

# Code Transformation for Distributed Python ML Code

## 1 Python Abstract Syntax

<i>module</i>	::=	<i>stmt</i> * <i>type_ignore</i>	(MODULEDEF)
<i>stmt</i>	::=	(@ <i>expr</i> )* <b>def</b> <i>id</i> ( <i>args</i> ) (-> <i>expr</i> )? : (# <b>type:s</b> )? <i>stmt</i> *	(FUNDEF)
		(@ <i>expr</i> )* <b>async def</b> <i>id</i> ( <i>args</i> ) (-> <i>expr</i> )? : (# <b>type:s</b> )? <i>stmt</i> *	(ASYNCFUNDEF)
		(@ <i>expr</i> )* <b>class</b> <i>id</i> ( <i>expr</i> * <i>keyword</i> *) : <i>stmt</i> *	(CLASSDEF)
		<b>return</b> <i>expr</i> ?	(RETURN)
		<b>delete</b> <i>expr</i> *	(DELETE)
		<i>expr</i> * = <i>expr</i> (# <b>type:s</b> )?	(ASSIGN)
		<i>expr</i> <i>binop</i> = <i>expr</i>	(AUGASSIGN)
		<i>expr</i> : <i>expr</i> (= <i>expr</i> )?	(ANNASSIGN)
		(# <b>type:s</b> )? <b>for</b> <i>expr</i> <b>in</b> <i>expr</i> : <i>stmt</i> * ( <b>else</b> : <i>stmt</i> )?	(FORLOOP)
		(# <b>type:s</b> )? <b>async for</b> <i>expr</i> <b>in</b> <i>expr</i> : <i>stmt</i> * ( <b>else</b> : <i>stmt</i> )?	(ASYNCFORLOOP)
		<b>while</b> ( <i>expr</i> ) : <i>stmt</i> * ( <b>else</b> : <i>stmt</i> )?	(WHILELOOP)
		<b>if</b> ( <i>expr</i> ) : <i>stmt</i> * ( <b>else</b> : <i>stmt</i> )?	(IF)
		(# <b>type:s</b> )? <b>with</b> <i>with_item</i> * : <i>stmt</i> *	(WITH)
		(# <b>type:s</b> )? <b>async with</b> <i>with_item</i> * : <i>stmt</i> *	(ASYNCWITH)
		<b>match</b> <i>expr</i> : <i>match_case</i> *	(MATCH)
		<b>raise</b> <i>expr</i> ? ( <b>from</b> <i>expr</i> )?	(RAISE)
		<b>try</b> : <i>stmt</i> * <i>exc_handler</i> * ( <b>else</b> : <i>stmt</i> )? ( <b>finally</b> : <i>stmt</i> )?	(TRY)
		<b>assert</b> <i>expr</i> <i>expr</i> ?	(ASSERT)
		<b>import</b> <i>alias</i> *	(IMPORT)
		<b>from</b> <i>i id</i> ? <b>import</b> <i>alias</i> *	(IMPORTFROM)
		<b>global</b> <i>id</i> *	(GLOBAL)
		<b>nonlocal</b> <i>id</i> *	(NONLOCAL)
		<i>expr</i>	(EXPRSTMT)
		<b>pass</b>	(PASS)
		<b>break</b>	(BREAK)
		<b>continue</b>	(CONTINUE)

<i>expr</i>	<code>::=</code>	<code>expr boolop expr</code>	(BOOLOP)
		<code>expr := expr</code>	(NAMEDEXPR)
		<code>expr binop expr</code>	(BINARYOP)
		<code>unop expr</code>	(UNARYOP)
		<code>lambda args : expr</code>	(LAMBDA)
		<code>expr if expr else expr</code>	(IFEXPR)
		<code>{(expr : expr)* (**expr)*}</code>	(DICTIONARY)
		<code>{expr*}</code>	(SET)
		<code>[expr*]</code>	(LIST)
		<code>(expr*)</code>	(TUPLE)
		<code>{expr:expr comprehension*}</code>	(DICTIONARYCOMP)
		<code>{expr comprehension*}</code>	(SETCOMP)
		<code>[expr comprehension*]</code>	(LISTCOMP)
		<code>(expr comprehension*)</code>	(GENERATORCOMP)
		<code>await expr</code>	(AWAIT)
		<code>yield expr?</code>	(YIELD)
		<code>yield from expr</code>	(YIELDFROM)
		<code>expr (compop expr)*</code>	(COMPOP)
		<code>expr (expr* keyword*)</code>	(CALL)
		<code>{expr (!i)? (:expr)?}</code>	(FORMATTEDVALUE)
		<code>expr*</code>	(JOINEDSTR)
		<code>constant</code>	(CONSTANT)
		<code>expr.id</code>	(ATTRIBUTE)
		<code>expr[expr]</code>	(SUBSCRIPT)
		<code>*expr</code>	(STARRED)
		<code>**expr</code>	(DOUBLESTARRED)
		<code>id</code>	(NAME)
		<code>expr? (:expr)? (:expr)?</code>	(SLICE)
<i>boolop</i>	<code>::=</code>	<code>and   or</code>	(BOOLOPERATOR)
<i>binop</i>	<code>::=</code>	<code>+   -   *   @   /   **   &lt;&lt;   &gt;&gt;       ^   &amp;   //   %</code>	(BINOPERATOR)
<i>unop</i>	<code>::=</code>	<code>~   not   +   -</code>	(UNOPERATOR)
<i>compop</i>	<code>::=</code>	<code>==   !=   &lt;   &lt;=   &gt;   &gt;=   is   is not   in   not in</code>	(COMPOP)
<i>comprehension</i>	<code>::=</code>	<code>for expr in expr (if expr)*</code>	(COMPREHENSION)
		<code>async for expr in expr (if expr)*</code>	(ASYNCCOMPREHEN)
<i>exc.handler</i>	<code>::=</code>	<code>except expr? (as id)? : stmt*</code>	(EXCHANDLER)
<i>args</i>	<code>::=</code>	<code>(arg (= expr)?)*, (arg (= expr)?)*, arg?, (arg (= expr)?)*, arg?</code>	(ARGUMENTS)
<i>arg</i>	<code>::=</code>	<code>id expr? s?</code>	(ARGUMENT)
<i>keyword</i>	<code>::=</code>	<code>id? = expr</code>	(KEYWORD)
<i>alias</i>	<code>::=</code>	<code>id (.id)* (as id)?</code>	(ALIAS)
<i>with.item</i>	<code>::=</code>	<code>expr (as expr)?</code>	(WITHITEM)

