# Code Transformation for Distributed Python ML Code

## 1 Python Abstract Syntax

```
stmt^* type\_ignore
                                                                                 (ModuleDef)
module
         ::=
              (@expr)^* def id (args) (-> expr)? : (#type:s)? stmt^*
                                                                                 (FunDef)
stmt
              (@expr)^* async def id (args) (\rightarrow expr)? : (\#type:s)? stmt^*
                                                                                 (AsyncFunDef)
              (@expr)^* class id (expr^* keyword^*) : stmt^*
                                                                                 (ClassDef)
             return expr?
                                                                                 (Return)
             delete expr^*
                                                                                 (Delete)
             expr^* = expr (\#type:s)?
                                                                                 (Assign)
              expr \ binop = \ expr
                                                                                 (Augassign)
             expr : expr (= expr)?
                                                                                 (AnnAssign)
              (#type:s)? for expr in expr : stmt^* (else : stmt^*)?
                                                                                 (FORLOOP)
              (#type:s)? async for expr in expr : stmt^* (else : stmt^*)?
                                                                                 (ASYNCFORLOOP)
             while (expr) : stmt^* (else : stmt^*)?
                                                                                 (WHILELOOP)
              if (expr) : stmt^* (else : stmt^*)?
                                                                                 (IF)
              (#type:s)? with with\_item^* : stmt^*
                                                                                 (WITH)
             (#type:s)? async with with_{-}item^* : stmt^*
                                                                                 (AsyncWith)
             match expr : match\_case^*
                                                                                 (Match)
             raise expr? (from expr)?
                                                                                 (Raise)
             try : stmt^* exc\_handler^* (else : stmt^*)? (finally : stmt^*)?
                                                                                 (TRY)
              assert expr expr?
                                                                                 (Assert)
              import alias^*
                                                                                 (Import)
             from i\ id? import alias^*
                                                                                 (IMPORTFROM)
              global id^*
                                                                                 (GLOBAL)
             nonlocal id^*
                                                                                 (NonLocal)
                                                                                 (ExprStmt)
              expr
             pass
                                                                                 (Pass)
                                                                                 (Break)
             break
                                                                                 (CONTINUE)
              continue
```

```
(BOOLOP)
                     expr boolop expr
expr
                ::=
                     expr :=
                              expr
                                                                                         (NAMEDEXPR)
                     expr binop expr
                                                                                         (BINARYOP)
                                                                                         (UNARYOP)
                     unop expr
                     lambda args : expr
                                                                                         (LAMBDA)
                                                                                         (IFEXPR)
                     expr if expr else expr
                     \{(expr : expr)^* (**expr)^*\}
                                                                                         (DICTIONARY)
                     \{expr^*\}
                                                                                         (Set)
                      [expr^*]
                                                                                         (List)
                      (expr^*)
                                                                                         (Tuple)
                     \{expr: expr comprehension^*\}
                                                                                         (DICTIONARY COMP)
                      \{expr \ comprehension^*\}
                                                                                         (SetComp)
                      [expr comprehension^*]
                                                                                         (LISTCOMP)
                      (expr \ comprehension^*)
                                                                                         (GENERATORCOMP)
                     await expr
                                                                                         (AWAIT)
                     yield expr?
                                                                                         (Yield)
                                                                                         (YIELDFROM)
                     yield from expr
                     expr (compop \ expr)^*
                                                                                         (COMPOP)
                     expr (expr^* keyword^*)
                                                                                         (Call)
                     \{expr (!i)? (:expr)?\}
                                                                                         (FORMATTED VALUE
                     expr^*
                                                                                         (JoinedStr)
                     constant
                                                                                         (Constant)
                     expr.id
                                                                                         (Attribute)
                     expr[expr]
                                                                                         (Subscript)
                     *expr
                                                                                         (Starred)
                     **expr
                                                                                         (DoubleStarred)
                                                                                         (Name)
                     expr? (:expr)? (:expr)?
                                                                                         (SLICE)
                     and | or
                                                                                         (BOOLOPERATOR)
boolop
                ::=
binop
                     + | - | * | @ | / | ** | << | >> | | | ^ | & | // | %
                                                                                         (BINOPERATOR)
                ::=
unop
                ::=
                     \sim | not | + | -
                                                                                         (Unoperator)
                     == | != | < | <= | > | >= | is | is not | in | not in
compop
                ::=
                                                                                         (COMPOPERATOR)
comprehension
                     for expr in expr (if expr)*
                                                                                         (Comprehension)
                ::=
                     async for expr in expr (if expr)*
                                                                                         (ASYNCCOMPREHEN
                     except expr? (as id)? : stmt*
exc\_handler
                                                                                         (EXCHANDLER)
                ::=
args
                     (arg (= expr)?)^*, (arg (= expr)?)^*, arg?, (arg (= expr)?)^*, arg?
                                                                                         (Arguments)
                ::=
arg
                ::=
                     id expr? s?
                                                                                         (Argument)
keyword
                     id? = expr
                                                                                         (Keyword)
                ::=
                     id (.id)^* (as id)?
alias
                                                                                         (ALIAS)
                ::=
with\_item
                     expr (as expr)?
                                                                                         (WITHITEM)
                ::=
```

```
case pattern (if expr)? : stmt^*
                                                       (MATCHCASE)
match\_case
             ::=
pattern
             ::=
                  expr
                                                       (MATCHVALUE)
                  constant
                                                       (MATCHSINGLETON)
                                                       (MATCHSEQUENCE)
                  [pattern*]
                  *(id)?
                                                       (MATCHSTAR)
                  \{(expr: pattern)^* id?\}
                                                       (MATCHMAPPING)
                  expr (pattern^* (id = pattern)*)
                                                       (MATCHCLASS)
                  (pattern as)? id
                                                       (MatchAs)
                  pattern | pattern
                                                       (MATCHOR)
                                                       (MATCHWILDCARD)
                                                       (NoneLiteral)
constant
             ::=
                  None
                                                       (IntLiteral)
                  i
                                                       (FLOATLITERAL)
                  c
                                                       (ComplexLiteral)
                                                       (STRINGLITERAL)
                                                       (BOOLEANLITERAL)
                  (constant^*)
                                                       (TupleLiteral)
                                                       (Ellipsis)
                  i^*
                                                       (TypeIgnore)
type\_ignore
             ::=
                  Id
id
              \in
              \in
                  Str
s
                  {True, False}
              \in
i
              \in
                  \mathbb{Z}
f
              \in \mathbb{R}
                  \mathbb{C}
c
              \in
```

Note: there may be more constant terms such as an immutable container containing only constant elements. Please update the constant rule if you know such terms.

