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Assignment 3 - Derivatives of Complex Functions
Due February 12th

1. (0.5 pts each) Use the limit definition of derivative to prove the following:

   a) $f(z) = z^n$, $f'(z) = nz^{n-1}$ for $n \in \mathbb{N}$

   b) $h(z) = af(z) + bg(z)$, $h'(z) = af'(z) + bg'(z)$ with $a, b \in \mathbb{C}$ constant

2. (1 pt) Let $p(z) = a_0 + a_1 z + ... + a_n z^n$ with $a_0, ..., a_n \in \mathbb{C}$ constant. Show that p is entire and $p'(z) = a_1 + 2a_2 z + ... + na_n z^{n-1}$.

3. (1 pt) Use the complex L'Hopital's rule to compute $\lim\limits_{z \to i} \frac{z^3 - z^2 + z - 1}{z^3 + z^2 + z + 1}$

4. (1 pt) Let $f(x + iy) = x^2 + iy^2$. Use the cartesian Cauchy-Riemann equations to determine where f is differentiable, where it is analytic and what the derivative is where it exists.

5. (1 pt) Let $f(re^{i\theta}) = re^{i2\theta}$. Use the polar Cauchy-Riemann equations to determine where f is differentiable, where it is analytic and what the derivative is where it exists.