<i>match_case</i>	<code>::= case pattern (if expr)? : stmt*</code>	(MATCHCASE)
<i>pattern</i>	<code>::= expr</code>	(MATCHVALUE)
	<i>constant</i>	(MATCHSINGLETON)
	[ <i>pattern</i> *]	(MATCHSEQUENCE)
	<i>*(id)?</i>	(MATCHSTAR)
	{( <i>expr</i> : <i>pattern</i> )* <i>id</i> ?}	(MATCHMAPPING)
	<i>expr</i> ( <i>pattern</i> * ( <i>id</i> = <i>pattern</i> )*)	(MATCHCLASS)
	( <i>pattern</i> as)? <i>id</i>	(MATCHAS)
	<i>pattern</i>   <i>pattern</i>	(MATCHOR)
	-	(MATCHWILDCARD)
<i>constant</i>	<code>::= None</code>	(NONELITERAL)
	<i>i</i>	(INTLITERAL)
	<i>f</i>	(FLOATLITERAL)
	<i>c</i>	(COMPLEXLITERAL)
	<i>s</i>	(STRINGLITERAL)
	<i>b</i>	(BOOLEANLITERAL)
	( <i>constant</i> *)	(TUPLELITERAL)
	...	(ELLIPSIS)
<i>type_ignore</i>	<code>::= i*</code>	(TYPEIGNORE)
<i>id</i>	$\in Id$	
<i>s</i>	$\in Str$	
<i>b</i>	$\in \{\text{True}, \text{False}\}$	
<i>i</i>	$\in \mathbb{Z}$	
<i>f</i>	$\in \mathbb{R}$	
<i>c</i>	$\in \mathbb{C}$	

Note: there may be more constant terms such as an immutable container containing only constant elements. Please update the *constant* rule if you know such terms.

## 2 Transformation for TF2 Python Code

### 2.1 Restrictions

1. All import statements must be placed at the top of a module.
2. The `tensorflow` module must be assigned to a variable only using an import statement.
3. Members of the `tensorflow` module must not be aliased.
4. `print` function call expressions must not introduce side-effects.
5. A dataset and an optimizer object must only be created once via an assignment statement with a function call expression and must not be aliased.
6. Variables storing a dataset or a optimizer object must not be reassigned to store another value that is not a dataset or an optimizer object.
7. A dataset and an optimizer object must not be created conditionally.
8. `optimizer.apply_gradients` function call expressions must be expression statements or a direct RHS of an assignment statement.
9. Global variables storing an `optimizer` object and referred in functions must be defined before the functions' definitions and must not be changed after their initializations.
10. A checkpoint object must only be created once via an assignment statement with a function call expression and must not be aliased.

### 2.2 Rules

#### 2.2.1 Types and Auxiliary Functions

$\tau$	$::=$	<i>Module</i>	(MODULES)
		<i>Stmt</i>	(STATEMENTS)
		<i>Expr</i>	(EXPRESSIONS)
		<i>Comprehension</i>	(COMPREHENSIONS)
		<i>ExcHandler</i>	(EXCEPTION HANDLERS)
		<i>Alias</i>	(ALIASES)
		<i>WithItem</i>	(WITHITEMS)
		<i>Pattern</i>	(PATTERNS)
		<i>Id</i>	(IDENTIFIERS)
		<i>Str</i>	(STRINGS)
		$\tau$ list	(LIST OF ELEMENTS TYPED $\tau$ )

$\sigma \in \Sigma = Str \xrightarrow{\text{fin}} Id$       ENVIRONMENT STORING MAPPINGS FROM STRINGS TO IDENTIFIERS

<b>.1</b>	:	$\tau \times \tau \rightarrow \tau$	GET THE FIRST ELEMENT OF THE GIVEN PAIR
<b>@</b>	:	$\tau \text{ list} \rightarrow \tau \text{ list} \rightarrow \tau \text{ list}$	CONCATENATE TWO LISTS
<b>::</b>	:	$\tau \rightarrow \tau \text{ list} \rightarrow \tau \text{ list}$	APPEND AN ELEMENT TO A LIST (RIGHT-ASSOCIATIVE)

### 2.2.2 Transformation Rules

$trans_M : Module \rightarrow Module$
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$$trans_M \llbracket stmt^* \text{ type\_ignore} \rrbracket = trans_{\bar{S}} \llbracket stmt^* \rrbracket(\sigma).1 \text{ type\_ignore}$$

$trans_{\bar{S}} : Stmt \text{ list} \rightarrow \Sigma \rightarrow (Stmt \text{ list} \times \Sigma)$
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$$\begin{aligned} trans_{\bar{S}} \llbracket stmt_1 \text{ stmt}_2 \dots stmt_n \rrbracket(\sigma) &= \text{LET } stmt_1^{*'}, \sigma_1 = trans_S \llbracket stmt_1 \rrbracket(\sigma) \text{ IN} \\ &\quad \text{LET } stmt_2^{*'}, \sigma_2 = trans_S \llbracket stmt_2 \rrbracket(\sigma_1) \text{ IN} \\ &\quad \dots \\ &\quad \text{LET } stmt_n^{*'}, \sigma_n = trans_S \llbracket stmt_n \rrbracket(\sigma_{n-1}) \text{ IN} \\ &\quad (stmt_1^{*'} @ stmt_2^{*'} @ \dots @ stmt_n^{*'}, \sigma_n) \end{aligned}$$

$trans_S : Stmt \rightarrow \Sigma \rightarrow (Stmt \text{ list} \times \Sigma)$
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$$\begin{aligned} trans_S \llbracket (@expr_1)^* \text{ def } id (args) (-> expr_2)? : (\#type:s)? stmt^* \rrbracket(\sigma) &= \\ ([(@expr_1)^* \text{ def } id (args) (-> expr_2)? : (\#type:s)? trans_{\bar{S}} \llbracket stmt^* \rrbracket(\sigma).1], \sigma) \end{aligned}$$

$$\begin{aligned} trans_S \llbracket (@expr_1)^* \text{ async def } id (args) (-> expr_2)? : (\#type:s)? stmt^* \rrbracket(\sigma) &= \\ ([(@expr_1)^* \text{ async def } id (args) (-> expr_2)? : (\#type:s)? trans_{\bar{S}} \llbracket stmt^* \rrbracket(\sigma).1], \sigma) \end{aligned}$$

$$\begin{aligned} trans_S \llbracket (@expr_1)^* \text{ class } id (expr_2^* keyword^*) : stmt^* \rrbracket(\sigma) &= \\ ([(@expr_1)^* \text{ class } id (expr_2^* keyword^*) : trans_{\bar{S}} \llbracket stmt^* \rrbracket(\sigma).1], \sigma) = \end{aligned}$$

$$trans_S \llbracket \text{return } expr? \rrbracket(\sigma) = ([\text{return } (trans_E \llbracket expr \rrbracket(\sigma))?, \sigma)$$

$$trans_S \llbracket \text{delete } expr^* \rrbracket(\sigma) = ([\text{delete } expr^*], \sigma)$$