## 2 Transformation for TF2 Python Code

### 2.1 Restrictions

- 1. All import statements must be placed at the top of a module.
- 2. The tensorflow module must be assigned to a variable only using an import statement.
- 3. Members of the tensorflow module must not be aliased.
- 4. print function call expressions must not introduce side-effects.
- 5. A dataset and an optimizer object must only be created once via an assignment statement with a function call expression and must not be aliased.
- 6. Variables storing a dataset or a optimizer object must not be reassigned to store another value that is not a dataset or an optimizer object.
- 7. A dataset and an optimizer object must not be created conditionally.
- 8. optimizer.apply\_gradients function call expressions must be expression statements or a direct RHS of an assignment statement.
- 9. Global variables storing an optimizer object and referred in functions must be defined before the functions' definitions and must not be changed after their initializations.
- 10. A checkpoint object must only be created once via an assignment statement with a function call expression and must not be aliased.

## 2.2 Rules

#### 2.2.1 Types and Auxiliary Functions

```
\tau ::=
        Module
                         (Modules)
        Stmt
                         (STATEMENTS)
                         (Expressions)
        Expr
        Comprehension
                         (Comprehensions)
        ExcHandler
                         (EXCEPTION HANDLERS)
        Alias
                         (ALIASES)
        With Item
                         (WITHITEMS)
        Pattern
                         (Patterns)
        Id
                         (IDENTIFIERS)
        Str
                         (STRINGS)
        \tau list
                         (List of elements typed \tau)
```

 $\sigma \in \Sigma = Str^{\text{fin}} Id$  Environment storing mappings from strings to identifiers

```
._1 : \tau \times \tau \to \tau Get the first element of the given pair

@ : \tau list \to \tau list \to \tau list Concatenate two lists

:: \tau \to \tau list \to \tau list Append an element to a list (right-associative)
```

#### 2.2.2 Transformation Rules

```
trans_M: Module \rightarrow Module
trans_{M}[\![stmt^{*}\ type\_ignore]\!] = trans_{\overline{S}}[\![stmt^{*}\ ]\!](\sigma).\_1 \ type\_ignore
   trans_{\overline{S}}: Stmt list \rightarrow \Sigma \rightarrow (Stmt list \times \Sigma)
                                                                                                                                                                                   LET stmt_1^{*\prime}, \sigma_1 = trans_S[stmt_1](\sigma) IN
trans_{\overline{S}}[\![stmt_1 \quad stmt_2 \dots stmt_n \ ]\!](\sigma) =
                                                                                                                                                                                     LET stmt_2^{*\prime}, \sigma_2 = trans_S[stmt_2](\sigma_1) IN
                                                                                                                                                                                     LET stmt_n^{*\prime}, \sigma_n = trans_S[\![ stmt_n ]\!](\sigma_{n-1}) IN
                                                                                                                                                                                     (stmt_1^{*\prime} \ @ \ stmt_2^{*\prime} \ @ \ \dots \ @ \ stmt_n^{*\prime}, \ \sigma_n)
    trans_S: Stmt \rightarrow \Sigma \rightarrow (Stmt \ \textbf{list} \times \Sigma)
trans_S \llbracket (@expr_1)^* \text{ def } id \text{ } (args) \text{ } (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } stmt^* \rrbracket (\sigma) = (-> expr_2)? \text{ } : \text{ } (\#type:s
               ([(@expr_1)^* \quad \mathsf{def} \quad id \quad (args) \quad ( \rightarrow \ expr_2)? \quad : \quad (\mathtt{\#type} : s)? \quad trans_{\overline{S}}[\![ \ stmt^* \ ]\!](\sigma).\_1], \ \sigma)
([(@expr_1)^* \text{ async def } id \text{ } (args) \text{ } (-> expr_2)? \text{ } : \text{ } (\#type:s)? \text{ } trans_{\overline{S}}[\![ stmt^* ]\!](\sigma).\_1], \sigma)
trans_{S} \llbracket (@expr_{1})^{*}  class id (expr_{2}^{*} keyword^{*}) : stmt^{*} \rrbracket (\sigma) =
              ([(@expr_1)^* \ class \ id \ (expr_2^* \ keyword^*) : trans_{\overline{S}}[[stmt^*]](\sigma).1], \sigma) =
trans_S[\![ return \ expr? \ ]\!](\sigma) = ([return \ (trans_E[\![ expr \ ]\!](\sigma))?], \sigma)
trans_S[\![ delete \ expr^* \ ]\!](\sigma) = ([delete \ expr^*], \sigma)
```

