Homework 4:
I pledge my honor that I have abided by the Stevens honor system.
2.1: 10, 26, 30, 42; 2.2: 4, 8a, 12, 16abc, 20; 2.3: 10, 16, 20

2.1:
10. Determine whether these statements are true or false.

a) $\emptyset \in \{\emptyset\}$ True
b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$ True
c) $\{\emptyset\} \in \{\emptyset\}$ False
d) $\{\emptyset\} \in \{\{\emptyset\}\}$ True
e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$ True
f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$ False
g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$ True

26. Show that if $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$.

1. $A \subseteq C \equiv \forall x\,(x \in A \rightarrow x \in C)$ definition of $\subseteq$
2. $B \subseteq D \equiv \forall y\,(y \in B \rightarrow y \in D)$ definition of $\subseteq$
3. $(x, y) \in A \times B$ relation from A to B
4. $(x, y) \in C \times D$ relation from C to D
5. $A \times B \subseteq C \times D$

30. Suppose that $A \times B = \emptyset$, where A and B are sets. What can you conclude?
Either A, B, or both are empty sets ($\emptyset$).

42. Translate each quantification into English and determine the truth-value.

a) $\exists x \in \mathbf{R}\,(x^3 = -1)$: There exists a real number, x, such that $x^3 = -1$.
True ($x = -1$: $(-1)^3 = -1$)
b) $\exists x \in \mathbf{Z}\,(x + 1 > x)$: There exists an integer x such that $x + 1 > x$.
True ($x = 1$: $1 + 1 = 2 > 1$)
c) $\forall x \in \mathbf{Z}\,(x - 1 \in \mathbf{Z})$: For all integers x, $x - 1$ is also an integer.
True: Domain is $\mathbf{Z}$, so Range is $\mathbf{Z}$.
d) $\forall x \in \mathbf{Z}\,(x^2 \in \mathbf{Z})$: For all integers x, x^2 is also an integer.
True: Domain is $\mathbf{Z}$ $(-\infty, \infty)$, Range is $\mathbf{Z}$ $(0, \infty)$

2.2:
4. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find:

a. $A \cup B$: $\{x \mid x \in A \vee x \in B\} \Rightarrow \{a, b, c, d, e, f, g, h\}$
b. $A \cap B$: $\{x \mid x \in A \wedge x \in B\} \Rightarrow \{a, b, c, d, e\}$
c. $A - B$: $\{x \mid x \in A \wedge x \notin B\} \Rightarrow \{\ \}$
d. $B - A$: $\{x \mid x \in B \wedge x \notin A\} \Rightarrow \{f, g, h\}$

8. a. Prove $A \cup A = A$.
$\forall x\,(x \in (A \cup A) \leftrightarrow x \in A)$

1. $x \in A \cup A$ definition of $\cup$

2. $x \in \{y | y \in A \wedge (y \in A \vee y \in A)\}$ set builder notation
3. $x \in A \vee x \in A$ idempotence of $\vee$
4. $x \in A$

12. Prove $A \cup (A \cap B) = A$.
    $\forall x\ (x \in A \vee (x \in A \wedge x \in B) \leftrightarrow x \in A)$
    1. $x \in A \cup (A \cap B)$ definition of $\cup, \cap$
    2. $x \in \{z \mid z \in A \vee (z \in A \wedge z \in B)\}$ set builder
    3. $x \in A \vee (x \in A \wedge x \in B)$ $x \in A$ in both cases, therefore.
    4. $A \cup (A \cap B) \subseteq A$ switch directions…
    5. $y \in \{z \mid z \in A \vee (z \in A \wedge z \in B)\}$ set builder
    6. $y \in A \vee y \in (A \cap B)$ definition of $\cup, \cap$
    7. $y \in A \cup (A \cap B)$ $y \in A$ in both cases
    8. $A \subseteq A \cup (A \cap B)$ definition of absorption law
    9. $A \cup (A \cap B) = A$

16. Let A and B be sets. Show that:
    a. $(A \cap B) \subseteq A$
       $\forall x\ (x \in (A \cap B) \rightarrow x \in A)$
       1. $x \in (A \cap B)$ definition of $\cap$
       2. $x \in \{y \mid y \in A \wedge y \in B)$ set builder
       3. $x \in A \wedge x \in B$ A intersect B is a subset of A
       4. $(A \cap B) \subseteq A$
    b. $A \subseteq (A \cup B)$
       $\forall x\ (x \in A \rightarrow x \in (A \cup B))$
       1. $x \in A \vee x \in B$ definition of $\vee$
       2. $x \in A \cup B$ definition of $\cup$
       3. $x \in A \rightarrow x \in A \cup B$ if an element exists in a subset, then it exists in the set
       4. $x \in A \rightarrow x \in A \vee x \in B$ definition of $\vee$
       5. $A \subseteq (A \cup B)$ x is in A or B, A is a subset or equal to $(A \cup B)$
    c. $A - B \subseteq A$
       $\forall x\ ((x \in A \wedge x \notin B) \rightarrow x \in A)$
       1. $x \in A - B$ definition of difference
       2. $x \in \{y \mid y \in A \wedge y \notin B\}$ set builder
       3. $x \in A \wedge x \notin B$ if x isn't in B, then it's just in A
       4. $x \in A$ the difference of A and B is a subset of A
       5. $A - B \subseteq A$

20. Show that if A and B are sets with $A \subseteq B$, then:
    a. $A \cup B = B$
       If $A \subseteq B$, then $A \in B \wedge (x \in A \vee x \in B)$.
       Therefore, $x \in B$.
       Therefore, $A \cup B = B$
    b. $A \cap B = A$

If $A \subseteq B$, then $A \in B \wedge (x \in A \wedge x \in B)$
Therefore, $A = B \rightarrow A \cap B = A$

2.3:

10. Determine whether each one of these functions from {a, b, c, d} to itself is one-to-one.

a.	f(a) = b, f(b) = a, f(c) = c, f(d) = d	True
b.	f(a) = b, f(b) = b, f(c) = d, f(d) = c	False
c.	f(a) = d, f(b) = b, f(c) = c, f(d) = d	False

16. Consider these functions from the set of students in a discrete mathematics class. Under what conditions is the function one-to-one if it assigns to a student his or her:

a. Mobile phone number: every phone has its own phone number.
b. Student id number: the school gives each student a unique id number.
c. Final grade in the class: no two students get the same grade in the class.
d. Hometown: no two students are from the same town.

20. Give an example of a function form **N** to **N** that is:

a. One-to-one but not onto: f(x) = x + 1
b. Onto but not one-to-one: f(x) = x/2
c. Both onto and one-to-one (different from identity):

$$f(x) = \begin{cases} x+1 & \textit{when } x \textit{ is even} \\ x-1 & \textit{when } x \textit{ is odd} \end{cases}$$

d. Neither one-to-one nor onto: f(x) = c where c is a constant.