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180 minutes (3 hours) to complete. Closed book. No cooperation. No electronic communication. No calculators.

There are 54 points in this paper. To get a full mark, you need to score 48 points or more. Please write your answers on these question sheets in the space provided. If you run out of space, use extra paper.

Name:

Pledge:

Please write your name and pledge before turning the page.

A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	$\sum$

B1	B2	B3	B4	B5	B6	$\sum$

Part A. In this part, only provide answers (you can, but don't have to, provide any supporting work). Each question is worth **2 points**.

(A1) [2pt] Below are some complex numbers that lie on the circle $|z| = 1$. Sort them in *counterclockwise* order, starting with 1. (Only the answer is required.)

$$1, \qquad -\tfrac{1}{2} + \tfrac{\sqrt{3}}{2}i, \qquad ie^{2i}, \qquad -i, \qquad \exp(\tfrac{60}{7}\pi i).$$

(A2) [2pt] For each of the following points, write whether it is inside, on, or outside the circle of radius 2 centered at $1 + i$. (Only the answer is required.)

$$0, \qquad -1 - i, \qquad \pi i, \qquad 1 - i, \qquad e^{i\frac{\pi}{11}}.$$

(A3) [2pt] Which of the following equations have *infinitely* many complex solutions? (Only the answer is required.)

$$\cos z = 20 + 17i, \qquad e^z = 20 + 17i, \qquad \operatorname{Log} z = 20 + 17i, \qquad z^8 = 20 + 17i.$$

(A4) [2pt] Compute $\int_C \cos \frac{z}{2}\, dz$, where C is a contour that starts at $z_0 = 0$ and ends at $z_1 = 2017\pi$. (Only the answer is required.)

(A5) [2pt] Find the integral $\int_C \frac{f(z)}{(z-2)^4}dz$, where f is a given analytic function and C is the circle $|z-2|=1$. (Only the answer is required.)

(A6) [2pt] Compute $\displaystyle\int_0^{\pi} e^{t-it}dt.$ (Only the answer is required.)

(A7) [2pt] Find radius of convergence of the following series. (Only the answer is required.)

$$\frac{1}{10}+\frac{3i}{10}z^2+\frac{(3i)^2}{10}z^4+\frac{(3i)^3}{10}z^6+\ldots=\sum_{n=0}^{\infty}\frac{1}{10}\cdot(3i)^n z^{2n}.$$

(A8) [2pt] Find radius of convergence of the Taylor series for the function $\frac{1}{e^z-1}$ at the point $z_0=7+5i$.

(A9) [2pt] Find Laurent series for the function $\frac{1}{1+3z}$ in the annulus $|z| > 1/3$. (Only the answer is required.)

(A10) [2pt] For the following functions, determine type of singularity at $z = 0$ (removable, pole of order n, essential, or non-isolated). (Only the answer is required.)

$$\frac{\tan z}{z^3}, \qquad \cos\tfrac{1}{z}, \qquad \frac{1}{\sin z}.$$

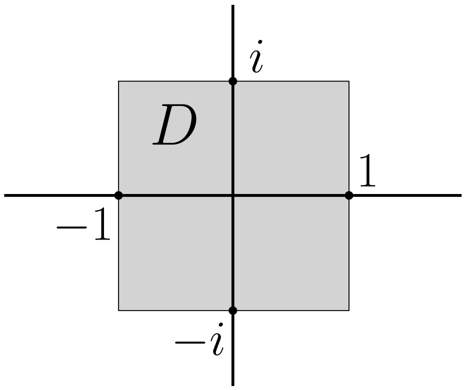
(A11) [2pt] Find $\mathrm{Res}[\frac{\cos z}{z^{2017}}, 0]$. (Only the answer is required.)