#### A strict form of assignment statements

```
trans_S \parallel id_r = expr_1 (expr_{11} \dots expr_{1n} \quad (id_1 = )? \ expr_{21} \dots (id_k = )? \ expr_{2k}) (\#type:s)? \parallel (\sigma) = 
   IF \sigma("tensor_flow") = id_t AND expr_1 = id_t.data.Dataset.expr_3 THEN
       ([id_r = expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... (id_k = )? expr_{2k}) (\#type:s)?],
           \sigma["dataset"\mapsto id_r])
   ELIF \sigma ("tensor_flow") = id_t AND expr_1 = id_t.train.Checkpoint THEN
       ([id_r = expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... (id_k = )? expr_{2k}) (\#type:s)?],
           \sigma["checkpoint"\mapsto id_r])
   ELIF \sigma(\text{"tensor_flow"}) = id_t \text{ AND } expr_1 = id_t.\text{optimizers.Adam THEN}
       IF id_i = learning_rate WHEN 1 \le i \le k THEN
           ([id_r = expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... id_i = expr_{2i} * hvd.size()
              ... (id_k = )? expr_{2k}) (#type:s)?], \sigma["optimizer" \mapsto id_r])
       ELSE
           ([id_r = expr_1 \ (expr_{11} * hvd.size() \ ... \ expr_{1n} \ (id_1 = )? \ expr_{21} \ ... \ (id_k = )? \ expr_{2k}) \ (\#type:s)?],
              \sigma["optimizer" \mapsto id_r])
   ELIF \sigma ("optimizers") = id_t AND expr_1 = id_t. Adam THEN
       IF id_i = learning_rate WHEN 1 \le i \le k THEN
           ([id_r = expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... id_i = expr_{2i} * hvd.size())
               ... (id_k = )? expr_{2k}) (#type:s)?], \sigma["optimizer" \mapsto id_r])
       ELSE
           ([id_r = expr_1 \ (expr_{11} * hvd.size() ... \ expr_{1n} \ (id_1 = )? \ expr_{21} ... \ (id_k = )? \ expr_{2k}) \ (\#type:s)?],
              \sigma["optimizer" \mapsto id_r])
   ELIF \sigma("optimizer") = id_t AND expr_1 = id_t.apply_gradients THEN
       IF id_i = grads\_and\_vars WHEN 1 \le i \le k THEN
           LET id_z = NewID() IN
           ([id_z = expr_{2i},
           id_r = expr_1 (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots id_i = id_z \dots (id_k = )? expr_{2k}) (#type:s)?,
           global hvd_broadcast_done,
           if not hvd_broadcast_done: [hvd.broadcast\_variables([x[1] for x in <math>id_z], root\_rank=0),
              hvd.broadcast_variables(id_t.variables(), root_rank=0),
              hvd\_broadcast\_done = True ]], \sigma)
       ELSE
           LET id_z = \text{NewID}() IN
           ([id_z = expr_{11},
           id_r = expr_1 \ (id_z \ expr_{12} \ ... \ expr_{1n} \ \ (id_1 = )? \ expr_{21} \ ... \ (id_k = )? \ expr_{2k}) \ (\#type:s)?,
           global hvd_broadcast_done,
           if not hvd_broadcast_done: [hvd.broadcast_variables([x[1] for x in <math>id_z], root_rank=0),
              hvd.broadcast_variables(id_t.variables(), root_rank=0),
              hvd\_broadcast\_done = True ]], \sigma)
   ELIF \sigma("checkpoint") = id_t AND expr_1 = id_t.save THEN
       ([if hvd.rank() == 0: [id_r = expr_1 (expr_{11} ... expr_{1n} (id_1 =)? expr_{21} ... (id_k =)? expr_{2k}) (#type:s)?]],
           \sigma)
   ELIF \sigma(\text{``keras''}) = id_k \text{ AND } expr_1 = id_k.\text{models.Sequential THEN}
       ([id_r = expr_1 (expr_{11} ... expr_{1n} (id_2 = )? expr_{21} ... (id_k = )? expr_{2k})], \sigma["model"\mapsto id_r])
   ELSE ([id_r = trans_E[[expr_1 (expr_{11} ... expr_{1n} (id_2 = )? expr_{21} ... (id_k = )? expr_{2k})](\sigma) (#type:s)?],
           \sigma)
```

```
trans_{S} \llbracket expr_{0} = expr_{1} \text{ (#type:}s)? \rrbracket (\sigma) =
      IF \sigma(\text{``os''}) = id_{os} AND expr_0 = id_{os}.\text{environ [`CUDA_VISIBLE_DEVICES']}
      THEN ([], \sigma)
A general form of assignment statements except for the strict form
 trans_S[\![expr_1^*]\!] = expr_2 (#type:s)? [\![\sigma]\!] = ([expr_1^*]\!] = trans_E[\![expr_2]\!] (\sigma) (#type:s)?], \sigma)
 trans_S[\![expr_1\ binop = expr_2\ ]\!](\sigma) = ([expr_1\ binop = trans_E[\![expr_2\ ]\!](\sigma)], \sigma)
 trans_S \llbracket expr_1 : expr_2 (= expr_3)? \rrbracket (\sigma) = ([expr_1 : expr_2 (= trans_E \llbracket expr_3 \rrbracket (\sigma))?], \sigma)
      IF expr_1 = id_1 AND \sigma(\text{"tensor\_flow"}) = id_2
      AND expr_3 = id_2.data.Dataset.expr_4 (expr_5^* keyword^*) THEN
          ([expr_1: expr_2 = expr_3 (\#type:s)?], \sigma["dataset" \mapsto id_1])
      ELSE ([expr_1 : expr_2 (= trans_E [expr_3 ](\sigma))?], \sigma)
