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Assignment 9 - Solutions

1. a) $\int_0^3 (t-2i)^2\,dt = \int_0^3 t^2 - 4it - 4\,dt = [\frac{t^3}{3} - 2it^2 - 4t]_0^3 = -3 - 18i$

   b) Let $z(t) = ti$, $t \in [0,3]$, $z'(t) = i$

   $\int_{L_1} (z-2i)^2\,dz = \int_0^3 (ti-2i)^2 i\,dt = -i\int_0^3 t^2 - 4t + 4\,dt$
   $= -i[\frac{t^3}{3} - 2t^2 + 4t]_0^3 = -3i$

   c) Let $z(t) = 3 + ti$, $t \in [0,3]$, $z'(t) = i$

   $\int_{L_2} (z-2i)^2\,dz = \int_0^3 (3+ti-2i)^2 i\,dt = i\int_0^3 (5-12i) + (4+6i)t - t^2\,dt$
   $= i[(5-12i)t + (2+3i)t^2 - \frac{t^3}{3}]_0^3 = 9 + 24i$

   d) Let $z(t) = t + 3i$, $t \in [0,3]$, $z'(t) = 1$

   $\int_{L_3} (z-2i)^2\,dz = \int_0^3 (t+i)^2\,dt = i\int_0^3 t^2 + 2it - 1\,dt$
   $= [\frac{t^3}{3} + it^2 - t]_0^3 = 6 + 9i$

2. a) Let $z(t) = e^{it}$, $t \in [0, 2\pi]$, $z'(t) = ie^{it}$

   $\int_C \frac{1}{z}\,dz = \int_0^{2\pi} \frac{1}{e^{it}} ie^{it}\,dt = \int_0^{2\pi} i\,dt = 2\pi i$

   b) Let $z(t) = e^{it}$, $t \in [0, 2\pi]$, $z'(t) = ie^{it}$

   $\int_C \frac{z+1}{z}\,dz = \int_0^{2\pi} \frac{e^{it}+1}{e^{it}} ie^{it}\,dt = \int_0^{2\pi} ie^{it} + i\,dt = 2\pi i$

3. In 1., our function is a polynomial, and thus entire, so the Cauchy-Goursat theorem tells us the integral should be independant of the path. As such, we can predict that adding our results for 1.a) and 1.c) should give us the same as adding our results for 1.b) and 1.d).

   In 2. we have $\frac{z+1}{z} = 1 + \frac{1}{z}$ so $\int_C \frac{z+1}{z}\,dz = \int_C 1\,dz + \int \frac{1}{z}\,dz$. Since C is closed and 1 is entire, the Cauchy-Goursat theorem allows us to predict that $\int_C 1\,dz = 0$ and our result for 2.a) and 2.b) should be the same.