Homework 7:
I pledge my honor that I have abided by the Stevens Honor System.
5.1: 14, 18(just prove), 5.2: 4
5.1:
14. Prove that, for every positive integer n, $\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2$.
Base Case:
$P(1) = 1 * 2^1$
$= 2$
$= (1-1) * 2^{1+1} + 2$

Inductive Case:
Ind. Hip.:
$$\sum_{k=1}^{m} k2^k = (m-1)2^{m+1} + 2$$
Inductive Step:
$$\sum_{k=1}^{m=1} k2^k = (m+1)2^{m+1} + \sum_{k=1}^{m} k2^k$$

$= (m+1) * 2^{m+1} + (m-1) * 2^{m+1} + 2$	By Inductive Hyp.
$= (m+1+m-1) * 2^{m+1} + 2$	Arithmetic
$= (2m) * 2^{m+1} + 2$	Arithmetic
$= m * 2^{m+2} + 2$	Arithmetic
$= ((m+1)-1) * 2^{(m+1)+1} + 2$	Arithmetic

When $P(m)$ is true, $P(m+1)$ is also true. $P(1)$ is also true, so $P(n)$ is true for all positive integers n.

18. Let $P(n)$ be the statement that $n! < n^n$, where integer $n > 1$.
Base Case:
$P(2) = 2! < 2^2 \Rightarrow 2 < 4$
Inductive Case:
Inductive Hypothesis: $k! < k^k$ for $k > 1$
Inductive Step: $(k+1) < (k+1)^{k+1}$
$(k+1)!$

$= (k+1)k!$	Def. !
$< (k+1)k^k$	by Inductive Hyp.
$< (k+1)(k+1)^k$	Arithmetic
$= (k+1)^{k+1}$	Arithmetic

5.2:

4. Let $P(n)$ be the statement that a postage of n cents can be formed using just 4-cent stamps and 7-cent stamps. The parts of this exercise outline a strong induction proof that $P(n)$ is true for $n \geq 18$.

a) Show statements $P(18)$, $P(19)$, $P(20)$, and $P(21)$ are true, completing the basis step of the proof.
   $P(18) = 4 + 7 + 7$
   $P(19) = 4 + 4 + 4 + 7$
   $P(20) = 4 + 4 + 4 + 4 + 4$
   $P(21) = 7 + 7 + 7$
b) What is the inductive hypothesis of the proof?
   Inductive Hypothesis:
   $18 \leq n \leq k, \quad k \geq 21$ for n cents postage.
c) What do you need to prove in the inductive step?
   Inductive step:
   Prove, assuming the Inductive Hyp. is true, that you can form $(k + 1)$ cents of postage.
d) Complete the inductive step for $k \geq 21$.
   $P(k - 3)$ is true, since $k \geq 21$.
   If you add one 4-cent postage stamp, $P(k + 1)$ is true.
e) Explain why these steps show that this statement is true whenever $n \geq 18$.
   Both the base case and the inductive case are true, so by the principle of Strong Induction, the statement is true for all integers $n \geq 18$.