#### A strict form of assignment statements

$$trans_S \llbracket id_r = expr_1 (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k}) (\#type:s)? \rrbracket(\sigma) =$$

**IF**  $\sigma(\text{"tensor\_flow"}) = id_t$  **AND**  $expr_1 = id_t.\text{estimator.Estimotor}$  **THEN**

$([id_r = expr_1 (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k}) (\#type:s)?], \sigma[\text{"classifier"} \mapsto id_r])$

**ELIF**  $\sigma(\text{"tensor\_flow"}) = id_t$  **AND**  $expr_1 = id_t.\text{train.MomentumOptimizer}$  **THEN**

**IF**  $id_i = \text{learning\_rate}$  **WHEN**  $1 \leq i \leq k$  **THEN**

$([id_r = expr_1 (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots id_i = expr_{2i} * \text{hvd.size}() \dots (id_k = )? expr_{2k}) (\#type:s)?,$

$id_r = \text{hvd.DistributedOptimizer}(id_r, \text{backward\_passes\_per\_step}=1)],$

$\sigma[\text{"optimizer"} \mapsto id_r])$

**ELSE**

$([id_r = expr_1 (expr_{11} * \text{hvd.size}() \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k}) (\#type:s)?,$

$id_r = \text{hvd.DistributedOptimizer}(id_r, \text{backward\_passes\_per\_step}=1)],$

$\sigma[\text{"optimizer"} \mapsto id_r])$

**ELIF**  $\sigma(\text{"checkpoint"}) = id_t$  **AND**  $expr_1 = id_t.\text{save}$  **THEN**

$([\text{if } \text{hvd.rank}() == 0: [id_r = expr_1 (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k}) (\#type:s)?], \sigma)$

**ELSE**  $([id_r = trans_E \llbracket expr_1 (expr_{11} \dots expr_{1n} (id_2 = )? expr_{21} \dots (id_k = )? expr_{2k}) \rrbracket(\sigma) (\#type:s)?], \sigma)$

$trans_S \llbracket expr_0 = expr_1 \text{ (\#type:s)? } \rrbracket(\sigma) =$   
**IF**  $\sigma(\text{"os"}) = id_{os}$  **AND**  $expr_0 = id_{os}.\text{environ} \llbracket \text{'CUDA_VISIBLE_DEVICES'} \rrbracket$   
**THEN**  $(\llbracket, \sigma)$

**A general form of assignment statements except for the strict form**

$trans_S \llbracket expr_1^* = expr_2 \text{ (\#type:s)? } \rrbracket(\sigma) = ([expr_1^* = trans_E \llbracket expr_2 \rrbracket(\sigma) \text{ (\#type:s)?}], \sigma)$

$trans_S \llbracket expr_1 \text{ binop } = expr_2 \rrbracket(\sigma) = ([expr_1 \text{ binop } = trans_E \llbracket expr_2 \rrbracket(\sigma)], \sigma)$

$trans_S \llbracket expr_1 : expr_2 (= expr_3)? \rrbracket(\sigma) = ([expr_1 : expr_2 (= trans_E \llbracket expr_3 \rrbracket(\sigma))?), \sigma)$   
**IF**  $expr_1 = id_1$  **AND**  $\sigma(\text{"tensor\_flow"}) = id_2$   
**AND**  $expr_3 = id_2.\text{data.Dataset.expr}_4 \text{ (expr}_5^* \text{ keyword*)}$  **THEN**  
 $([expr_1 : expr_2 = expr_3 \text{ (\#type:s)?}], \sigma[\text{"dataset"} \mapsto id_1])$   
**ELSE**  $([expr_1 : expr_2 (= trans_E \llbracket expr_3 \rrbracket(\sigma))?), \sigma)$

$trans_S \llbracket \text{(\#type:s)? for } expr_1 \text{ in } expr_2 : stmt_1^* \text{ (else : } stmt_2^*)? \rrbracket(\sigma) =$   
 $([\text{(\#type:s)? for } expr_1 \text{ in } trans_E \llbracket expr_2 \rrbracket(\sigma) :$   
 $trans_{\bar{S}} \llbracket stmt_1^* \rrbracket(\sigma).\_1 \text{ (else : } trans_{\bar{S}} \llbracket stmt_2^* \rrbracket(\sigma).\_1?), \sigma)$

$trans_S \llbracket \text{(\#type:s)? async for } expr_1 \text{ in } expr_2 : stmt_1^* \text{ (else : } stmt_2^*)? \rrbracket(\sigma) =$   
 $([\text{(\#type:s)? async for } expr_1 \text{ in } trans_E \llbracket expr_2 \rrbracket(\sigma) :$   
 $trans_{\bar{S}} \llbracket stmt_1^* \rrbracket(\sigma).\_1 \text{ (else : } trans_{\bar{S}} \llbracket stmt_2^* \rrbracket(\sigma).\_1?), \sigma)$

$trans_S \llbracket \text{while } (expr) : stmt_1^* \text{ (else : } stmt_2^*)? \rrbracket(\sigma) =$   
 $([\text{while } (trans_E \llbracket expr \rrbracket(\sigma)) : trans_{\bar{S}} \llbracket stmt_1^* \rrbracket(\sigma).\_1 \text{ (else : } trans_{\bar{S}} \llbracket stmt_2^* \rrbracket(\sigma).\_1?), \sigma)$

$trans_S \llbracket \text{if } (expr) : stmt^* \text{ (else : } stmt^*)? \rrbracket(\sigma) =$   
 $([\text{if } (trans_E \llbracket expr \rrbracket(\sigma)) : trans_{\bar{S}} \llbracket stmt^* \rrbracket(\sigma).\_1 \text{ (else : } trans_{\bar{S}} \llbracket stmt^* \rrbracket(\sigma).\_1?), \sigma)$

