Green's Theorem:

Let C be a positively oriented, piecewise-smooth, simple closed curve in the plane and let **"D be the region bounded by C "**. If P and Q have continuous partial derivatives on an open region that contains D, then

$$\oint_C P dx + Q dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

Corollary of Green's Theorem:

Let C be a positively oriented, piecewise-smooth, simple closed curve in the plane and let **"D be the region bounded by C "**. Then

$$A = (1) \oint_C x\,dy = (2) \oint_C -y\,dx = (3)\ \frac{1}{2} \oint_C x\,dy - y\,dx$$

Stokes' Theorem Let S be an oriented piecewise-smooth surface that is bounded by a simple, closed, piecewise-smooth boundary curve C with positive orientation. Let $\mathbf{F}$ be a vector field whose components have continuous partial derivatives on an open region in $\mathbb{R}^3$ that contains S.

Then

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \iint_S \operatorname{curl} \mathbf{F}\cdot d\mathbf{S} \left[= \iint_D \operatorname{curl} \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_u)\, dA \ (\mathbf{r} : \text{surface; D, surface parameter domain}) \right]$$

The Divergence Theorem Let E be a simple solid region and let S be the boundary surface of E, given with positive (outward) orientation. Let F be a vector field whose component functions have continuous partial derivatives on an open region that contains E.

Then

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \operatorname{div}\mathbf{F}\, dV$$