

**[[A]]:**  $V^* \rightarrow S \rightarrow (S, V_{\perp})_{\perp}$

**[[I]]:**  $S \rightarrow S_{\perp}$

**[[C]]:**  $S \rightarrow B_{\perp}$

**[[E]]:**  $S \rightarrow V_{\perp}$

## Algorithm

**[[a]]**( $v^*$ )( $s$ ) = **pop\_al\_context**(**[[i]]**( $s'$ )) where

```
a = (e*, i)
s = (w*, c', h)
s' = (w*, c :: c', h)
c = (ρ, ⊥, 0)
ρ = bind(ε, e*, v*)
```

where

```
pop_al_context(
    w*, (_, v⊥, _) :: c', h
) = (w*, c', h), v⊥
```

## Instruction

**[[i1; i2]]**( $s$ ) = if  $v_{\perp} = \perp$  then **[[i2]]**( $s'$ ) else  $s'$  where

```
s' = [[i1]](s)
    = (w*, c :: c', h)
c = (_, v⊥, _)
```

**[[if c then i1 else i2]]**( $s$ ) = if **[[c]]**( $s$ ) then **[[i1]]**( $s$ ) else **[[i2]]**( $s$ )

**[[either i1 or i2]]**( $s$ ) = **[[i1]]**( $s$ )

**[[enter e1: e2 after i]]**( $s$ ) = **cleanup**(**[[i]]**( $s'$ )) where

```
v = [[e1]](s)
v* = [[e2]](s)
s = (w*, c :: c*, h)
s' = (w' :: w*, c' :: c*, h)
c = (ρ, v⊥, n)
c' = (ρ, v⊥, n + 1)
w' = (v, ε, v* ++ [ConstructorV (“end”, [])])
```

where

**cleanup**( $s$ ) = if  $n > 0$  then **cleanup**( $s_{res}$ ) else  $s_{res}$  where

```
s = (w :: w*, c :: c*, h)
s' = (w' :: w*, c :: c*, h)
w = (v_ctx, v_val*, v :: v_instr*)
w' = (v_ctx, v_val*, v_instr*)
v = ConstructV (str, v'*)
s_res = [[lookup(p, str)]](v'*)(s').1
      = (w_res*, c :: c*, h)
c = (ρ, v⊥, n)
```

**[[assert c]]**( $s$ ) = if **[[c]]**( $s$ ) then  $s$  else  $\perp$

**[[push e]]**( $s$ ) =  $s'$  where

```
v = [[e]](s)
s = (w :: w*, c*, h)
s' = (w' :: w*, c*, h)
w = (v1, v2*, v3*)
w' = (v1, v :: v2*, v3*)
```

**[[pop e]]**( $s$ ) =  $s'$  where

```
s = (w :: w*, c :: c*, h)
s' = (w' :: w*, c' :: c*, h)
w = (v1, v :: v2*, v3*)
w' = (v1, v2*, v3*)
c = (ρ, v⊥, n)
c' = (ρ', v⊥, n)
ρ' = bind(ρ, e, v)
```

**[[pop all e]]**( $s$ ) =  $s'$  where

```
s = (w :: w*, c :: c*, h)
s' = (w' :: w*, c' :: c*, h)
w = (v1, v2*, v3*)
w' = (v1, ε, v3*)
c = (ρ, v⊥, n)
c' = (ρ', v⊥, n)
ρ' = bind(ρ, e, v2*)
```

**[[let e1 e2]]**( $s$ ) =  $s'$  where

```
v = [[e2]](s)
s = (w*, c :: c*, h)
s' = (w*, c' :: c*, h)
c = (ρ, v⊥, n)
c' = (ρ', v⊥, n)
ρ' = bind(ρ, e1, v)
```

**[[trap]]**( $s$ ) =  $\perp$

**[[nop]]**( $s$ ) =  $s$

**[[return e]]**( $s$ ) =  $s'$  where

```
v = [[e]](s)
s = (w*, c :: c*, h)
s' = (w*, c' :: c*, h)
c = (ρ, _, n)
c' = (ρ, v, n)
```

**[[execute e]]**( $s$ ) = **[[lookup(p, str)]]**( $v^*$ )( $s$ ).1 where

```
v = [[e]](s)
    = ConstructV (str, v*)
```

**[[execute all e]]**( $s$ ) =  $s_n$  where

```
v = [[e]](s)
    = v_0 ... v_{n-1}
v_i = ConstructV (str_i, v_i*)
s_0 = s
s_{i+1} = [[lookup(p, str_i)]](v_i*)(s_i).1
```

**[[exit]]**( $s$ ) =  $s'$  where

```
s = (w :: w', c*, h)
s' = (w', c'*, h)
c'* = decrease_first_al_context_whose_number_is_nonzero(c*)
```

