if trans_E\llbracket expr_3\rrbracket(\sigma))*
               : ExcHandler \rightarrow \Sigma \rightarrow ExcHandler
trans_H \llbracket \text{ except } expr? \text{ (as } id)? : stmt^* \rrbracket (\sigma) = \text{except } expr? \text{ (as } id)? : trans_{\overline{S}} \llbracket stmt^* \rrbracket (\sigma)._1
 trans_{\overline{A}} : Alias list \rightarrow \Sigma \rightarrow \Sigma
trans_{\overline{A}} \llbracket \ alias_1 \ \dots \ alias_{n-1} \ alias_n \ \rrbracket(\sigma) = trans_A \llbracket \ alias_n \ \rrbracket(trans_A \llbracket \ alias_{n-1} \ \rrbracket(\dots \ trans_A \llbracket \ alias_1 \ \rrbracket(\sigma)))
 trans_A : Alias \rightarrow \text{CG} \rightarrow
trans_A \llbracket id_1 . id_2 ... . id_n \rrbracket (cg) =
LET (ns, vs) = CG IN
     LET node_1 = \{ id_1, \cdot \}, node_i = \{ id_i, node_{i-1} \} WHEN 2 \le i \le n IN
     LET var_k = id_1 \dots id_k WHEN 1 \le k \le n IN
(ns@[node_1, ..., node_n], vs@[var_1 \mapsto node_1, ..., var_n \mapsto node_n])
trans_A \llbracket id_1 . id_2 ... . id_n \text{ as } id_{as} \rrbracket (cg) =
LET (ns, vs) = CG IN
     LET node_1 = \{ id_1, \cdot \}, node_i = \{ id_i, node_{i-1} \} WHEN 2 \le i \le n IN
(ns@[node_1, ..., node_n], vs@[id_{as} \mapsto node_n])
 trans_{\overline{W}}: WithItem list \rightarrow \Sigma \rightarrow (WithItem list \times \Sigma)
```

```
trans_{\overline{W}} \llbracket with\_item_1 \ with\_item_2 \ ... \ with\_item_n \ \rrbracket(\sigma) =
     LET with\_item_1', \sigma_1 = trans_W \llbracket with\_item_1 \rrbracket (\sigma) IN
     LET with\_item_2', \sigma_2 = trans_W \llbracket with\_item_2 \rrbracket (\sigma_1) IN
inden ...
     LET with\_item_n', \sigma_n = trans_W \llbracket with\_item_n \rrbracket (\sigma_{n-1}) IN
     (with\_item_1' :: with\_item_2' :: ... :: [with\_item_n'], \sigma_n)
                 : WithItem \rightarrow \Sigma \rightarrow (WithItem \times \Sigma)
trans_W \llbracket expr \rrbracket(\sigma) = (trans_E \llbracket expr \rrbracket(\sigma), \sigma)
trans_W \llbracket expr_1 \text{ as } expr_2 \rrbracket (smodenv) =
     IF \sigma("tensor_flow") = id_1 AND expr_1 = id_1.GradientTape() AND expr_2 = id_2 THEN
          (expr_1 \text{ as } expr_2, \sigma[\text{"gradient\_tape"} \mapsto id_2])
     ELSE (trans_E \llbracket expr_1 \rrbracket (\sigma) \text{ as } expr_2, \sigma)
 trans_C : MatchCase \rightarrow \Sigma \rightarrow MatchCase
trans_{C}[\![ case pattern (if expr)? : stmt^{*}[\![](\sigma) =
     case trans_P \llbracket pattern \rrbracket (\sigma) (if trans_E \llbracket expr \rrbracket (\sigma))? : trans_{\overline{S}} \llbracket stmt^* \rrbracket (\sigma)._1
 trans_P: Pattern \rightarrow \Sigma \rightarrow Pattern
trans_P \llbracket expr \rrbracket (\sigma) = trans_E \llbracket expr \rrbracket (\sigma)
trans_P \llbracket constant \rrbracket (\sigma) = constant
trans_P \llbracket [pattern^*] \rrbracket (\sigma) = [trans_P \llbracket pattern \rrbracket (\sigma)^*]
trans_P \llbracket *(id)? \rrbracket (\sigma) = *(id)?
trans_P[\![\{(expr: pattern)^* id?\}]\!](\sigma) = \{(expr: trans_P[\![pattern]\!](\sigma))^* id?\}
trans_P[\![expr\ (pattern_1^*\ (id = pattern_2)^*)\ ]\!](\sigma) = expr\ (trans_P[\![pattern_1]\!](\sigma)^*\ (id = trans_P[\![pattern_2]\!](\sigma))^*)
trans_P[\![ (pattern as)? id ]\!](\sigma) = (trans_P[\![ pattern ]\!](\sigma) as)? id
trans_P \llbracket pattern_1 \mid pattern_2 \rrbracket (\sigma) = trans_P \llbracket pattern_1 \rrbracket (\sigma) \mid trans_P \llbracket pattern_2 \rrbracket (\sigma)
trans_P[\![ \ \_ \ ]\!](\sigma) = \_
```

## 3 Identifying training loop

#### 3.1 Restrictions

- 1. Training loop must be defined in only one file.