 trans_S[\![ (\#type:s)? for \ expr_1 \ in \ expr_2 \ : \ stmt_1^* \ (else : \ stmt_2^*)? ]\!](\sigma) =
      ([(#type:s)? for expr_1 in trans_E \llbracket expr_2 \rrbracket (\sigma) :
          trans_{\overline{S}}[stmt_1^*](\sigma).1 (else : trans_{\overline{S}}[stmt_2^*](\sigma).1)?], \sigma)
 trans_S[ (#type:s)? async for expr_1 in expr_2 : stmt_1^* (else : stmt_2^*)? ](\sigma) =
      ([(#type:s)? async for expr_1 in trans_E \llbracket expr_2 \rrbracket (\sigma) :
          trans_{\overline{S}}[\![stmt_1^*]\!](\sigma)...1 (else : trans_{\overline{S}}[\![stmt_2^*]\!](\sigma)...1)?], \sigma)
 trans_{S} while (expr) : stmt_{1}^{*} (else : stmt_{2}^{*})? (\sigma) =
      (\llbracket \text{while } (trans_{\overline{S}} \llbracket \ expr \ \rrbracket(\sigma)) \ : \ trans_{\overline{S}} \llbracket \ stmt_1^* \ \rrbracket(\sigma).\_1 \ (\text{else } : \ trans_{\overline{S}} \llbracket \ stmt_2^* \ \rrbracket(\sigma).\_1)? \rrbracket, \sigma)
 trans_{S}[\![ if (expr) : stmt^{*} (else : stmt^{*})? \![ \![ \![ \![ \![ \!]
      ([\text{if } (trans_E[\![ expr ]\!](\sigma)) : trans_{\overline{S}}[\![ stmt^* ]\!](\sigma).\_1 (\text{else } : trans_{\overline{S}}[\![ stmt^* ]\!](\sigma).\_1)?], \sigma)
 trans_{S}[\![ (\#type:s)? with with\_item^* : stmt^* ]\!](\sigma) =
      LET with\_item^{*\prime}, \sigma_1 = trans_{\overline{W}} \llbracket with\_item^* \rrbracket (\sigma) IN
      LET stmt^{*\prime}, \sigma_2 = trans_{\overline{S}}[stmt^*](\sigma_1) IN
      IF \sigma_1 \setminus \sigma = ["gradient\_tape" \mapsto id] THEN
          ([(\#type:s)? with with\_item^{*\prime} : stmt^{*\prime},
          id = hvd.DistributedGradientTape(id), \sigma_2
      ELSE ([(#type:s)? with with\_item^{*\prime} : stmt^{*\prime}], \sigma_2)
 trans_{S}[\![ (\#type:s)?  async with with\_item^* : stmt^* ]\!](\sigma) =
      LET with_item*', \sigma_1 = trans_{\overline{W}} [\![\![ with_item^* ]\!]](\sigma) IN
      LET stmt^{*\prime}, \sigma_2 = trans_{\overline{S}} [stmt^*](\sigma_1) IN
      IF \sigma_1 \setminus \sigma = ["gradient\_tape" \mapsto id] THEN
          ([(\#type:s)? async with with\_item^{*\prime} : stmt^{*\prime},
          id = hvd.DistributedGradientTape(id), \sigma_2
      ELSE ([(#type:s)? async with with\_item^{*\prime} : stmt^{*\prime}], \sigma_2)
```

```
trans_{S}[\![match\ expr\ :\ match\_case^*\ ]\!](\sigma) =
    ([match trans_E[\![expr]\!](\sigma) : (trans_C[\![match\_case]\!](\sigma))^*], \sigma)
trans_S[ raise expr_1? (from expr_2)? [(\sigma) = ([raise expr_1? (from <math>expr_2)?], \sigma)
trans_S \llbracket try : stmt_1^* = exc\_handler^* \text{ (else : } stmt_2^*)? \text{ (finally : } stmt_3^*)? \rrbracket (\sigma) =
    ([try : trans_{\overline{S}}[stmt_1^*](\sigma)._1 (trans_H[exc\_handler](\sigma))*
    (\texttt{else} \ : \ trans_{\overline{S}} \llbracket \ stmt_2^* \ \rrbracket (\sigma).\_1)? \quad (\texttt{finally} \ : \ trans_{\overline{S}} \llbracket \ stmt_3^* \ \rrbracket (\sigma).\_1)? \rbrack, \ \sigma)
trans_{S}[\![ assert \ expr_{1} \ expr_{2}? \ ]\!](\sigma) = ([assert \ trans_{E}[\![ \ expr_{1} \ ]\!](\sigma) \ expr_{2}?], \sigma)
trans_S[\![\!] import alias^*[\![\!](\sigma)=
    LET \sigma_1 = trans_{\overline{A}} [alias^*](\sigma) IN
    IF \sigma_1 \setminus \sigma = [\text{"tensor\_flow"} \mapsto id] THEN
        ([import alias^*,
        import horovod.tensorflow as hvd,
        hvd_broadcast_done = False,
        hvd.init(),
        gpus = id.config.experimental.list_physical_devices('GPU'),
        for gpu in gpus: id.config.experimental.set_memory_growth(gpu, True),
        if gpus: id.config.experimental.set_visible_devices(gpus[hvd.local_rank()], 'GPU')], \sigma_1)
    ELSE ([import alias^*], \sigma_1)
trans_S[\![ import \ alias^* ]\!](\sigma) =
    LET \sigma_1 = trans_{\overline{A}} [alias^*](\sigma) IN
    IF \sigma_1 \setminus \sigma = [\text{"tensor\_flow"} \mapsto id] THEN
        ([import alias*,
        import horovod.tensorflow as hvd,
        hvd.init(),
        gpus = id.config.experimental.list_physical_devices('GPU'),
        for gpu in gpus: id.config.experimental.set_memory_growth(gpu, True),
        if gpus: id.config.experimental.set_visible_devices(gpus[hvd.local_rank()], 'GPU')], \sigma_1)
    ELSE ([import alias^*], \sigma_1)
```

```
trans_{S}[\![ from \ i \ id? import \ alias^* ]\!](\sigma) =
    LET \sigma_1 = trans_{\overline{A}} [\![ alias^* ]\!](\sigma) IN
