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Assignment 8 - Solutions

1. a) $\cos(1+i) = \frac{1}{2}(e^{i(1+i)} + e^{-i(1+i)})$
$= \frac{1}{2}(e^{-1}(\cos 1 + i\sin 1) + e(\cos -1 + i\sin -1))$
$= \frac{1}{2}\cos 1(e^{-1}+e) + \frac{i}{2}\sin 1(e^{-1}-e)$

   b) $\sin(1+i) = \frac{1}{2i}(e^{i(1+i)} - e^{-i(1+i)})$
$= \frac{-i}{2}(e^{-1}(\cos 1 + i\sin 1) - e(\cos -1 + i\sin -1)$
$= \frac{-i}{2}\cos 1(e^{-1}-e) + \frac{1}{2}\sin 1(e^{-1}+e)$

   c) $\tan(1+i) = \frac{\sin(1+i)}{\cos(1+i)} = \frac{\sin 1(e^{-1}+e) - i\cos 1(e^{-1}-e)}{\cos 1(e^{-1}+e) + i\sin 1(e^{-1}-e)}$
$= \frac{(\cos 1(e^{-1}+e) + i\sin 1(e^{-1}-e)(\sin 1(e^{-1}+e) + i\cos 1(e^{-1}-e)}{\sin^2 1(e^{-1}+e)^2 + \cos^2 1(e^{-1}+e)^2 n}$
$= \frac{\cos 1\sin 1(e^{-1}+e)^2 - \cos 1\sin 1(e^{-1}-e)^2 + i\cos^2 1(e^{-1}+e)(e^{-1}-e) + i\sin^2 1(e^{-1}+e)(e^{-1}-e)}{\sin^2 1(e^{-1}+e)^2 + \cos^2 1(e^{-1}+e)^2 n}$
$= \frac{4\cos 1\sin 1 + i(e^{-2}-e^2)}{\sin^2 1(e^{-1}+e)^2 + \cos^2 1(e^{-1}+e)^2 n}$

2. a) $\cosh(1+i) = \frac{1}{2}(e^{1+i} + e^{-(1+i)})$
$= \frac{1}{2}(e(\cos 1 + i\sin 1) + e^{-1}(\cos -1 + i\sin -1)$
$= \frac{1}{2}\cos 1(e+e^{-1}) + \frac{i}{2}\sin 1(e-e^{-1})$

   b) $\sinh(1+i) = \frac{1}{2}(e^{1+i} - e^{-(1+i)})$
$= \frac{1}{2}(e(\cos 1 + i\sin 1) - e^{-1}(\cos -1 + i\sin -1))$
$= \frac{1}{2}\cos 1(e-e^{-1}) + \frac{i}{2}\sin 1(e+e^{-1})$

   c) $\tanh(1+i) = \frac{\sinh(1+i)}{\cosh(1+i)} = \frac{\cos 1(e-e^{-1}) + i\sin 1(e+e^{-1})}{\cos 1(e+e^{-1}) + i\sin 1(e-e^{-1})}$
$= \frac{(\cos 1(e-e^{-1}) + i\sin 1(e+e^{-1}))(\cos 1(e+e^{-1}) - i\sin 1(e-e^{-1}))}{\cos^2 1(e+e^{-1}) + \sin^2 1(e-e^{-1})^2}$
$= \frac{\cos^2 1(e+e^{-1})(e-e^{-1}) + \sin^2 1(e+e^{-1})(e-e^{-1}) + i\cos 1\sin 1(e+e^{-1})^2 - i\cos 1\sin 1(e-e^{-1})^2}{\cos^2 1(e+e^{-1}) + \sin^2 1(e-e^{-1})^2}$
$= \frac{(e^2-e^{-2}) + i4\cos 1\sin 1}{\cos^2 1(e+e^{-1}) + \sin^2 1(e-e^{-1})^2}$

3. a) $\cos(L_1) = [-1,1]$, $\cos(L_2) = [1,\infty)$, $\cos(R) = \mathbb{C}$

   To see that $\cos(R) = \mathbb{C}$, note that R is closed, simply connected and its boundaries are L_1 and L_2, and the only closed, simply connected region bounded by $\cos(L_1)$ and $\cos(L_2)$ is $\mathbb{C}$.

   b) $\cosh(L_1) = [1,\infty)$, $\cos(L_2) = [-1,1]$, $\cos(R) = \mathbb{C}$

   Same reasoning as before to show $\cosh(R) = \mathbb{C}$.