HW #6

reading in text: pp. 138-150, 165-167, , class notes

1. Find the value of the constant k so that :

$$f(x) = \begin{cases} k x^2 (1-x^3) & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

is a proper density function of a continuous random variable.

2. The continuous random variable X has pdf $f(x) = x/2$ for $0 \le x \le 2$. Two independent determinations (observations) of X are made.
a) What is the probability that both of these determinations will be greater than 1.0?
b) If three independent determinations (observations) are made, what is the probability that exactly two out of the three are larger than 1.0?

3. The diameter of an electric cable, say X, is assumed to be a continuous random variable with pdf
$f(x) = 6x(1-x)$ for $0 \le x \le 1$.
a) Check that the above is a pdf and sketch it.
b) Determine a number b such that $P(X<b) = 2P(X>b)$.
c) Compute $P(X \le 1/2 \mid 1/3 < X < 2/3)$.

4. Suppose that X is a uniformly distributed random variable over [-a, a], where $a > 0$.
Whenever possible, determine the value of a so that the following are satisfied:
a) $P(X>1) = 1/3$ b) $P(X>1) = 1/2$ c) $P(X< 1/2) = 0.7$

5. If rainfall duration at the Toronto Airport is exponentially distributed with mean value 2.725 hours, what is the probability that the duration of a particular rainfall event there is:
a) at least 2 hours long?
b) at most 3 hours long?
c) between 2 and 3 hours long?

6. Particles arrive independently at a detector at the average rate of 3 per second.
a) Find the probability that you have to wait no more than 2 seconds for an arrival?
b) Nothing has arrived for the past 6 seconds. Find the probability that nothing will arrive in the next 6 seconds. (Do this problem twice, once with the exponential and once with the Poisson to illustrate theat they both work.)

7. Assume that the number of accidents in a factory can be represented by a Poisson process averaging 2 accidents per week (assume a week is 5 days long). What is the probability that
a) the time from now to the next accident will be more than 3 days?
b) the time from now to the third accident will be more than 5 days?

(8.) Let X be a random variable with

$$f(x) = \begin{cases} \frac{k}{x^4} & x > 1 \\ 0 & x \leq 1 \end{cases}$$

a) Find the value for k which makes $f(x)$ a legitimate pdf

b) Find the cdf (cumulative distribution function for X)

c) Use the cdf from (b) to determine the probability that X is greater than 2, and also the $P(2 < X < 3)$

d) What is the probability that X is within one standard deviation of its mean?