$trans_S \llbracket \text{(\#type:s)? with } with\_item^* : stmt^* \rrbracket(\sigma) =$   
**LET**  $with\_item^{*'}, \sigma_1 = trans_{\bar{W}} \llbracket with\_item^* \rrbracket(\sigma)$  **IN**  
**LET**  $stmt^{*'}, \sigma_2 = trans_{\bar{S}} \llbracket stmt^* \rrbracket(\sigma_1)$  **IN**  
**IF**  $\sigma_1 \setminus \sigma = [\text{"gradient\_tape"} \mapsto id]$  **THEN**  
 $([\text{(\#type:s)? with } with\_item^{*'} : stmt^{*'},$   
 $id = \text{hvd.DistributedGradientTape}(id)], \sigma_2)$   
**ELSE**  $([\text{(\#type:s)? with } with\_item^{*'} : stmt^{*'}], \sigma_2)$

$trans_S \llbracket \text{(\#type:s)? async with } with\_item^* : stmt^* \rrbracket(\sigma) =$   
**LET**  $with\_item^{*'}, \sigma_1 = trans_{\bar{W}} \llbracket with\_item^* \rrbracket(\sigma)$  **IN**  
**LET**  $stmt^{*'}, \sigma_2 = trans_{\bar{S}} \llbracket stmt^* \rrbracket(\sigma_1)$  **IN**  
**IF**  $\sigma_1 \setminus \sigma = [\text{"gradient\_tape"} \mapsto id]$  **THEN**  
 $([\text{(\#type:s)? async with } with\_item^{*'} : stmt^{*'},$   
 $id = \text{hvd.DistributedGradientTape}(id)], \sigma_2)$   
**ELSE**  $([\text{(\#type:s)? async with } with\_item^{*'} : stmt^{*'}], \sigma_2)$

$trans_S \llbracket \text{match } expr : \text{match\_case}^* \rrbracket(\sigma) =$   
 $([\text{match } trans_E \llbracket expr \rrbracket(\sigma) : (trans_C \llbracket \text{match\_case} \rrbracket(\sigma))^*], \sigma)$

$trans_S \llbracket \text{raise } expr_1? \text{ (from } expr_2?) \rrbracket(\sigma) = ([\text{raise } expr_1? \text{ (from } expr_2?)?], \sigma)$

$trans_S \llbracket \text{try} : stmt_1^* \text{ exc\_handler}^* \text{ (else} : stmt_2^*)? \text{ (finally} : stmt_3^*)? \rrbracket(\sigma) =$   
 $([\text{try} : trans_{\bar{S}} \llbracket stmt_1^* \rrbracket(\sigma).1 \text{ (trans}_H \llbracket \text{exc\_handler} \rrbracket(\sigma))^* \text{ (else} : trans_{\bar{S}} \llbracket stmt_2^* \rrbracket(\sigma).1? \text{ (finally} : trans_{\bar{S}} \llbracket stmt_3^* \rrbracket(\sigma).1?], \sigma)$

$trans_S \llbracket \text{assert } expr_1 \text{ } expr_2? \rrbracket(\sigma) = ([\text{assert } trans_E \llbracket expr_1 \rrbracket(\sigma) \text{ } expr_2?], \sigma)$

$trans_S \llbracket \text{import } alias^* \rrbracket(\sigma) =$   
**LET**  $\sigma_1 = trans_{\bar{A}} \llbracket alias^* \rrbracket(\sigma)$  **IN**  
**IF**  $\sigma_1 \setminus \sigma = [\text{"tensor\_flow"} \mapsto id]$  **THEN**  
 $([\text{import } alias^*,$   
 $\text{import horovod.tensorflow as hvd,}$   
 $\text{hvd.broadcast\_done} = \text{False,}$   
 $\text{hvd.init(),}$   
 $\text{gpus} = id.\text{config.experimental.list\_physical\_devices('GPU'),}$   
 $\text{for gpu in gpus: } id.\text{config.experimental.set\_memory\_growth(gpu, True),}$   
 $\text{if gpus: } id.\text{config.experimental.set\_visible\_devices(gp[ hvd.local\_rank()], 'GPU')], \sigma_1)$   
**ELSE**  $([\text{import } alias^*], \sigma_1)$

$trans_S \llbracket \text{from } i \text{ id? import } alias^* \rrbracket(\sigma) =$   
**LET**  $\sigma_1 = trans_{\bar{A}} \llbracket alias^* \rrbracket(\sigma)$  **IN**  
**IF**  $id = \text{tensorflow}$  **AND**  $\sigma_1 \setminus \sigma = [\text{"keras"} \mapsto id]$  **THEN**  
 $([\text{from } i \text{ id? import } alias^*,$   
 $\text{import horovod.tensorflow.keras as hvd,}$   
 $\text{hvd.init(),}$   
 $\text{gpus} = id.\text{config.experimental.list\_physical\_devices('GPU'),}$   
 $\text{for gpu in gpus: } id.\text{config.experimental.set\_memory\_growth(gpu, True),}$   
 $\text{if gpus: } id.\text{config.experimental.set\_visible\_devices(gp[ hvd.local\_rank()], 'GPU')], \sigma_1)$   
**ELSE**  $([\text{from } i \text{ id? import } alias^*], \sigma)$

$trans_S \llbracket \text{global } id^* \rrbracket(\sigma) = ([\text{global } id^*], \sigma)$

$trans_S \llbracket \text{nonlocal } id^* \rrbracket(\sigma) = ([\text{nonlocal } id^*], \sigma)$

**A strict form of expr statements**

```

transS[[ expr1 (expr11 ... expr1n (id1 = )? expr21 ... (idk = )? expr2k) ]](σ) =
  IF σ("optimizer") = idt AND expr1 = idt.apply_gradients THEN
    IF idi = grads_and_vars WHEN 1 ≤ i ≤ k THEN
      LET idz = NewID() IN
        ([idz = expr2i,
          expr1 (expr11 ... expr1n (id1 = )? expr21 ... idi = idz ... (idk = )? expr2k),
          global hvd.broadcast_done,
          if not hvd.broadcast_done: [ hvd.broadcast_variables([x[1] for x in idz], root_rank=0),
            hvd.broadcast_variables(idt.variables(), root_rank=0),
            hvd.broadcast_done = True ]], σ)
    ELSE
      LET idz = NewID() IN
        ([idz = expr11,
          expr1 (idz expr12 ... expr1n (id1 = )? expr21 ... (idk = )? expr2k),
          global hvd.broadcast_done,
          if not hvd.broadcast_done: [ hvd.broadcast_variables([x[1] for x in idz], root_rank=0),
            hvd.broadcast_variables(idt.variables(), root_rank=0),
            hvd.broadcast_done = True ]], σ)
    ELIF expr1 = print THEN
      ([if hvd.rank() == 0:,
        [expr1 (expr11 ... expr1n (id1 = )? expr21 ... (idk = )? expr2k)]], σ)
    ELIF σ("checkpoint") = idt AND expr1 = idt.save THEN
      ([if hvd.rank() == 0: [expr1 (expr11 ... expr1n (id1 = )? expr21 ... (idk = )? expr2k)]], σ)
    ELIF σ("classifier") = idt AND expr1 = idt.train THEN
      IF idi = hooks WHEN 1 ≤ i ≤ k THEN
        ([expr1 (expr11 ... expr1n (id1 = )? expr21 ...
          idi = expr2i + [hvd.BroadcastGlobalVariablesHook(0)]
          ... (idk = )? expr2k)], σ)
      ELSE
        ([expr1 (expr11 ... expr1n (id1 = )? expr21 ... (idk = )? expr2k
          hooks = [hvd.BroadcastGlobalVariablesHook(0)])], σ)
    ELSE
      [transE[[ expr1 (expr11 ... expr1n (id1 = )? expr21 ... (idk = )? expr2k) ]](σ)], σ)

