5.6-5.8

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1) For problem 7 of Assignment 8, determine whether or not G and H are independent. Justify your ans. math.

$$f_G(g) = \begin{cases} \tfrac{3}{4}\, g(2-g) & 0<g<2 \\ 0 & \text{else} \end{cases} \qquad f_H(h) = \begin{cases} \tfrac{3}{8}\, h^2 & g<h\le 2 \\ 0 & \text{else} \end{cases}$$

$$f_{G,H}(g,h) = \tfrac{3}{4}\cdot g \quad \text{for } 0<g\le 2 \text{ and } g<h\le 2$$

$$f_G(g)\cdot f_H(h) = \left(\tfrac{3}{4}\, g(2-g)\right)\left(\tfrac{3}{8}\, h^2\right) = \tfrac{9}{32}\, g(2-g)h^2 \neq f_{G,H}(g,h)$$

$\therefore$ G and H are <u>not</u> independent because $f_{G,H}(g,h) \neq f_G(g) f_H(h)$

2) $$f_G(g) = \begin{cases} \tfrac{1}{8} & 4<g\le 12 \\ 0 & \text{else} \end{cases} \qquad f_H(h) = \begin{cases} k\cdot h^2 & 1<h\le 3 \\ 0 & \text{else} \end{cases}$$ *Independent

a) Determine the value of k

$$\int_1^3 kh^2\,dh = 1 \rightarrow 1 = \frac{k}{3}h^3\Big|_1^3 \rightarrow k\left(\frac{27}{3}\right) - k\left(\frac{1}{3}\right) = 1 \rightarrow \frac{1}{k} = \frac{26}{3} \rightarrow \boxed{k = \frac{3}{26}}$$

b) $f_{G,H}(g,h) = f_G(g) f_H(h)$ because $f_G(g)$ and $f_H(h)$ are independent

$$f_{G,H}(g,h) = \begin{cases} \left(\tfrac{1}{8}\right)\left(\tfrac{3}{26}\right)h^2 & 4<g\le 12,\ 1<h\le 3 \\ 0 & \text{else} \end{cases}$$

c) $$\int_1^3\int_4^{12} \left(\tfrac{1}{8}\right)\left(\tfrac{3}{26}\right)h^2\,dg\,dh \rightarrow \left(\tfrac{1}{8}\right)\left(\tfrac{3}{26}\right)h^2\, g\Big|_4^{12} = \tfrac{3}{208}h^2\cdot 12 - \tfrac{3}{208}h^2\cdot 4 = \tfrac{3}{26}h^2$$

$$\rightarrow \int_1^3 \tfrac{3}{26}h^2\,dh = \tfrac{3}{26}\left(\tfrac{1}{3}\right)h^3\Big|_1^3 = \tfrac{1}{26}h^3\Big|_1^3 = \frac{27}{26} - \frac{1}{26} = \frac{26}{26} = 1 \checkmark$$ is valid!

*From Class Notes

3) $$f_{G,H}(g,h) = \begin{cases} \tfrac{c}{g} & 27\le h\le g\le 33 \\ 0 & \text{else} \end{cases} \qquad c \approx 1.7185$$

$$f_G(g) = \int_{27}^{g} \frac{c}{g}\,dh = c\left(1 - \frac{27}{g}\right) \quad 27<g\le 33; \quad = 0 \quad \text{else}$$

$$f_H(h) = \int_h^{33} \frac{c}{g}\,dg = c\left(\ln(g)\right)\Big|_h^{33} = c\left(\ln(33) - \ln(h)\right) \quad 27<h\le 33; \quad = 0 \quad \text{else}$$

$$f_G(g)\cdot f_H(h) = c\left(1-\tfrac{27}{g}\right)\cdot c\left(\ln(33)-\ln(h)\right) \neq \tfrac{c}{g} \rightarrow f_{G,H}(g,h)$$ So, G & H are <u>not</u> indep.

a) $$E[G] = \int_{-\infty}^{\infty} g f_G(g)\,dg \rightarrow \int_{27}^{33} gc\left(1-\tfrac{27}{g}\right)dg = c\int_{27}^{33} g\left(1-\tfrac{27}{g}\right)dg = c\int_{27}^{33} g - 27\,dg = c\left(\frac{g^2}{2} - 27g\right)\Big|_{27}^{33}$$

$$= 1.7185\left(\left(\frac{33^2}{2} - 27(33)\right) - \left(\frac{27^2}{2} - 27(27)\right)\right) = \boxed{30.933}$$

$$E[H] = \int_{-\infty}^{\infty} h f_H(h)\,dh \rightarrow \int_{27}^{33} h\cdot c\left(\ln(33)-\ln(h)\right) = c\int_{27}^{33} h\left(\ln(33)-\ln(h)\right)dh$$

$$= 1.7185\left(16.855\ldots\right) = \boxed{28.966}$$

b) $E[GH] = \iint gh f_G(g) f_H(h)\, dg\, dh = c^2 \int_{27}^{33} \int_{27}^{33} gh(1 - 27/g)(\ln(33) - \ln(h))\, dg\, dh$

$= c^2 \int_{27}^{33} -18h(\ln(h) - \ln(33))\, dh = c^2 (303.399\ldots) = \boxed{896.01}$

$Cov[G,H] = E[GH] - E[G]E[H] = \boxed{0.00472}$

c) $Var[G] = E[G^2] - E[G]^2 = 1.7185\left(\left(\frac{33^{2^2}}{2} - 27(33^3)\right) - \left(\frac{27^{2^2}}{2} - 27(27^3)\right)\right) - 30.933 = \boxed{545630}$

$Var[H] = E[H^2] - E[H]^2 = 1.7185(-1.08722\ldots \times 10^6) - 28.966 = \boxed{-116530000000}$

d) $Var[G+H] = Var[G] + Var[H] + 2\,Cov(G,H) = \boxed{-1.1653 \times 10^{11}}$