- 2. Training loop type is either distributed gradient tape or distributed optimizer.
- 3. Each model must have only one type of training loop.
- 4. Function must not be assigned to the variables or passed as an argument.
- 5. Training loop must not be defined conditionally.

#### 3.2 Rules

#### 3.2.1 Summary

 $\sigma \in \Sigma = Id \stackrel{\text{fin}}{\to} Summary$  Environment storing mappings from ids to summaries

#### 3.2.2 Training Loop

```
summary_S \llbracket (@expr_1)^*  async def id (args) (-> expr_2)? : (\#type:s)? stmt^* \rrbracket (\sigma) =
                LET \sigma', tl = summary_{\overline{S}} [stmt^*] IN
                (\sigma[id \mapsto \texttt{FuncSummary}\ tl], \perp)
    summary_S \llbracket (@expr)^* \text{ class } id \text{ } (expr_{11} \dots expr_{1n} \text{ } (id_1 = )? \text{ } expr_{21} \dots (id_k = )? \text{ } expr_{2k}) \text{ } : \text{ } stmt^* \rrbracket (\sigma) = summary_S \llbracket (@expr)^* \text{ } class \text{ } id \text{ } (expr_{11} \dots expr_{1n} \text{ } (id_1 = )? \text{ } expr_{21} \dots (id_k = )? \text{ } expr_{2k}) \text{ } : \text{ } stmt^* \rrbracket (\sigma) = summary_S \llbracket (@expr)^* \text{ } class \text{ } id \text{ } (expr_{11} \dots expr_{1n} \text{ } (id_1 = )? \text{ } expr_{21} \dots (id_k = )? \text{ } expr_{2k}) \text{ } : \text{ } stmt^* \rrbracket (\sigma) = summary_S \llbracket (@expr_{11} \dots expr_{1n} \text{ } (id_1 = )? \text{ } expr_{2n} \text{ } ) \text{ } : \text{ } stmt^* \rrbracket (\sigma) = sum_S \rrbracket (\sigma
               IF expr_{1i} = id_c.keras.Model WHEN 1 \le i \le n AND
                           \sigma(id_c) = \text{ModuleSummary Id}("tensorflow") \quad [] \perp \textbf{THEN} \quad (\sigma[id \mapsto \text{ClassSummary } Model], \perp)
               ELIF expr_{2i} = id_c.keras.Model WHEN 1 \le i \le k AND
                           \sigma(id_c) = \text{ModuleSummary Id}("tensorflow") [] \perp \textbf{THEN} (\sigma[id \mapsto \texttt{ClassSummary } Model], \bot)
               ELSE (\sigma[id \mapsto \text{ClassSummary } \bot], \bot)
A strict form of assignment statements
    summary_S \llbracket id_r = id_c \text{ () (#type:}s)? \rrbracket (\sigma) =
               IF \sigma(id_c) = \text{ClassSummary } Model \text{ THEN } (\sigma[id_r \mapsto \text{ValueSummary "model_instance"}], \bot)
                ELSE (\sigma, \perp)
    summary_S[\![ (\#type:s)? \text{ for } expr_1 \text{ in } expr_2 : stmt_1^* \text{ (else : } stmt_2^*)? ]\!](\sigma) =
               LET __, tl = summary_{\overline{S}} [\![ stmt_1^* ]\!](\sigma) IN
                (\sigma, tl)
    summary_S \llbracket \text{ (#type:}s)? \text{ async for } expr_1 \text{ in } expr_2 : stmt_1^* \text{ (else : } stmt_2^*)? \rrbracket (\sigma) =
                LET __, tl = summary_{\overline{S}} [\![ stmt_1^* ]\!](\sigma) IN
                (\sigma, tl)
    summary_S[\![ while (expr) : stmt_1^* (else : stmt_2^*)? ]\![(\sigma) =
                LET _-, tl = summary_{\overline{S}} [\![ stmt_1^* ]\!](\sigma) IN
                (\sigma, tl)
    summary_S[\![ (\#type:s)? with with\_item^* : stmt^* ]\!](\sigma) =
                LET __, tl = summary_{\overline{S}} [\![ stmt^* ]\!](\sigma) IN
                (\sigma, summary_{\overline{W}} \llbracket with\_item^* \rrbracket (\sigma) \sqcup tl)