    IF id = \text{tensorflow AND } \sigma_1 \setminus \sigma = [\text{"keras"} \mapsto id] THEN
       ([from i id? import alias^*,
       import horovod.tensorflow.keras as hvd,
       hvd.init(),
       gpus = id.config.experimental.list_physical_devices('GPU'),
       for gpu in gpus: id.config.experimental.set_memory_growth(gpu, True),
       if gpus: id.config.experimental.set_visible_devices(gpus[hvd.local_rank()], 'GPU')], <math>\sigma_1)
    ELSE ([from i id? import alias^*], \sigma)
 trans_S[\![global\ id^*]\!](\sigma) = ([global\ id^*], \sigma)
 trans_S[\![ nonlocal id^* ]\!](\sigma) = ([nonlocal id^*], \sigma)
A strict form of expr statements
 trans_{S}[expr_{1} (expr_{11} ... expr_{1n} (id_{1} = )? expr_{21} ... (id_{k} = )? expr_{2k})](\sigma) =
    IF \sigma("optimizer") = id_t AND expr_1 = id_t.apply_gradients THEN
        IF id_i = grads\_and\_vars WHEN 1 \le i \le k THEN
           LET id_z = \text{NewID}() IN
           ([id_z = expr_{2i},
           expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... id_i = id_z ... (id_k = )? expr_{2k}),
           global hvd_broadcast_done,
           if not hvd_broadcast_done: [hvd.broadcast_variables([x[1] for x in id_z], root_rank=0),
               hvd.broadcast_variables(id_t.variables(), root_rank=0),
               hvd\_broadcast\_done = True ]], \sigma)
        ELSE
           LET id_z = \text{NewID}() IN
           ([id_z = expr_{11},
           expr_1 (id_z expr_{12} \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k}),
           global hvd_broadcast_done,
           if not hvd_broadcast_done: [hvd.broadcast_variables([x[1] for x in id_z], root_rank=0),
               hvd.broadcast_variables(id_t.variables(), root_rank=0),
               hvd\_broadcast\_done = True ]], \sigma)
    ELIF expr_1 = print THEN
        ([if hvd.rank() == 0:,
           [expr_1 (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k})]], \sigma)
    ELIF \sigma("checkpoint") = id_t AND expr_1 = id_t.save THEN
        ([if hvd.rank() == 0: [expr_1 (expr_{11} ... expr_{1n} (id_1 =)? expr_{21} ... (id_k =)? expr_{2k})]], \sigma)
    ELIF \sigma(\text{``model''}) = id_t \text{ AND } expr_1 = id_t.\text{fit THEN}
        IF id_i = verbose WHEN 1 \le i \le k THEN
           IF id_j = callbacks WHEN 1 \le j \le k THEN
               ([expr_1 \ (expr_{11} \ ... \ expr_{1n} \ (id_1 = )? \ expr_{21} \ ...
```

```
id_i = 1 if hvd.rank() == 0 else 0
                     id_i = expr_{2i} + [hvd.callbacks.BroadcastGlobalVariablesCallback(0)]
                 ... (id_k = )? expr_{2k}), \sigma)
             ELSE
                 ([expr_1 \ (expr_{11} \ \dots \ expr_{1n} \ \ (id_1 = )? \ expr_{21} \ \dots
                     id_i = 1 if hvd.rank() == 0 else 0 ... (id_k = )? expr_{2k}
                     callbacks = expr_{2i} + [hvd.callbacks.BroadcastGlobalVariablesCallback(0)])], \sigma)
         ELSE
             IF id_j = callbacks WHEN 1 \le j \le k THEN
                 ([expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ...
                     id_i = expr_{2i} + [hvd.callbacks.BroadcastGlobalVariablesCallback(0)] ... (id_k = )? expr_{2k}
                     verbose = 1 if hvd.rank() == 0 else 0), \sigma)
             ELSE
                 ([expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... (id_k = )? expr_{2k})
                     verbose = 1 if hvd.rank() == 0 else 0
                     callbacks = expr_{2j} + [hvd.callbacks.BroadcastGlobalVariablesCallback(0)])], \sigma)
     ELIF \sigma(\text{``model''}) = id_t \text{ AND } expr_1 = id_t.\text{compile THEN}
         IF id_i = optimizer AND expr_{2i} = "adam" WHEN 1 \le i \le k THEN
             (id_z = tf.optimizers.Adam(learning_rate=0.001 * hvd.size())
             id_z = hvd.DistributedOptimizer(id_z)
             expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... id_i = id_z ... (id_k = )? expr_{2k}), \sigma)
         ELIF expr_{11} = "adam" THEN
             (id_z = tf.optimizers.Adam(learning_rate=0.001 * hvd.size())
             id_z = hvd.DistributedOptimizer(id_z)
             expr_1 (id_z ... expr_{1n} (id_1 = )? expr_{21} ... (id_k = )? expr_{2k})], \sigma)
         ELSE
             [trans_E \parallel expr_1 (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k}) \parallel (\sigma)], \sigma)
     ELSE
         [trans_E \parallel expr_1 (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k}) \parallel (\sigma) \mid, \sigma)