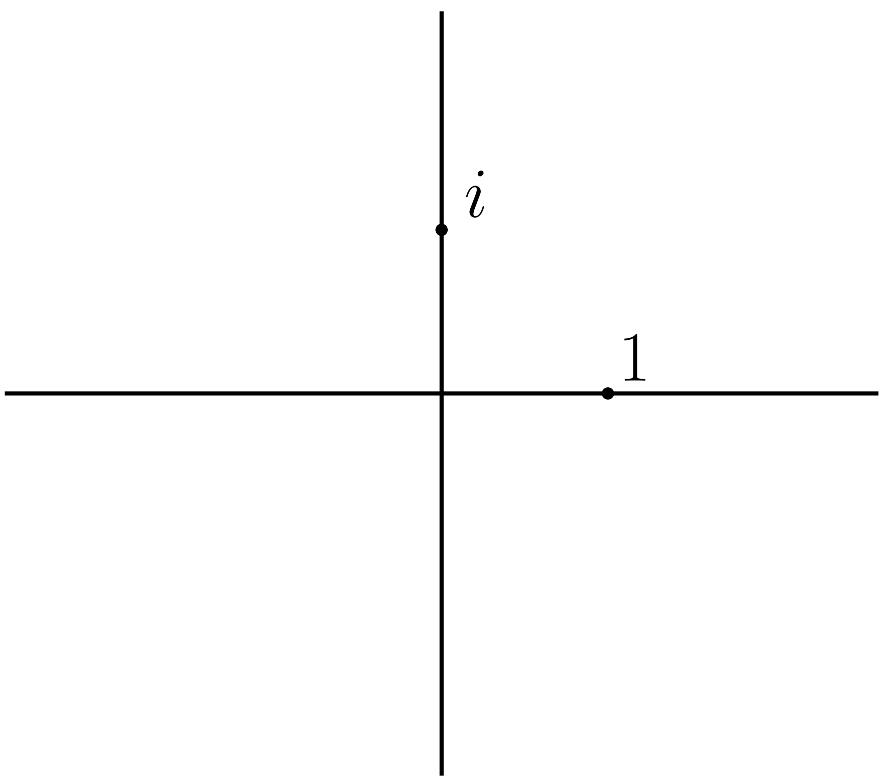
(A12) [2pt] How many solutions does the following equation have in the disk $|z| < 2$? (Only the answer is required.)

$$z^5 - 7iz^3 + 11z - i = 0.$$

Part B. In this part, show your work and provide explanations. Each question is worth **5 points**.

(B1) [5pt] Let D be the square region $\{z : -1 \le \operatorname{Re} z \le 1, -1 \le \operatorname{Im} z \le 1\}$. Find and sketch the image of D under $f(z) = 1/z$.





(B2) [5pt] Let $u(x, y) = xy + \sin x \sinh(Ay)$, where A is a real number. Find all values of A such that $u(x, y)$ is harmonic, and find the corresponding harmonic conjugates.

(B3) [5pt]

(a) Give Cauchy's integral formula for nth derivative. (Proof not required.)

(b) Use that formula for the first derivative and ML-inequality (or just Cauchy inequalities) to show that if a function f is bounded and entire, then it is constant.

(B4) [5pt]

(a) Find terms up to z^4 in the Maclaurin series of the functions $\frac{z}{1-2z}\sin(z^2)$ and $(1-\cos z)^2$.

(b) Use those Maclaurin series to find the limit

$$\lim_{z\to 0} \frac{\frac{z}{1-2z}\sin(z^2) - z^3}{(1-\cos z)^2}.$$

(c) How many times would you have to apply L'Hospital rule to get the same result? (You *are not asked* to actually do that.)

(B5) [5pt] Use residues to compute

$$\int_{C_R(1)} \frac{z}{\sin z} dz$$

for the following values of R: $R = 2$, $R = 3$, $R = 5$. (As always, $C_R(1)$ denotes a positively oriented circle of radius R centered at 1.)

(B6) [5pt] Use residues to compute

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+9)^2}.$$