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Course: CSE 276C
HW#: Homework 3
Date: 10/23/2021
Professor: Dr. Christensen

Problem 1

$V = 10^3 \sin\sqrt{\pi t}$; $R_1 = 1\,k\Omega$; $C_1 = 2\,mF$

$$\frac{dq}{dt} = \frac{V}{R_1} - \frac{q}{R_1C_1}$$

$$\frac{dq}{dt} = \frac{10^3 \sin\sqrt{\pi t}}{R_1} - \frac{q}{R_1C_1}$$

$$\frac{dq}{dt} = \frac{10^3 C_1 \sin\sqrt{\pi t} - q}{R_1C_1} \quad ; \quad q(0) = 4C \quad ; \quad h = 0.1$$

Determine $q(t)$ at $t = 0.1$

a.) Euler's Method

$t_0 = 0 \qquad t_1 = t_0 + h$

$q_0 = 4C \qquad t_1 = 0 + 0.1$

$t_1 = 0.1$

$$q_1 = q_0 + \frac{dq}{dt}h \quad ; \quad \left.\frac{dq}{dt}\right|_{t=t_i} = f(t_i, q_i) \quad ; \quad \forall i \in \{0, 1, 2, \dots, n\}$$

$$= q_0 + f(t_0, q_0)h$$

$$= 4 + \left[\frac{10^3 C_1 \sin\sqrt{\pi t_0} - q_0}{R_1C_1}\right]h$$

$$= 4 + \left[\frac{10^3 C_1 \sin\sqrt{\pi(0)} - 4}{R_1C_1}\right]0.1$$

$$= 4 + \left[\frac{0 - 4}{R_1C_1}\right]0.1$$

$$= 4 - \frac{2}{5R_1C_1}$$

$$\boxed{q_1 = 4 - \frac{2}{5(1000)(0.002)} = 3.8\,C}$$; hence, using Euler's Method, $q(t)$ at $t = 0.1s$ is $3.8\,C$.

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Problem 1 (Part 2)

b.) 4^{th} Order Runge-Kutta's Method

$$\left.\frac{dq(t)}{dt}\right|_{t=t_i} = f(t_i, q_i) \quad ; \forall i \in \{0, 1, 2, \dots, n\} \quad ; t_0 = 0_s \quad ; h = 0.1_s$$

$$t_1 = t_0 + h = 0 + 0.1$$

$$t_1 = 0.1_s$$

$$K_1 = f(t_0, q_0)$$

$$= \frac{10^3 \sin\sqrt{\pi t_0}}{R_1} - \frac{q_0}{R_1 C_1}$$

$$= \frac{10^3 \sin\sqrt{\pi(0)}}{(1000)} - \frac{4}{(1000)(0.002)}$$

$$K_1 = -2 \text{ A}$$

$$K_2 = f(t_0 + \tfrac{1}{2}h, q_0 + \tfrac{1}{2}K_1 h) \; ; \; t_0 + \tfrac{1}{2}h = 0 + \tfrac{1}{2}(0.1) \; ; \; q_0 + \tfrac{1}{2}K_1 h = 4 + \tfrac{1}{2}(-2)(0.1)$$

$$= 0.05_s \qquad = 3.9 \text{ C}$$

$$= \frac{10^3 \sin\sqrt{\pi(0.05)}}{1000} - \frac{3.9}{1000(0.002)}$$

$$K_2 = -1.563962 \text{ A}$$

$$K_3 = f(t_0 + \tfrac{1}{2}h, q_0 + \tfrac{1}{2}K_2 h) \; ; \; t_0 + \tfrac{1}{2}h = 0.05_s \; ; \; q_0 + \tfrac{1}{2}K_2 h = 4 + \tfrac{1}{2}(-1.663962)(0.1)$$

$$K_3 = \frac{10^3 \sin\sqrt{\pi(0.05)}}{1000} - \frac{3.921802}{1000(0.002)} = -1.574863 \text{ A} \qquad = 3.921802 \text{ C}$$

$$K_4 = f(t_0 + h, q_0 + K_3 h) \; ; \; t_0 + h = 0.1_s \; ; \; q_0 + K_3 h = 4 + (-1.574863)\,0.1$$

$$= 3.842514 \text{ C}$$

$$K_4 = \frac{10^3 \sin\sqrt{\pi(0.1)}}{1000} - \frac{3.842514}{1000(0.002)}$$

$$K_4 = -1.389648 \text{ A}$$

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Problem 1 (Part 3)

$$q_1 = q_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)h$$

$$= 4 + \frac{1}{6}\left[-2 + 2(-1.563962) + 2(-1.574863) + (-1.389648)\right]0.1$$

$\boxed{q_1 = 3.838878 \text{ C} \quad ; \text{ hence, using } 4^{th} \text{ Order Runge-Kutta's Method, } q(t) \text{ at } t = 0.1s \text{ is } 3.838878 \text{ C.}}$

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Problem 3

$$\frac{dy}{dx} = \frac{1}{x^2(1-y)} \; ; \; \text{interval } [0, 1] \; ; \; y(1) = -1$$

Step 1:

a.) $\frac{dy}{dx} = \frac{1}{x^2(1-y)}$ $\Rightarrow$ $(1-y)\,dy = \frac{1}{x^2}dx$

$\cancel{y' = \frac{1}{x^2(1-y)}}$

$$\int (1-y)\,dy = \int \frac{1}{x^2}dx$$

$\cancel{y - \frac{y^2}{2}}$

$$y - \frac{y^2}{2} = -\frac{1}{x} + C \; ; \; y = -1 \; ; \; x = 1$$

$$(-1) - \frac{(-1)^2}{2} = -\frac{1}{1} + C$$

$$C = -\frac{1}{2}$$

$\cancel{\frac{dy}{dx} = \frac{1}{x^2(1-y)}}$

$\cancel{\frac{dx}{dy} = x^2(1-y)}$

$\cancel{x' = x^2(1-y)}$

$\cancel{x' = x^2 - x^2 y}$

$\cancel{x' = x^2 - x^2 y}$

$\cancel{x^2 y = -x' + x^2}$

$\cancel{y = -\frac{x'}{x^2} + 1}$

$\cancel{y = 1 - \frac{x'}{x^2}}$

Step 2:

$$y - \frac{y^2}{2} = -\frac{1}{x} - \frac{1}{2}$$

$$2y - y^2 = -\frac{2}{x} - 1$$

$$y^2 - 2y = 1 + \frac{2}{x}$$

$$y(y-2) = 1 + \frac{2}{x} \quad \cancel{x = 0}$$

$\cancel{y(y-2) = \infty}$

$\cancel{y^2 - 2y - \infty = 0}$

$\cancel{y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-\infty)}}{2(1)}}$

$\cancel{y = \frac{2 \pm \sqrt{\infty}}{2}}$

$\cancel{\boxed{y = \infty \; , \; y = -\infty}}$

Step 3:

$$y(y-2) = 1 + \frac{2}{x}$$

$$y^2 - 2y - \left(1 + \frac{2}{x}\right) = 0$$

$$y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1-\frac{2}{x})}}{2}$$

$$y = \frac{2 \pm \sqrt{4 + 4(1+\frac{2}{x})}}{2}$$

$$y = \frac{2 \pm 2\sqrt{2 + \frac{2}{x}}}{2}$$

$$y = 1 \pm \sqrt{2(1+\frac{1}{x})}$$

Step 4:

$$\boxed{\text{if } x = 0, \; y = \infty \; , \; y = -\infty}$$