A general form of expr statements except for the strict form
 trans_{S}[\![expr]\!](\sigma) = ([trans_{E}[\![expr]\!](\sigma)], \sigma)
 trans_S[\![ pass ]\!](\sigma) = ([pass], \sigma)
 trans_S[\![ break ]\!](\sigma) = ([break], \sigma)
 trans_S[\![ continue ]\!](\sigma) = ([continue], \sigma)
             : Expr \rightarrow \Sigma \rightarrow Expr
 trans_{E} \llbracket expr_{1} \ boolop \ expr_{2} \ \rrbracket(\sigma) = trans_{E} \llbracket expr_{1} \ \rrbracket(\sigma) \ boolop \ trans_{E} \llbracket expr_{2} \ \rrbracket(\sigma)
 trans_{E} \llbracket expr_{1} := expr_{2} \rrbracket (\sigma) = expr_{1} := trans_{E} \llbracket expr_{2} \rrbracket (\sigma)
```

```
trans_{E}[\![expr_{1} \ binop \ expr_{2} \ ]\!](\sigma) = trans_{E}[\![expr_{1} \ ]\!](\sigma) \ binop \ trans_{E}[\![expr_{2} \ ]\!](\sigma)
trans_{E}[\![unop\ expr\ ]\!](\sigma) = unop\ trans_{E}[\![expr\ ]\!](\sigma)
trans_{E}[\![ ] ] lambda args: expr[\![ (\sigma) ] ] lambda args: trans_{E}[\![ ] ] expr[\![ (\sigma) ]
trans_{E}[\![ expr_{1} ]\!] if expr_{2}  else expr_{3} ]\!] (\sigma) =
     trans_{E} \llbracket expr_{1} \rrbracket (\sigma) if trans_{E} \llbracket expr_{2} \rrbracket (\sigma) else trans_{E} \llbracket expr_{3} \rrbracket (\sigma)
trans_{E} \llbracket expr_{1} \rrbracket (\sigma) if trans_{E} \llbracket expr_{2} \rrbracket (\sigma) else trans_{E} \llbracket expr_{3} \rrbracket (\sigma)
trans_{E}[\{(expr_{1}: expr_{2})^{*} \ (**expr_{3})^{*}\}] (\sigma) = \{(expr_{1}: trans_{E}[[expr_{2}]](\sigma))^{*} \ (**expr_{3})^{*}\}]
trans_E[\![\{expr^*\}]\!](\sigma) = \{(trans_E[\![expr]\!](\sigma))^*\}
trans_E \llbracket [expr^*] \rrbracket (\sigma) = \llbracket (trans_E \llbracket expr \rrbracket (\sigma))^* \rrbracket
trans_E[\![ (expr^*) \ ]\!](\sigma) = ((trans_E[\![ expr \ ]\!](\sigma))^*)
trans_{\mathbb{E}}[\![expr_1:expr_2:comprehension^*]\!](\sigma) = \{expr_1:trans_{\mathbb{E}}[\![expr_2:l](\sigma):(trans_{\mathbb{E}}[\![expr_2:l](\sigma):trans_{\mathbb{E}}[\![expr_2:l](\sigma):](\sigma))^*\}
trans_E[\![\{expr\ comprehension^*\}\]](\sigma) = \{trans_E[\![\ expr\]](\sigma)\ (trans_O[\![\ comprehension]\!](\sigma))^*\}
trans_E[\![expr\ comprehension^*]\!](\sigma) = [trans_E[\![expr\ ]\!](\sigma)\ (trans_O[\![comprehension]\!](\sigma))^*]
trans_{E} \llbracket (expr \ comprehension^*) \ \rrbracket (\sigma) = (trans_{E} \llbracket \ expr \ \rrbracket (\sigma) \ (trans_{O} \llbracket comprehension \rrbracket (\sigma))^*)
trans_{E}[\![\![\ await\ expr\ ]\!](\sigma) = await\ trans_{E}[\![\![\ expr\ ]\!](\sigma)
trans_E yield expr? \|(\sigma) = yield (trans_E \| expr \|(\sigma))?
trans_{E}[\![\ yield\ from\ expr\ ]\!](\sigma) = yield\ from\ trans_{E}[\![\ expr\ ]\!](\sigma)
trans_{\mathbb{E}} \llbracket expr_1 \ (compop \ expr_2)^* \ \rrbracket(\sigma) = trans_{\mathbb{E}} \llbracket expr_1 \ \rrbracket(\sigma) \ (compop \ trans_{\mathbb{E}} \llbracket expr_2 \rrbracket(\sigma))^*
```

```
trans_E \llbracket expr_1 \ (expr_{11} \ ... \ expr_{1n} \ (id_1 = )? \ expr_{21} \ ... \ (id_k = )? \ expr_{2k}) \ \rrbracket(\sigma) =
     IF \sigma("dataset") = id_t AND expr_1 = id_t.take THEN
           IF id_i = \text{count} WHEN 1 \le i \le k THEN
                expr_1 (expr_{11} ... expr_{1n} (id_1 = )? expr_{21} ... id_i = expr_{2i} // hvd.size() ... (id_k = )? expr_{2k})
           ELSE
                 expr_1 (expr_{11} // hvd.size() ... expr_{1n} (id_1 = )? expr_{21} ... (id_k = )? expr_{2k})
     ELSE
           trans_{E}[\![expr_{1}]\!](\sigma) (trans_{E}[\![expr_{11}]\!](\sigma) ... trans_{E}[\![expr_{1n}]\!](\sigma)
                (id_1 = )? trans_E[\![expr_{21}]\!](\sigma) \dots (id_k = )? trans_E[\![expr_{2k}]\!](\sigma))
trans_{E}[\![\{expr_{1} \ (!i)? \ (:expr_{2})?\}]\!](\sigma) = \{expr_{1} \ (!i)? \ (:expr_{2})?\}
trans_E \llbracket expr^* \rrbracket (\sigma) = expr^*
trans_E[\![ constant \ ]\!](\sigma) = constant
trans_E \llbracket expr.id \rrbracket (\sigma) = (trans_E \llbracket expr \rrbracket (\sigma)).id
trans_{E} \llbracket expr_{1} \llbracket expr_{2} \rrbracket \rrbracket (\sigma) = trans_{E} \llbracket expr_{1} \rrbracket (\sigma) \llbracket trans_{E} \llbracket expr_{2} \rrbracket (\sigma) \rrbracket
trans_{E}[\![ *expr ]\!](\sigma) = *expr
trans_E \llbracket **expr \rrbracket (\sigma) = **expr
trans_E \llbracket id \rrbracket (\sigma) = id
