1. Remove all unit productions from the below grammar:

$$S \rightarrow ACA \mid CA \mid AA \mid AC \mid A \mid C$$
$$A \rightarrow aAa \mid aa \mid B \mid C$$
$$B \rightarrow bB \mid C$$
$$C \rightarrow cC \mid c$$

$B \rightarrow bB \mid cC \mid c$

$A \rightarrow aAa \mid aa \mid bB \mid cC \mid c$

$S \rightarrow ACA \mid CA \mid AA \mid AC \mid aAa \mid aa \mid bB \mid cC \mid c$

$C \rightarrow cC \mid c$

reorder →

$S \rightarrow ACA \mid CA \mid AA \mid AC \mid aAa \mid aa \mid bB \mid cC \mid c$

$A \rightarrow aAa \mid aa \mid bB \mid cC \mid c$

$B \rightarrow bB \mid cC \mid c$

$C \rightarrow cC \mid c$

2. Convert the below grammar to Chomsky Normal Form:

$$S \rightarrow abAB$$
$$A \rightarrow aAB \mid \lambda$$
$$B \rightarrow BAa \mid A \mid \lambda$$

1. Remove lambdas
2. Remove Unit Productions
3. CNF

1. Remove Lambdas $N = \{A, B\}$ $A = (A \mid \lambda)$ $B = (B \mid \lambda)$

$S \rightarrow abAB$ $S \rightarrow ab(A \mid \lambda)(B \mid \lambda)$

$A \rightarrow aAB \mid \lambda$ $A \rightarrow a(A \mid \lambda)(B \mid \lambda) \mid \cancel{\lambda}$

$B \rightarrow BAa \mid A \mid \lambda$ $B \rightarrow (B \mid \lambda)(A \mid \lambda)a \mid (A \mid \lambda) \mid \cancel{\lambda} \mid (B \mid \lambda)$

$S \rightarrow ab((A \mid \lambda)(B \mid \lambda))$

$A \rightarrow a((A \mid \lambda)(B \mid \lambda))$

$B \rightarrow ((B \mid \lambda)(A \mid \lambda))a \mid (A \mid \lambda) \mid (B \mid \lambda)$

2. Remove Unit Productions

$X = a$

$Y = b$

$W = XY$

$V = AB$

$Z = BA$

$S \rightarrow abAB \mid abA \mid abB \mid ab$

$A \rightarrow aAB \mid aA \mid aB \mid a$

$B \rightarrow BAa \mid Ba \mid Aa \mid a \mid aAB \mid aA \mid aB$

3. CNF

$S \rightarrow WV \mid WA \mid WB \mid XY$

$A \rightarrow XV \mid XA \mid XB \mid a$

$B \rightarrow ZX \mid BX \mid AX \mid XV \mid XA \mid XB \mid a$

3. Using the CYK algorithm, determine if the below grammar generates the string "bbaa":

$$S \to AB \mid CD \mid a \mid b$$
$$A \to a$$
$$B \to SA$$
$$C \to DS$$
$$D \to a \mid b$$

(1) b: S D

a: S A D

(2) bb: $(SD)(SD) = (\cancel{SS})(\cancel{SD})(\overset{C}{DS})(\cancel{DD})$: C

ba: $(SD)(SAD) = (\cancel{SS})(\overset{B}{SA})(\cancel{SD})(\overset{C}{DS})(\cancel{DA})(\cancel{DD})$: BC

aa: $(SAD)(SAD) = (\cancel{SS})(\overset{B}{SA})(\cancel{SD})(\cancel{AS})(\cancel{AA})(\cancel{AD})(\overset{C}{DS})(\cancel{DA})(\cancel{DD})$: BC

(3) bba: → bb, a: $(C)(SAD) = (\cancel{CS})(\cancel{CA})(\underset{S}{CD})$: S

baa: → b, aa: $(SD)(BC) = (\cancel{SB})(\cancel{SC})(\cancel{DB})(\cancel{DC})$

ba, a: $(BC)(SAD) = (\cancel{BS})(\cancel{BA})(\cancel{BD})(\cancel{CS})(\cancel{CA})(\underset{S}{CD})$: S

(4) bbaa: b, baa: (SD)(S) → SS, DS → C

bb, aa: (C)(B,C) → CB, CC = ∅

bbb, a: (S)(S,A,D) → (SS)(SA)(SD) → B

$\boxed{\text{S is not generated @ final step}}$ → bbaa is not generated

4. Show that the below language is not context-free:

$a^n b^m a^m$ where $0 \le n \le m$

$\underbrace{a^n}_{x} \underbrace{b^m}_{y} \underbrace{a^m}_{z}$ Pump x up to increase # of leading a's so n > m

→ Breaks $0 \le n \le m$ so not Context Free