MA 222 A SPRING 2016

EXAM #3 : SOLUTIONS

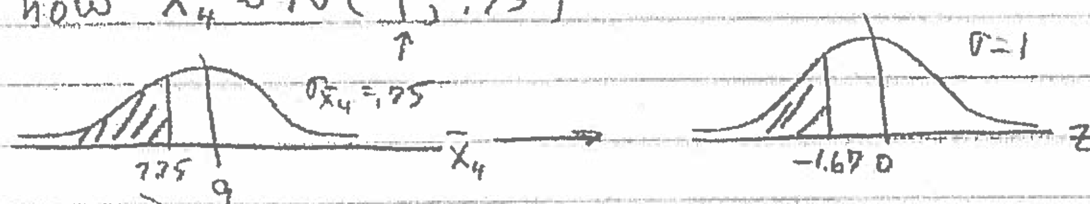
(1.) a) $X \sim N(\underset{\mu}{10}, \underset{\sigma_x}{1.5})$ $\therefore \bar{X}_4 \sim N(10, \frac{1.5}{\sqrt{4}}) = N(10, .75)$

Find $P(\bar{X}_4 < 7.75)$

$z = \frac{7.75 - 10}{.75} = -3$

$\therefore P(\bar{X}_4 < 7.75) \equiv P(Z < -3) = .5 - .4987 = .0013$

b) now $\bar{X}_4 \sim N(9, .75)$



$z = \frac{7.75 - 9}{.75} = -1.67$

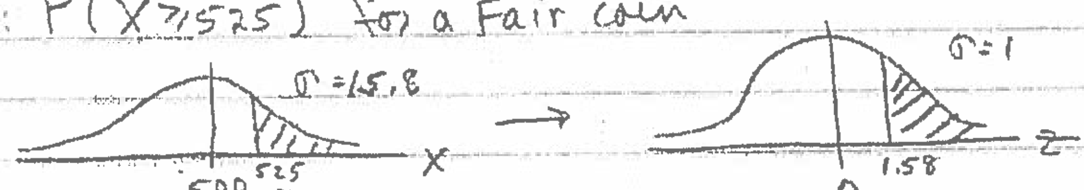
$\therefore P(\bar{X}_4 < 7.75) \equiv P(Z < -1.67) = .5 - .4525 = .0475$

(2.) X = #H's in 1000 tosses of a fair coin

$X \sim bin(\underset{n}{1000}, \underset{p}{.5})$ ... can approx: $X \sim N(np, \sqrt{npq})$

" " $X \sim N(500, 15.8)$

we seek: $P(X \geq 525)$ for a Fair coin



$z = \frac{525 - 500}{15.8} = 1.58$

$\therefore P(X \geq 525) \equiv P(Z \geq 1.58) = .5 - .4429 = .0571$

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(3.) One way to do this is just to make a list of all possible outcomes with their probabilities

W	win \$1	with prob. $\frac{18}{38}$	$= .473$
LW	win \$0	" " $\frac{20}{38} \cdot \frac{18}{38}$	$= .24$
LL	lose \$2	" " $\frac{20}{38} \cdot \frac{20}{38}$	$= .277$

so: $E(\text{winnings}) = (1)(.473) + 0(.24) + (-2)(.277) = -\$.08$

(4.) Marginal dist. for X is: $P(X=0) = \frac{1}{6}$ $P(X=1) = \frac{1}{4}$ $P(X=2) = \frac{7}{12}$

Find $V(X)$, using $V(X) = E(X^2) - [E(X)]^2$

where $E(X) = 0(\frac{1}{6}) + 1(\frac{1}{4}) + 2(\frac{7}{12}) = \frac{17}{12}$

and $E(X^2) = 0^2(\frac{1}{6}) + 1^2(\frac{1}{4}) + 2^2(\frac{7}{12}) = \frac{31}{12}$

$\therefore V(X) = E(X^2) - [E(X)]^2 = \frac{31}{12} - (\frac{17}{12})^2 = 2.58 - 2 = .58$

(5.) Let Y = the number of names selected by both Person A and Person B

Define:

$$X_i = \begin{cases} 1 & \text{if the } i^{th} \text{ name is selected by both people} \\ 0 & \text{" " " " NOT " " " "} \end{cases}$$

where: $P(X_i = 1) = P(i^{th}\text{ name is selected by Person A AND selected by Person B})$

where $P(i^{th}\text{ name selected by Person A}) = 1 - P(\text{NOT selected by person A})$

$$= 1 - \left(\tfrac{9}{10}\right)\left(\tfrac{8}{9}\right)\left(\tfrac{7}{8}\right) = .3$$

and $\therefore$ P(selected by Person B) is ALSO .3

$$\therefore P(X_i = 1) = (.3)(.3) = .09$$

$$\therefore E(X_i) = 1(.09) + 0(.91) = .09$$

and since

$$Y = X_1 + X_2 + \dots + X_{10}$$

$$E(Y) = E(X_1 + X_2 + \dots + X_{10})$$

$$= E(X_1) + E(X_2) + \dots + E(X_{10})$$

$$= 10(.09)$$

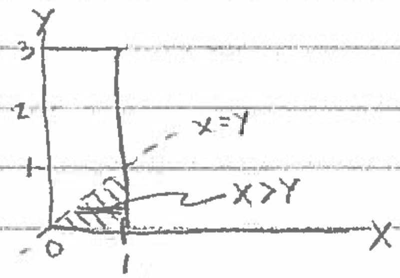
$= .9$ names on average will be selected by both Person A and Person B

EXTRA CREDIT PROBLEM

$X \sim U(0,1) \Rightarrow f_X(x) = 1 \quad 0 \le x \le 1$

$Y \sim U(0,3) \Rightarrow g_Y(y) = \frac{1}{3} \quad 0 \le y \le 3$

FIND $P(X>Y)$



JOINT DIST. OF X and Y is $h_{X,Y}(x,y) = f_X(x) \cdot g_Y(y)$

since X and Y are independent

so $h_{X,Y}(x,y) = 1 \cdot \frac{1}{3} = \frac{1}{3}$ for $0 \le x \le 1$ and $0 \le y \le 3$

and $P(X>Y) = \int_0^1 \int_y^1 \frac{1}{3}\, dx\, dy = \int_0^1 \left[\frac{x}{3} \Big|_y^1 \right] dy$

$= \int_0^1 \left(\frac{1}{3} - \frac{y}{3} \right) dy = \left(\frac{y}{3} - \frac{y^2}{6} \right)\Big|_0^1 = \frac{1}{6}$

NOTE: in this case, since $h_{X,Y}(x,y)$ is a flat plane of height $= \frac{1}{3}$ above the x-y axis, you can also compute the volume under the plane by multiplying the area of the triangle times the height:

$\left[\frac{1}{2}(1)(1) \right] \cdot \frac{1}{3} = \frac{1}{6}$