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55 minutes to complete. Closed book. No cooperation. No electronic communication. *No calculators.*

There are 30 points in this paper. To get a full mark, you need to score 27 points or more. Please write your answers on these question sheets in the space provided. If you run out of space, use extra paper.

Name:

Pledge:

Please write your name and pledge before turning the page.

	A1	A2	A3	A4	A5	A6	B1	B2	B3		$\sum$

Part A. In this part, only answers are required, so you don't have to (but you can) provide any explanations and show any work. Each question is worth **2 or 3 points**.

(A1) [2pt] Which of the following limits converge to a finite complex number? (Answer only.)

$$\lim_{n\to\infty} \tfrac{n+(1+i)^n}{n^2}, \qquad \lim_{n\to\infty} \tfrac{n+i^n}{n^2}, \qquad \lim_{n\to\infty} \tfrac{n^2+i^n}{n^2}.$$

(A2) [2pt] Find radius of convergence of the following power series. (Answer only.)

$$1 + 3 + 5^2 z^2 + 3^3 z^3 + 5^4 z^4 + \ldots = \sum_{n=0}^{\infty} (4 + (-1)^n)^n z^n.$$

(A3) [2pt] Which of the following complex power functions have only finitely many values for a given $z \neq 0$? (Answer only.)

$$z^{-2017}, \qquad z^{\sqrt{2}}, \qquad z^{i}, \qquad z^{\frac{3}{4}i}, \qquad z^{\frac{3}{4}}.$$

(A4) [3pt] Arrange the following numbers in the order of increasing absolute value. (Answer only.)

$$(2+i)^8, \qquad \sinh(2017i), \qquad e^{4-20i}, \qquad \operatorname{Log}(5e^{2017i}).$$

(A5) [3pt] Suppose C is a contour with endpoints z_0 and z_1 which does not pass through 0. For which of the following functions $f(z)$ is the integral $\int_C f(z)dz$ path independent? (Answer only.)

$$\operatorname{Log} z, \qquad \cos z^3, \qquad \tfrac{1}{e^z}, \qquad \tfrac{1}{z}, \qquad \bar{z}.$$

(A6) [3pt] Find

$$\frac{1}{2\pi i}\int_C \frac{\operatorname{Log} z}{(z-2i)^2}dz,$$

where $\operatorname{Log} z$ is the principal value of the logarithm, and C is a circle of radius 1 centered at $2i$ traversed in the positive direction. (Answer only. Simplify the answer.)

Part B. In this part, show your work and provide explanations.

(B1) [4pt] Find all solutions of the equation $\cos z = 2ie^{-iz}$. (Give the answer in the form $x + iy$.)

(B2) [4pt] Find and sketch image of the region $\{x+iy : 0 < x < 2,\ 1 < y < 7\}$ under the mapping $f(z) = e^z$.

(B3) [7pt] Use Cauchy Integral Theorem and Cauchy Integral Formula (and its version for derivative) to evaluate the following integral:

$$\int_{C_R(0)} \left(\frac{e^z}{2z - \pi i} + \frac{e^{3z}}{(z-4)^3} \right) dz$$

for $R = 1$, for $R = 2$, and for $R = 10$.

(Reminder: $C_R(0)$ is a circle of radius R centered at 0 traversed in the positive direction.)