trans_{E}[\![expr_{1}?\ (:expr_{2})?\ (:expr_{3})?\ ]\!](\sigma) = trans_{E}[\![expr_{1}\ ]\!](\sigma)?\ (:trans_{E}[\![expr_{2}\ ]\!](\sigma))?\ (:trans_{E}[\![expr_{3}\ ]\!](\sigma))?
               : Comprehension \rightarrow \Sigma \rightarrow Comprehension
trans_O \llbracket for expr_1 in expr_2 (if expr_3)* \rrbracket (\sigma) = for expr_1 in trans_E \llbracket expr_2 \rrbracket (\sigma) (if trans_E \llbracket expr_3 \rrbracket (\sigma))*
 trans_H : ExcHandler \rightarrow \Sigma \rightarrow ExcHandler
trans_H \llbracket \text{ except } expr? \text{ (as } id)? : stmt^* \rrbracket (\sigma) = \text{except } expr? \text{ (as } id)? : trans_{\overline{S}} \llbracket stmt^* \rrbracket (\sigma).\_1
 trans_{\overline{A}} : Alias list \rightarrow \Sigma \rightarrow \Sigma
trans_{\overline{A}} \llbracket \ alias_1 \ \dots \ alias_{n-1} \ alias_n \ \rrbracket (\sigma) = trans_A \llbracket \ alias_n \ \rrbracket (trans_A \llbracket \ alias_{n-1} \ \rrbracket (\dots \ trans_A \llbracket \ alias_1 \ \rrbracket (\sigma)))
                      Alias \rightarrow
                                         \Sigma \rightarrow
                                                       \sum
 trans_A
```

```
trans_A \llbracket id \rrbracket (\sigma) =
     LET id = \text{tensorflow THEN } \sigma[\text{"tensor\_flow"} \mapsto id]
     LET id = \text{keras} THEN \sigma[\text{"keras"} \mapsto id]
     LET id = os THEN \sigma["os" \mapsto id]
     LET id = \text{optimizers} THEN \sigma[\text{"optimizers"} \mapsto id]
     ELSE \sigma
trans_A \llbracket id_1 \text{ as } id_2 \rrbracket (\sigma) =
    LET id_1 = tensorflow THEN \sigma["tensor_flow"\mapsto id_2]
     LET id_1 = tensorflow.keras THEN \sigma["keras" \mapsto id_2]
     LET id_1 = os THEN \sigma["os" \mapsto id_2]
     LET id = \text{optimizers THEN } \sigma[\text{"optimizers"} \mapsto id]
     ELSE \sigma
trans_A \llbracket id_1 . id_2 (.id_3)^* \text{ (as } id_2)? \rrbracket (\sigma) = \sigma
 trans_{\overline{W}}: WithItem list \rightarrow \Sigma \rightarrow (WithItem list \times \Sigma)
trans_{\overline{W}} \llbracket with\_item_1 \ with\_item_2 \ ... \ with\_item_n \ \rrbracket (\sigma) =
     LET with\_item_1', \sigma_1 = trans_W \llbracket with\_item_1 \rrbracket (\sigma) IN
     LET with_{-}item_{2}', \sigma_{2} = trans_{W} \llbracket with_{-}item_{2} \rrbracket (\sigma_{1}) IN
     LET with\_item_n', \sigma_n = trans_W \llbracket with\_item_n \rrbracket (\sigma_{n-1}) IN
     (with\_item_1' :: with\_item_2' :: ... :: [with\_item_n'], \sigma_n)
               : WithItem \rightarrow \Sigma \rightarrow (WithItem \times \Sigma)
trans_W \llbracket expr \rrbracket (\sigma) = (trans_E \llbracket expr \rrbracket (\sigma), \sigma)
trans_W \llbracket expr_1 \text{ as } expr_2 \rrbracket (smodenv) =
     IF \sigma("tensor_flow") = id_1 AND expr_1 = id_1.GradientTape() AND expr_2 = id_2 THEN
         (expr_1 \text{ as } expr_2, \sigma[\text{"gradient\_tape"} \mapsto id_2])
     ELSE (trans_E \llbracket expr_1 \rrbracket (\sigma) \text{ as } expr_2, \sigma)
 trans_C: MatchCase \rightarrow \Sigma \rightarrow MatchCase
trans_C[\![ case pattern (if expr)? : stmt^* ]\!](\sigma) =
     case trans_{P}[\![pattern\ ]\!](\sigma) (if trans_{E}[\![expr\ ]\!](\sigma))? : trans_{\overline{S}}[\![stmt^*\ ]\!](\sigma)._1
             : Pattern \rightarrow \Sigma \rightarrow
                                                    Pattern
 trans_P
trans_P \llbracket expr \rrbracket (\sigma) = trans_E \llbracket expr \rrbracket (\sigma)
trans_P \llbracket constant \rrbracket (\sigma) = constant
trans_P[\![pattern^*]]\!](\sigma) = [trans_P[\![pattern]\!](\sigma)^*]
```

```
trans_{P} \llbracket *(id)? \rrbracket (\sigma) = *(id)?
trans_{P} \llbracket \{(expr : pattern)^{*} \ id?\} \rrbracket (\sigma) = \{(expr : trans_{P} \llbracket pattern \rrbracket (\sigma))^{*} \ id?\}
trans_{P} \llbracket expr \ (pattern_{1}^{*} \ (id = pattern_{2})^{*}) \rrbracket (\sigma) = expr \ (trans_{P} \llbracket pattern_{1} \rrbracket (\sigma)^{*} \ (id = trans_{P} \llbracket pattern_{2} \rrbracket (\sigma))^{*})
trans_{P} \llbracket (pattern \ as)? \ id \rrbracket (\sigma) = (trans_{P} \llbracket pattern \ \rrbracket (\sigma) \ as)? \ id
trans_{P} \llbracket pattern_{1} \mid pattern_{2} \rrbracket (\sigma) = trans_{P} \llbracket pattern_{1} \rrbracket (\sigma) \mid trans_{P} \llbracket pattern_{2} \rrbracket (\sigma)
trans_{P} \llbracket - \rrbracket (\sigma) = -
```