```

**A general form of expr statements except for the strict form**

```
transS[[ expr ]](σ) = ([transE[[ expr ]](σ)], σ)
```

```
transS[[ pass ]](σ) = ([pass], σ)
```

```
transS[[ break ]](σ) = ([break], σ)
```

```
transS[[ continue ]](σ) = ([continue], σ)
```



$$\boxed{trans_E : Expr \rightarrow \Sigma \rightarrow Expr}$$

$$trans_E \llbracket expr_1 \text{ boolop } expr_2 \rrbracket(\sigma) = trans_E \llbracket expr_1 \rrbracket(\sigma) \text{ boolop } trans_E \llbracket expr_2 \rrbracket(\sigma)$$

$$trans_E \llbracket expr_1 := expr_2 \rrbracket(\sigma) = expr_1 := trans_E \llbracket expr_2 \rrbracket(\sigma)$$

$$trans_E \llbracket expr_1 \text{ binop } expr_2 \rrbracket(\sigma) = trans_E \llbracket expr_1 \rrbracket(\sigma) \text{ binop } trans_E \llbracket expr_2 \rrbracket(\sigma)$$

$$trans_E \llbracket unop \text{ expr } \rrbracket(\sigma) = unop \text{ } trans_E \llbracket expr \rrbracket(\sigma)$$

$$trans_E \llbracket \text{lambda } args : expr \rrbracket(\sigma) = \text{lambda } args : trans_E \llbracket expr \rrbracket(\sigma)$$

$$trans_E \llbracket expr_1 \text{ if } expr_2 \text{ else } expr_3 \rrbracket(\sigma) = \\ trans_E \llbracket expr_1 \rrbracket(\sigma) \text{ if } trans_E \llbracket expr_2 \rrbracket(\sigma) \text{ else } trans_E \llbracket expr_3 \rrbracket(\sigma)$$

$$trans_E \llbracket expr_1 \text{ if } expr_2 \text{ else } expr_3 \rrbracket(\sigma) = \\ trans_E \llbracket expr_1 \rrbracket(\sigma) \text{ if } trans_E \llbracket expr_2 \rrbracket(\sigma) \text{ else } trans_E \llbracket expr_3 \rrbracket(\sigma)$$

$$trans_E \llbracket \{(expr_1 : expr_2)^* (**expr_3)^*\} \rrbracket(\sigma) = \{(expr_1 : trans_E \llbracket expr_2 \rrbracket(\sigma))^* (**expr_3)^*\}$$

$$trans_E \llbracket \{expr^*\} \rrbracket(\sigma) = \{(trans_E \llbracket expr \rrbracket(\sigma))^*\}$$

$$trans_E \llbracket [expr^*] \rrbracket(\sigma) = [(trans_E \llbracket expr \rrbracket(\sigma))^*]$$

$$trans_E \llbracket (expr^*) \rrbracket(\sigma) = ((trans_E \llbracket expr \rrbracket(\sigma))^*)$$

$$trans_E \llbracket \{expr_1 : expr_2 \text{ comprehension}^*\} \rrbracket(\sigma) = \{expr_1 : trans_E \llbracket expr_2 \rrbracket(\sigma) \text{ } (trans_O \llbracket comprehension \rrbracket(\sigma))^*\}$$

$$trans_E \llbracket \{expr \text{ comprehension}^*\} \rrbracket(\sigma) = \{trans_E \llbracket expr \rrbracket(\sigma) \text{ } (trans_O \llbracket comprehension \rrbracket(\sigma))^*\}$$

$$trans_E \llbracket [expr \text{ comprehension}^*] \rrbracket(\sigma) = [(trans_E \llbracket expr \rrbracket(\sigma) \text{ } (trans_O \llbracket comprehension \rrbracket(\sigma))^*)]$$

$$trans_E \llbracket (expr \text{ comprehension}^*) \rrbracket(\sigma) = (trans_E \llbracket expr \rrbracket(\sigma) \text{ } (trans_O \llbracket comprehension \rrbracket(\sigma))^*)$$

$$trans_E \llbracket \text{await } expr \rrbracket(\sigma) = \text{await } trans_E \llbracket expr \rrbracket(\sigma)$$

$$trans_E \llbracket \text{yield } expr? \rrbracket(\sigma) = \text{yield } (trans_E \llbracket expr \rrbracket(\sigma))?$$

$$trans_E \llbracket \text{yield from } expr \rrbracket(\sigma) = \text{yield from } trans_E \llbracket expr \rrbracket(\sigma)$$

$$trans_E \llbracket expr_1 (compop expr_2)^* \rrbracket(\sigma) = trans_E \llbracket expr_1 \rrbracket(\sigma) (compop trans_E \llbracket expr_2 \rrbracket(\sigma))^*$$

$$\begin{aligned} trans_E \llbracket expr_1 (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k}) \rrbracket(\sigma) = \\ \text{IF } \sigma(\text{"dataset"}) = id_t \text{ AND } expr_1 = id_t.\text{take} \text{ THEN} \\ \quad \text{IF } id_i = \text{count} \text{ WHEN } 1 \leq i \leq k \text{ THEN} \\ \quad \quad expr_1 (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots id_i = expr_{2i} // \text{hvd.size}() \dots (id_k = )? expr_{2k}) \\ \quad \text{ELSE} \\ \quad \quad expr_1 (expr_{11} // \text{hvd.size}() \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k}) \\ \text{ELSE} \\ \quad trans_E \llbracket expr_1 \rrbracket(\sigma) (trans_E \llbracket expr_{11} \rrbracket(\sigma) \dots trans_E \llbracket expr_{1n} \rrbracket(\sigma) \\ \quad (id_1 = )? trans_E \llbracket expr_{21} \rrbracket(\sigma) \dots (id_k = )? trans_E \llbracket expr_{2k} \rrbracket(\sigma)) \end{aligned}$$