**[[ref x e]]**( $s$ ) =  $s'$  where

```
v = [[e2]](s)
s = (w*, c :: c*, h)
s' = (w*, c' :: c*, h')
c = (ρ, v⊥, n)
c' = (ρ', v⊥, n)
h', a = alloc(h, v)
ρ' = ρ + (x -> a)
```

**[[replace e1 [p\*] with e2]]**( $s$ ) =  $s'$  where

```
s = (w*, c :: c*, h)
s' = (w*, c :: c*, h')
c = (ρ, _, _)
a = [[e1]](s)
v = [[e2]](s)
v' = replace(h(a), p*, v)
h' = h + (a -> v')
```

where

**replace**( $v1$ , [],  $v2$ ) =  $v2$

**replace**( $v1$ ,  $p :: p'^*$ ,  $v2$ ) =  $v1$  + ( $p$  -> **replace**( $v1(p)$ ,  $p'^*$ ,  $v2$ ))

## Condition

**[[C]]:**  $S \rightarrow B_{\perp}$

**[[not c]]**( $s$ ) =  $\neg$ **[[c]]**( $s$ )

**[[c1 ⊕ c2]]**( $s$ ) = **[[c1]]**( $s$ ) ⊕ **[[c2]]**( $s$ )

**[[e1 ⊗ e2]]**( $s$ ) = **[[e1]]**( $s$ ) ⊗ **[[e2]]**( $s$ )

**[[e is of case t]]**( $s$ ) = **[[e]]**( $s$ ) == **ConstructV**( $t$ , \_)

## Expression

**[[E]]:**  $S \rightarrow V_{\perp}$

**[[n]]**( $s$ ) =  $n$

**[[t]]**( $s$ ) =  $t$

**[[e1 ⊕ e2]]**( $s$ ) = **[[e1]]**( $s$ ) ⊕ **[[e2]]**( $s$ )

**[[e\*]]**( $s$ ) = 1 where

```
n = |e*|
l(i) = [[e*[i]]](s) (for 0 <= i < n)
```

**[[e1^e2]]**( $s$ ) = 1 where

```
v = [[e1]](s)
n = [[e2]](s)
l(i) = v (for 0 <= i < n)
```

**[[e1 ++ e2]]**( $s$ ) = 1 where

```
l1, l2 = [[e1]](s), [[e2]](s)
n1 = |l1|
n2 = |l2|
l[i] = if i < n1 then l1[i] else l2[i-n2] (for 0 <= i < n1+n2)
```

**[[|e|]]**( $s$ ) = **[[|e|]]**( $s$ )

**[[{(t -> e)\*}]]**( $s$ ) =  $r$  where

```
n = |t*| = |e*|
r[t*[i]] = [[e*[i]]](s)
```

**[[e[p]]]**( $s$ ) = **[[e]]**( $s$ ) $[p]$

**[[e1[p\*] := e2]]**( $s$ ) = **replace**(**[[e1]]**( $s$ ),  $p^*$ , **[[e1]]**( $s$ ))

**[[e1[p\*] :+ e2]]**( $s$ ) = **append**(**[[e1]]**( $s$ ),  $p^*$ , **[[e1]]**( $s$ ))

where

**append**( $v1$ , [],  $v2$ ) =  $v1$  ++  $v2$

**append**( $v1$ ,  $p :: p'^*$ ,  $v2$ ) =  $v1$  + ( $p$  -> **append**( $v1(p)$ ,  $p'^*$ ,  $v2$ ))

**[[t(e\*)]]**( $s$ ) = **ConstructV**( $t$ ,  $v^*$ ) where

```
v*[i] = [[e*[i]]](s) (for 0 <= i < |e*|)
```

**[[e1, e2]]**( $s$ ) = ( $v1$ ,  $v2$ ) where

```
v1, v2 = [[e1]](s), [[e2]](s)
```

**[[f(e\*)]]**( $s$ ) =  $v_{\perp}$  where

```
v* = [[e*]](s)
v⊥ = [[lookup(p, f)]](v*)(s).2
```

**[[current context]]**( $s$ ) =  $v$  where

```
s = (w :: _, _, _)
w = (v, _, _)
```

**[[x]]**( $s$ ) =  $\rho(x)$  where

```
s' = (_, c :: _, _)
c = (ρ, _, _)
```

**[[e^{x\*}]]**( $s$ ) =  $v$  where

```
s = (w*, c :: c*, h)
c = (ρ, v⊥, n)
|ρ(x*[0])| = m
s_i = (w*, c_i :: c*, h) (for 0 <= i < m)
c_i = (ρ_i, v⊥, n) (for 0 <= i < m)
ρ_i = ρ + (x -> ρ(x)[i])* (for 0 <= i < m)
v[i] = [[e]](s_i) (for 0 <= i < m)
```