## 3 Identifying training loop

### 3.1 Restrictions

- 1. Training loop must be defined in only one file.
- 2. Training loop type is either distributed gradient tape or distributed optimizer.
- 3. Each model must have only one type of training loop.
- 4. Function must not be assigned to the variables or passed as an argument.
- 5. Training loop must not be defined conditionally.

### 3.2 Rules

## 3.2.1 Summary

```
ModuleSummary id \sigma_{\top} tl
                                                   (ModuleSummary)
Summary ::=
                                                   (FuncSummary)
                   FuncSummary tl
                                                   (ClassSummary)
                   {\tt ClassSummary}\ arg
                   ValueSummary s
                                                   (ValueSummary)
                   Model \mid \bot
                                                   (ARG)
arg
                   GradTape \mid Optimizer \mid \bot \quad (TrainingLoop)
                   \sigma \cup \top
\sigma \in \Sigma = Id \xrightarrow{\text{fin}} Summary
                                 Environment storing mappings from ids to summaries
```

#### 3.2.2 Training Loop

```
summary_M: Module \rightarrow \Sigma \rightarrow ModuleSummary
```

```
summary_M \llbracket id \ stmt^* \ type\_ignore \rrbracket =
      LET \sigma, tl = summary_{\overline{S}} [stmt^*](\sigma) IN
      {\tt ModuleSummary}\ id\ \sigma\ tl
   summary_{\overline{S}}: Stmt list \rightarrow \Sigma \rightarrow (\Sigma \times Stmt)
                                                                         tl)
 summary_{\overline{S}} [stmt_1 \ stmt_2 \dots stmt_n] (\sigma) = \mathbf{LET} \ \sigma_1, tl_1 = summary_{\overline{S}} [stmt_1] (\sigma) \mathbf{IN}
                                                                  LET \sigma_2, tl_2 = summary_{\overline{S}} [stmt_2](\sigma_1) IN
                                                                   LET \sigma_n, tl_n = summary_{\overline{S}} [stmt_n] (\sigma_{n-1}) IN
                                                                   (\sigma_n, tl_1 \sqcup tl_2 \sqcup \ldots tl_n)
  summary_S: Stmt \rightarrow \Sigma \rightarrow (\Sigma \times tl)
 summary_S[\![ (@expr_1)^* \ def \ id \ (args) \ (-> expr_2)? \ : \ (\#type:s)? \ stmt^* ]\!](\sigma) =
      LET \sigma', tl = summary_{\overline{S}} [\![ stmt^* ]\!] IN
      (\sigma[id \mapsto \text{FuncSummary } tl], \perp)
 summary_S \llbracket (@expr_1)^*  async def id (args) (-> expr_2)? : (\#type:s)? stmt^* \rrbracket (\sigma) =
      LET \sigma', tl = summary_{\overline{S}} [stmt^*] IN
      (\sigma[id \mapsto \text{FuncSummary } tl], \perp)
 summary_S[(@expr)^* \text{ class } id (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k}) : stmt^*](\sigma) =
      IF expr_{1i} = id_c.keras.Model WHEN 1 \le i \le n AND
          \sigma(id_c) = \text{ModuleSummary Id}(\text{"tensorflow"}) \quad [] \perp \text{THEN} \quad (\sigma[id \mapsto \text{ClassSummary } Model], \perp)
      ELIF expr_{2i} = id_c.keras.Model WHEN 1 \le i \le k AND
          \sigma(id_c) = \text{ModuleSummary Id}("tensorflow") \quad [] \perp \textbf{THEN} \quad (\sigma[id \mapsto \text{ClassSummary } Model], \perp)
      ELSE (\sigma[id \mapsto \text{ClassSummary } \bot], \bot)
A strict form of assignment statements
 summary_S \llbracket id_r = id_c \text{ () (\#type:}s)? \rrbracket (\sigma) =
      IF \sigma(id_c) = \text{ClassSummary } Model \text{ THEN } (\sigma[id_r \mapsto \text{ValueSummary "model_instance"}], \bot)
      ELSE (\sigma, \perp)
 summary_S \llbracket \text{ (#type:}s) ? \text{ for } expr_1 \text{ in } expr_2 : stmt_1^* \text{ (else : } stmt_2^*) ? \rrbracket (\sigma) =
      LET __, tl = summary_{\overline{S}}[stmt_1^*](\sigma) IN
      (\sigma, tl)
 summary_S \llbracket \text{ (#type:} s)? \text{ async for } expr_1 \text{ in } expr_2 : stmt_1^* \text{ (else : } stmt_2^*)? \rrbracket (\sigma) =
      LET __, tl = summary_{\overline{S}}[stmt_1^*](\sigma) IN
      (\sigma, tl)
 summary_{S} while (expr) : stmt_{1}^{*} (else : stmt_{2}^{*})? (\sigma) =
      LET __, tl = summary_{\overline{S}}[\![ stmt_1^* ]\!](\sigma) IN
      (\sigma, tl)
```

```
summary_S \llbracket \text{ (#type:}s)? \text{ with } with\_item^* : stmt^* \rrbracket (\sigma) =
     LET _-, tl = summary_{\overline{S}}[stmt^*](\sigma) IN
     (\sigma, summary_{\overline{W}} \llbracket with\_item^* \rrbracket (\sigma) \sqcup tl)
summary_S[\![ (\#type:s)?  async with with\_item^* : stmt^* ]\!](\sigma) =
     LET __, tl = summary_{\overline{S}}[\![ stmt^* ]\!](\sigma) IN
     (\sigma, summary_{\overline{W}} \llbracket with\_item^* \rrbracket (\sigma) \sqcup tl)
summary_S[\![try : stmt_1^*   exc\_handler^*   (else : stmt_2^*)?   (finally : stmt_3^*)? [\![](\sigma)
     (\sigma, summary_{\overline{S}} [\![ stmt_1^* ]\!](\sigma))
summary_S[\![\!] import \ alias^* ]\!](\sigma) = (summary_A[\![\!]\!] alias^* ]\!](\sigma), \perp)
summary_S[\![ from \ 0 \ id^* \ import \ alias^* \ ]\!](\sigma) =
     LET \sigma' = summary_{\overline{A}} [\![ alias^* ]\!](\sigma) IN
     LET [id_2 \mapsto ModuleSummary id_2 \sigma_2 \perp, id_2 \mapsto ModuleSummary id_2 \sigma_2 \perp,
          ... id_n \mapsto \text{ModuleSummary} \ id_n \ \sigma_n \perp \ = \ \sigma' \setminus \sigma \ \textbf{IN}
     \sigma \ ++ \ [id_2 \mapsto \texttt{ModuleSummary} \ (id^*+id_2) \ \sigma_2 \ \bot, \quad id_2 \mapsto \texttt{ModuleSummary} \ (id^*+id_2) \ \sigma_2 \ \bot,
          ... id_n \mapsto \text{ModuleSummary} (id^* + id_n) \sigma_n \perp]
summary_S \llbracket expr \rrbracket(\sigma) = (\sigma, summary_E \llbracket expr \rrbracket(\sigma))
summary_S \llbracket stmt \rrbracket (\sigma) = (\sigma, \bot)
 summary_{\overline{A}}: Alias list \rightarrow \Sigma \rightarrow \Sigma
summary_{\overline{A}} \llbracket alias_1 \dots alias_{n-1} alias_n \rrbracket (\sigma) =
     summary_A \llbracket alias_n \rrbracket (summary_A \llbracket alias_{n-1} \rrbracket (... summary_A \llbracket alias_1 \rrbracket (\sigma) ...))
trans_A : Alias \rightarrow \Sigma \rightarrow \Sigma
trans_A \llbracket id_1 id_2 \dots id_n \rrbracket (\sigma) =
     LET \sigma_n = [id_n \mapsto \text{ModuleSummary } id_n \top \bot]
     LET \sigma_{n-1} = [id_{n-1} \mapsto ModuleSummary id_{n-1} \quad \sigma_n \quad \bot]
     LET \sigma_2 = [id_2 \mapsto ModuleSummary id_2 \sigma_3 \perp]
     LET \sigma_1 = [id_1 \mapsto ModuleSummary id_1 \sigma_2 \perp]
     \sigma ++ \sigma_1
 summary_{\overline{W}}: WithItem list \rightarrow \Sigma \rightarrow tl
summary_{\overline{W}}[with\_item_1 \ with\_item_2 \ ... \ with\_item_n \ ](\sigma) =
     summary_{W} \llbracket with\_item_{1} \rrbracket(\sigma) \sqcup summary_{W} \llbracket with\_item_{2} \rrbracket(\sigma) \sqcup ... \quad summary_{W} \llbracket with\_item_{n} \rrbracket(\sigma)
 \overline{summary_W}: WithItem \rightarrow \Sigma \rightarrow tl
summary_W \llbracket expr_1 \text{ as } expr_2? \rrbracket (\sigma) = summary_E \llbracket expr_1 \rrbracket (\sigma)
```