    summary_S[\![ (\#type:s)? async with with\_item^* : stmt^* ]\!](\sigma) =
               LET __, tl = summary_{\overline{S}} [stmt^*](\sigma) IN
                (\sigma, summary_{\overline{W}} \llbracket with\_item^* \rrbracket (\sigma) \sqcup tl)
    summary_S[\![try : stmt_1^*   exc\_handler^*   (else : stmt_2^*)?   (finally : stmt_3^*)? ]\![(\sigma) =
               (\sigma, summary_{\overline{S}}[\![ stmt_1^* ]\!](\sigma))
    summary_S[\![\!] import \ alias^* ]\![\![\!](\sigma) = (summary_A[\![\!]\!] \ alias^* ]\![\![\!](\sigma), \bot)
```

```
summary_S \llbracket \text{ from } 0 \quad id^* \quad \text{import } alias^* \ \rrbracket(\sigma) =
      LET \sigma' = summary_{\overline{A}} [\![ alias^* ]\!](\sigma) IN
      LET [id_2 \mapsto \texttt{ModuleSummary} \ id_2 \ \sigma_2 \ \bot, \ id_2 \mapsto \texttt{ModuleSummary} \ id_2 \ \sigma_2 \ \bot,
           ... id_n \mapsto \texttt{ModuleSummary} \ id_n \ \sigma_n \perp ] = \sigma' \setminus \sigma \ \textbf{IN}
      \sigma \ ++ \ [id_2 \mapsto \texttt{ModuleSummary} \ (id^*+id_2) \ \sigma_2 \ \bot, \quad id_2 \mapsto \texttt{ModuleSummary} \ (id^*+id_2) \ \sigma_2 \ \bot,
           ... id_n \mapsto \texttt{ModuleSummary} \ (id^* + id_n) \ \sigma_n \ \bot]
 summary_S \llbracket expr \rrbracket(\sigma) = (\sigma, summary_E \llbracket expr \rrbracket(\sigma))
 summary_S[\![ stmt \ ]\!](\sigma) = (\sigma, \perp)
  summary_{\overline{A}}: Alias list \rightarrow \Sigma \rightarrow \Sigma
 summary_{\overline{A}}[\![ alias_1 \dots alias_{n-1} alias_n ]\!](\sigma) =
      summary_A \llbracket alias_n \rrbracket (summary_A \llbracket alias_{n-1} \rrbracket (... summary_A \llbracket alias_1 \rrbracket (\sigma) ...))
  trans_A: Alias \rightarrow \Sigma \rightarrow \Sigma
 trans_A \llbracket id_1 id_2 \dots id_n \rrbracket (\sigma) =
      LET \sigma_n = [id_n \mapsto ModuleSummary id_n \top \bot]
      LET \sigma_{n-1} = [id_{n-1} \mapsto ModuleSummary id_{n-1} \sigma_n \perp]
      LET \sigma_2 = [id_2 \mapsto ModuleSummary id_2 \sigma_3 \perp]
      LET \sigma_1 = [id_1 \mapsto ModuleSummary id_1 \quad \sigma_2 \quad \bot]
      \sigma ++ \sigma_1
   summary_{\overline{W}}: WithItem list \rightarrow \Sigma \rightarrow tl
 summary_{\overline{W}} \llbracket with\_item_1 \ with\_item_2 \ ... \ with\_item_n \ \rrbracket(\sigma) =
      summary_{W} \llbracket \ with\_item_1 \ \rrbracket (\sigma) \ \sqcup \ summary_{W} \llbracket \ with\_item_2 \ \rrbracket (\sigma) \ \sqcup \ \ldots \quad summary_{W} \llbracket \ with\_item_n \ \rrbracket (\sigma)
   summary_W: WithItem \rightarrow \Sigma \rightarrow tl
 summary_W \llbracket expr_1 \text{ as } expr_2? \rrbracket (\sigma) = summary_E \llbracket expr_1 \rrbracket (\sigma)
  summary_E : Expr \rightarrow \Sigma
A strict form of call expression
 summary_E \llbracket expr_1 (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k}) \rrbracket (\sigma) =
      IF \sigma(id_r) = ValueSummary "model_instance" AND
           expr_1 = id_r.fit THEN Optimizer
      ELIF \sigma(id_r) = ModuleSummary Id("tensorflow") \sigma' the AND
           expr_1 = id_r.GradientTape THEN GradTape
      ELIF \sigma(id_r) = FuncSummary tl AND
           expr_1 = idr THEN tl
      ELSE \perp
 summary_E \llbracket expr \rrbracket (\sigma) = \bot
```