$$trans_E \llbracket \{expr_1 \quad (!i)? \quad (:expr_2)?\} \rrbracket(\sigma) = \{expr_1 \quad (!i)? \quad (:expr_2)?\}$$

$$trans_E \llbracket expr^* \rrbracket(\sigma) = expr^*$$

$$trans_E \llbracket constant \rrbracket(\sigma) = constant$$

$$trans_E \llbracket expr.id \rrbracket(\sigma) = (trans_E \llbracket expr \rrbracket(\sigma)).id$$

$$trans_E \llbracket expr_1 [expr_2] \rrbracket(\sigma) = trans_E \llbracket expr_1 \rrbracket(\sigma) [trans_E \llbracket expr_2 \rrbracket(\sigma)]$$

$$trans_E \llbracket *expr \rrbracket(\sigma) = *expr$$

$$trans_E \llbracket **expr \rrbracket(\sigma) = **expr$$

$$trans_E \llbracket id \rrbracket(\sigma) = id$$

$$trans_E \llbracket expr_1? (:expr_2)? (:expr_3)? \rrbracket(\sigma) = trans_E \llbracket expr_1 \rrbracket(\sigma)? (:trans_E \llbracket expr_2 \rrbracket(\sigma))? (:trans_E \llbracket expr_3 \rrbracket(\sigma))?$$

$$\boxed{trans_O : Comprehension \rightarrow \Sigma \rightarrow Comprehension}$$

$$trans_O \llbracket \text{for } expr_1 \text{ in } expr_2 \text{ (if } expr_3)^* \rrbracket(\sigma) = \text{for } expr_1 \text{ in } trans_E \llbracket expr_2 \rrbracket(\sigma) \text{ (if } trans_E \llbracket expr_3 \rrbracket(\sigma))^*$$

$$\boxed{trans_H : ExcHandler \rightarrow \Sigma \rightarrow ExcHandler}$$

$$trans_H \llbracket \text{except } expr? \text{ (as } id)? : stmt^* \rrbracket(\sigma) = \text{except } expr? \text{ (as } id)? : trans_{\bar{S}} \llbracket stmt^* \rrbracket(\sigma).1$$

$$\boxed{trans_{\bar{A}} : Alias \text{ list} \rightarrow \Sigma \rightarrow \Sigma}$$

$$trans_{\bar{A}} \llbracket alias_1 \dots alias_{n-1} alias_n \rrbracket(\sigma) = trans_A \llbracket alias_n \rrbracket(trans_A \llbracket alias_{n-1} \rrbracket(\dots trans_A \llbracket alias_1 \rrbracket(\sigma)))$$

$$\boxed{trans_A : Alias \rightarrow \Sigma \rightarrow \Sigma}$$

$trans_A \llbracket id \rrbracket(\sigma) =$   
**LET**  $id = \text{tensorflow.compat.v1}$  **THEN**  $\sigma[\text{"tensorflow"} \mapsto id]$   
**LET**  $id = \text{keras}$  **THEN**  $\sigma[\text{"keras"} \mapsto id]$   
**LET**  $id = \text{os}$  **THEN**  $\sigma[\text{"os"} \mapsto id]$   
**ELSE**  $\sigma$

$trans_A \llbracket id_1 \text{ as } id_2 \rrbracket(\sigma) =$   
**LET**  $id_1 = \text{tensorflow.compat.v1}$  **THEN**  $\sigma[\text{"tensorflow"} \mapsto id_2]$   
**LET**  $id_1 = \text{tensorflow.keras}$  **THEN**  $\sigma[\text{"keras"} \mapsto id_2]$   
**LET**  $id_1 = \text{os}$  **THEN**  $\sigma[\text{"os"} \mapsto id_2]$   
**ELSE**  $\sigma$

$trans_A \llbracket id_1 . id_2 (.id_3)^* (\text{as } id_2)? \rrbracket(\sigma) = \sigma$

$trans_{\overline{W}} : WithItem \text{ list} \rightarrow \Sigma \rightarrow (WithItem \text{ list} \times \Sigma)$
---

$trans_{\overline{W}} \llbracket with\_item_1 with\_item_2 \dots with\_item_n \rrbracket(\sigma) =$   
**LET**  $with\_item_1', \sigma_1 = trans_W \llbracket with\_item_1 \rrbracket(\sigma)$  **IN**  
**LET**  $with\_item_2', \sigma_2 = trans_W \llbracket with\_item_2 \rrbracket(\sigma_1)$  **IN**  
 $\dots$   
**LET**  $with\_item_n', \sigma_n = trans_W \llbracket with\_item_n \rrbracket(\sigma_{n-1})$  **IN**  
 $(with\_item_1' :: with\_item_2' :: \dots :: [with\_item_n'], \sigma_n)$

$trans_W : WithItem \rightarrow \Sigma \rightarrow (WithItem \times \Sigma)$
--

$trans_W \llbracket expr \rrbracket(\sigma) = (trans_E \llbracket expr \rrbracket(\sigma), \sigma)$

$trans_W \llbracket expr_1 \text{ as } expr_2 \rrbracket(\text{smodenv}) =$   
**IF**  $\sigma[\text{"tensorflow"}] = id_1$  **AND**  $expr_1 = id_1.\text{GradientTape}()$  **AND**  $expr_2 = id_2$  **THEN**  
 $(expr_1 \text{ as } expr_2, \sigma[\text{"gradient\_tape"} \mapsto id_2])$   
**ELSE**  $(trans_E \llbracket expr_1 \rrbracket(\sigma) \text{ as } expr_2, \sigma)$

$trans_C : MatchCase \rightarrow \Sigma \rightarrow MatchCase$
--

$trans_C \llbracket \text{case pattern (if expr)? : stmt*} \rrbracket(\sigma) =$   
 $\text{case } trans_P \llbracket pattern \rrbracket(\sigma) \text{ (if } trans_E \llbracket expr \rrbracket(\sigma))? : trans_{\overline{S}} \llbracket stmt* \rrbracket(\sigma).1$

$trans_P : Pattern \rightarrow \Sigma \rightarrow Pattern$
--

$trans_P \llbracket expr \rrbracket(\sigma) = trans_E \llbracket expr \rrbracket(\sigma)$

