EXAM 2 - SOLUTIONS

1. P(takes at least 4 shots to first bulls-eye) = P(misses the first 3 shots)
$= (.4)^3 = .064$

2. a) $(.6)^2(.4)^6 = .00147$ b) $\binom{8}{2}(.6)^2(.4)^6 = .0413$

3. $f(x) = \frac{1}{10}$ $0 \le x \le 10$

$$E(x) = \int x f(x)dx = \int_0^{10} x\left(\tfrac{1}{10}\right)dx = \frac{x^2}{20}\Big|_0^{10} = 5$$

$$E(x^2) = \int x^2 f(x)dx = \int_0^{10} x^2\left(\tfrac{1}{10}\right)dx = \frac{x^3}{30}\Big|_0^{10} = \frac{1000}{30} = 33.33$$

$$\sigma_x = \sqrt{E(x^2)-[E(x)]^2} = \sqrt{33.33-5^2} = \sqrt{8.33} = 2.886$$

$\therefore P(5-2.886 < X < 5+2.886) = P(2.11 < X < 7.89) = .578$

4. X = # of accidents ~ Poisson ($\lambda = 4 \frac{\text{accidents}}{100 \text{ hours}}$)

a) using a λ in "# of accidents per hour" gives $\lambda = .04$ accidents/hour

METHOD 1 P(at least 1 accident in next 50 hours) = 1 − P(NO accidents in next 50 hours)

$$= 1 - \frac{\left[e^{-(.04)50}\right]\left[(.04)(50)\right]^0}{0!} = 1 - e^{-2} = 1 - .135 = .865$$

METHOD 2 OR: can use exponential: T = time to next accident

$T \sim \exp(\lambda = .04)$

P(at least 1 accident in next 50 hours) = $P(T < 50)$

$$= \int_0^{50} .04e^{-.04t}dt = -e^{-.04t}\Big|_0^{50} = .865$$

b) $P(T < 20) = \int_0^{20} .04e^{-.04t}dt = -e^{-.04(t)}\Big|_0^{20} = 1 - e^{-.8} = .55$

T = bulb lifetime $\sim \exp(\lambda)$

5. average lifetime $= 500$ days $= \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{500} = .002$

P(any single bulb is still operating after 100 days)

$= P(T > 100) = \int_{100}^{\infty} .002e^{-.002t}\,dt = -e^{-.002t}\Big|_{100}^{\infty}$

$= .818$

P(at least 9 are still operating after 100 days)

$= P(9 \text{ or } 10 \text{ are operating}) = \binom{10}{9}(.818)^9(.182)^1 + \binom{10}{10}(.818)^{10}(.182)^0$

$= .298 + .134 = .432$

so: $E(\$\text{pay}) = 1000(.432) + 300(1 - .432) =$ \$602.40

6. P(a "6" comes up for the 3RD time on exactly the 13TH toss)

$=$ P(2 6's in 12 tosses) P(a "6" on the 13th toss)

$= \binom{12}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{10} \cdot \frac{1}{6} = \binom{12}{2}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{10}$

$= (66)(.0046)(.1615)$

$= .049$