$trans_P \llbracket constant \rrbracket(\sigma) = constant$

$trans_P \llbracket [pattern^*] \rrbracket(\sigma) = [trans_P \llbracket pattern \rrbracket(\sigma)^*]$

$trans_P \llbracket *(id)? \rrbracket(\sigma) = *(id)?$

$$\text{trans}_P \llbracket \{(expr : pattern)^* id?\} \rrbracket(\sigma) = \{(expr : \text{trans}_P \llbracket pattern \rrbracket(\sigma))^* id?\}$$

$$\text{trans}_P \llbracket expr \ (pattern_1^* (id = pattern_2)^*) \rrbracket(\sigma) = expr \ (\text{trans}_P \llbracket pattern_1 \rrbracket(\sigma)^* (id = \text{trans}_P \llbracket pattern_2 \rrbracket(\sigma))^*)$$

$$\text{trans}_P \llbracket (pattern \ \text{as})? id \rrbracket(\sigma) = (\text{trans}_P \llbracket pattern \rrbracket(\sigma) \ \text{as})? id$$

$$\text{trans}_P \llbracket pattern_1 \mid pattern_2 \rrbracket(\sigma) = \text{trans}_P \llbracket pattern_1 \rrbracket(\sigma) \mid \text{trans}_P \llbracket pattern_2 \rrbracket(\sigma)$$

$$\text{trans}_P \llbracket - \rrbracket(\sigma) = -$$

### 3 Identifying trainig loop

#### 3.1 Restrictions

1. Training loop must be defined in only one file.
2. Training loop type is either **distributed gradient tape** or **distributed optimizer**.
3. Each model must have only one type of training loop.
4. Function must not be assigned to the variables or passed as an argument.
5. Training loop must not be defined conditionally.

#### 3.2 Rules

##### 3.2.1 Summary

$$\begin{aligned}
\text{Summary} &::= \text{ModuleSummary } id \ \sigma_{\top} \ tl && (\text{MODULESUMMARY}) \\
&\quad \text{FuncSummary } tl && (\text{FUNCSUMMARY}) \\
&\quad \text{ClassSummary } arg && (\text{CLASSSUMMARY}) \\
&\quad \text{ValueSummary } s && (\text{VALUESUMMARY}) \\
arg &::= Model \mid \perp && (\text{ARG}) \\
tl &::= GradTape \mid Optimizer \mid \perp && (\text{TRAININGLOOP}) \\
\sigma_{\top} &::= \sigma \cup \top
\end{aligned}$$

$$\sigma \in \Sigma = Id^{\text{fin}} \rightarrow \text{Summary} \quad \text{ENVIRONMENT STORING MAPPINGS FROM IDS TO SUMMARIES}$$

##### 3.2.2 Training Loop

$$\begin{aligned}
&\boxed{\text{summary}_M : Module \rightarrow \Sigma \rightarrow ModuleSummary} \\
&\text{summary}_M \llbracket id \ stmt^* \ type\_ignore \rrbracket = \\
&\quad \text{LET } \sigma, tl = \text{summary}_{\bar{S}} \llbracket stmt^* \rrbracket(\sigma) \ \text{IN} \\
&\quad \text{ModuleSummary } id \ \sigma \ tl
\end{aligned}$$

$$\boxed{\text{summary}_{\bar{S}} : Stmt \ \text{list} \rightarrow \Sigma \rightarrow (\Sigma \times tl)}$$

$$\begin{aligned}
summary_{\bar{S}} \llbracket stmt_1 \quad stmt_2 \dots stmt_n \rrbracket(\sigma) &= \text{LET } \sigma_1, tl_1 = summary_S \llbracket stmt_1 \rrbracket(\sigma) \text{ IN} \\
&\text{LET } \sigma_2, tl_2 = summary_{\bar{S}} \llbracket stmt_2 \rrbracket(\sigma_1) \text{ IN} \\
&\dots \\
&\text{LET } \sigma_n, tl_n = summary_{\bar{S}} \llbracket stmt_n \rrbracket(\sigma_{n-1}) \text{ IN} \\
&(\sigma_n, tl_1 \sqcup tl_2 \sqcup \dots tl_n)
\end{aligned}$$

$summary_S : Stmt \rightarrow \Sigma \rightarrow (\Sigma \times tl)$
--

$$\begin{aligned}
summary_S \llbracket (@expr_1)^* \text{ def } id (args) (-> expr_2)? : (\#type:s)? stmt^* \rrbracket(\sigma) = \\
\text{LET } \sigma', tl = summary_{\bar{S}} \llbracket stmt^* \rrbracket \text{ IN} \\
(\sigma[id \mapsto \text{FuncSummary } tl], \perp)
\end{aligned}$$

$$\begin{aligned}
summary_S \llbracket (@expr_1)^* \text{ async def } id (args) (-> expr_2)? : (\#type:s)? stmt^* \rrbracket(\sigma) = \\
\text{LET } \sigma', tl = summary_{\bar{S}} \llbracket stmt^* \rrbracket \text{ IN} \\
(\sigma[id \mapsto \text{FuncSummary } tl], \perp)
\end{aligned}$$

$$\begin{aligned}
summary_S \llbracket (@expr)^* \text{ class } id (expr_{11} \dots expr_{1n} (id_1 = )? expr_{21} \dots (id_k = )? expr_{2k}) : stmt^* \rrbracket(\sigma) = \\
\text{IF } expr_{1i} = id_c.keras.Model \text{ WHEN } 1 \leq i \leq n \text{ AND} \\
\sigma(id_c) = \text{ModuleSummary Id("tensorflow")} \sqcup \perp \text{ THEN } (\sigma[id \mapsto \text{ClassSummary } Model], \perp) \\
\text{ELIF } expr_{2i} = id_c.keras.Model \text{ WHEN } 1 \leq i \leq k \text{ AND} \\
\sigma(id_c) = \text{ModuleSummary Id("tensorflow")} \sqcup \perp \text{ THEN } (\sigma[id \mapsto \text{ClassSummary } Model], \perp) \\
\text{ELSE } (\sigma[id \mapsto \text{ClassSummary } \perp], \perp)
\end{aligned}$$

#### A strict form of assignment statements

$$\begin{aligned}
summary_S \llbracket id_r = id_c () (\#type:s)? \rrbracket(\sigma) = \\
\text{IF } \sigma(id_c) = \text{ClassSummary } Model \text{ THEN } (\sigma[id_r \mapsto \text{ValueSummary "model.instance"}], \perp) \\
\text{ELSE } (\sigma, \perp)
\end{aligned}$$

$$\begin{aligned}
summary_S \llbracket (\#type:s)? \text{ for } expr_1 \text{ in } expr_2 : stmt_1^* (\text{else} : stmt_2^*)? \rrbracket(\sigma) = \\
\text{LET } \_, tl = summary_{\bar{S}} \llbracket stmt_1^* \rrbracket(\sigma) \text{ IN} \\
(\sigma, tl)
\end{aligned}$$

$$\begin{aligned}
summary_S \llbracket (\#type:s)? \text{ async for } expr_1 \text{ in } expr_2 : stmt_1^* (\text{else} : stmt_2^*)? \rrbracket(\sigma) = \\
\text{LET } \_, tl = summary_{\bar{S}} \llbracket stmt_1^* \rrbracket(\sigma) \text{ IN} \\
(\sigma, tl)
\end{aligned}$$

$$\begin{aligned}
summary_S \llbracket \text{while } (expr) : stmt_1^* (\text{else} : stmt_2^*)? \rrbracket(\sigma) = \\
\text{LET } \_, tl = summary_{\bar{S}} \llbracket stmt_1^* \rrbracket(\sigma) \text{ IN} \\
(\sigma, tl)
\end{aligned}$$

$$\begin{aligned}
summary_S \llbracket (\#type:s)? \text{ with } with\_item^* : stmt^* \rrbracket(\sigma) = \\
\text{LET } \_, tl = summary_{\bar{S}} \llbracket stmt^* \rrbracket(\sigma) \text{ IN} \\
(\sigma, summary_{\bar{W}} \llbracket with\_item^* \rrbracket(\sigma) \sqcup tl)
\end{aligned}$$

$$\begin{aligned}
summary_S \llbracket (\#type:s)? \text{ async with } with\_item^* : stmt^* \rrbracket(\sigma) = \\
\text{LET } \_, tl = summary_{\bar{S}} \llbracket stmt^* \rrbracket(\sigma) \text{ IN} \\
(\sigma, summary_{\bar{W}} \llbracket with\_item^* \rrbracket(\sigma) \sqcup tl)
\end{aligned}$$

$summary_S \llbracket \text{try} : stmt_1^* \text{ exc\_handler}^* \text{ (else : } stmt_2^*)? \text{ (finally : } stmt_3^*)? \rrbracket (\sigma) =$   
 $(\sigma, summary_S \llbracket stmt_1^* \rrbracket (\sigma))$

$summary_S \llbracket \text{import alias}^* \rrbracket (\sigma) = (summary_{\bar{A}} \llbracket alias^* \rrbracket (\sigma), \perp)$

$summary_S \llbracket \text{from 0 id}^* \text{ import alias}^* \rrbracket (\sigma) =$   
 $\text{LET } \sigma' = summary_{\bar{A}} \llbracket alias^* \rrbracket (\sigma) \text{ IN}$   
 $\text{LET } [id_2 \mapsto \text{ModuleSummary } id_2 \sigma_2 \perp, \text{ id}_2 \mapsto \text{ModuleSummary } id_2 \sigma_2 \perp,$   
 $\dots id_n \mapsto \text{ModuleSummary } id_n \sigma_n \perp] = \sigma' \setminus \sigma \text{ IN}$   
 $\sigma ++ [id_2 \mapsto \text{ModuleSummary } (id^* + id_2) \sigma_2 \perp, \text{ id}_2 \mapsto \text{ModuleSummary } (id^* + id_2) \sigma_2 \perp,$   
 $\dots id_n \mapsto \text{ModuleSummary } (id^* + id_n) \sigma_n \perp]$

$summary_S \llbracket \text{expr} \rrbracket (\sigma) = (\sigma, summary_E \llbracket \text{expr} \rrbracket (\sigma))$

$summary_S \llbracket \text{stmt} \rrbracket (\sigma) = (\sigma, \perp)$

$summary_{\bar{A}} : Alias \text{ list} \rightarrow \Sigma \rightarrow \Sigma$

$summary_{\bar{A}} \llbracket alias_1 \dots alias_{n-1} alias_n \rrbracket (\sigma) =$   
 $summary_A \llbracket alias_n \rrbracket (summary_A \llbracket alias_{n-1} \rrbracket (\dots summary_A \llbracket alias_1 \rrbracket (\sigma) \dots))$

$trans_A : Alias \rightarrow \Sigma \rightarrow \Sigma$

$trans_A \llbracket id_1 id_2 \dots id_n \rrbracket (\sigma) =$   
 $\text{LET } \sigma_n = [id_n \mapsto \text{ModuleSummary } id_n \top \perp]$   
 $\text{LET } \sigma_{n-1} = [id_{n-1} \mapsto \text{ModuleSummary } id_{n-1} \sigma_n \perp]$   
 $\dots$   
 $\text{LET } \sigma_2 = [id_2 \mapsto \text{ModuleSummary } id_2 \sigma_3 \perp]$   
 $\text{LET } \sigma_1 = [id_1 \mapsto \text{ModuleSummary } id_1 \sigma_2 \perp]$   
 $\sigma ++ \sigma_1$

$summary_{\bar{W}} : WithItem \text{ list} \rightarrow \Sigma \rightarrow tl$

$summary_{\bar{W}} \llbracket with\_item_1 with\_item_2 \dots with\_item_n \rrbracket (\sigma) =$   
 $summary_W \llbracket with\_item_1 \rrbracket (\sigma) \sqcup summary_W \llbracket with\_item_2 \rrbracket (\sigma) \sqcup \dots summary_W \llbracket with\_item_n \rrbracket (\sigma)$

$summary_W : WithItem \rightarrow \Sigma \rightarrow tl$

$summary_W \llbracket \text{expr}_1 \text{ as } \text{expr}_2? \rrbracket (\sigma) = summary_E \llbracket \text{expr}_1 \rrbracket (\sigma)$

$summary_E : Expr \rightarrow \Sigma \rightarrow tl$

**A strict form of call expression**

```

summaryE[[ expr1 (expr11 ... expr1n (id1 = )? expr21 ... (idk = )? expr2k) ]](σ) =
  IF σ(idr) = ValueSummary "model_instance" AND
    expr1 = idr.fit THEN Optimizer
  ELIF σ(idr) = ModuleSummary Id("tensorflow") σ' tl AND
    expr1 = idr.GradientTape THEN GradTape
  ELIF σ(idr) = FuncSummary tl AND
    expr1 = idr THEN tl
  ELSE ⊥

```

```

summaryE[[ expr ]](σ) = ⊥

```