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Homework 11

4.1: 3, 4, 6 (w/ proofs), 20, 22, 24, 28, 4.2: 2a, 4a, 6a, 4.3: 4, 14, 16a, 28

3. Prove part (ii) of Theorem 1 is true.
if $a|b$, then $a|bc$ for all integers c.

1. $a \mid b$	assumption
2. $b = ad$	def $\mid$ from (1)
3. $bc = adc$	arith. from (2)
4. $bc = ad'$	arith. from (3)
5. $a \mid bc$	def $\mid$ from (4)

4. Prove part (iii) of Theorem 1 is true.
if $a|b$ and $b|c$, then $a|c$

1. $a \mid b$	assumption
2. $b \mid c$	assumption
3. $b = ak$	def $\mid$ from (1)
4. $c = bk'$	def $\mid$ from (2)
5. $c = (a \cdot k)k'$	arith. (3), (4)
6. $c = a(k \cdot k')$	associative multiplication (5)
7. $a \mid c$	def $\mid$ from (6)

6. Show that if a, b, c, d are ints, where $a \neq 0$, such that $a|c$ and $b|d$, then $ab|cd$

1. $a \mid c$	assumption
2. $b \mid d$	assumption
3. $c = ak$	def $\mid$ from (1)
4. $d = bk'$	def $\mid$ from (2)
5. $cd = (ak)(bk')$	arith (3), (4)
6. $cd = (ab)(kk')$	associative multiplication (5)
7. $ab \mid cd$	def $\mid$ from (6)

20. Evaluate these quantities:

a) $-17 \bmod 2 = 1 \rightarrow -17 = -9 \cdot 2 + 1$

b) $144 \bmod 7 = 4 \rightarrow 144 = 20 \cdot 7 + 4$

c) $-101 \bmod 13 = 3 \rightarrow -101 = -8 \cdot 13 + 3$

d) $199 \bmod 19 = 9 \rightarrow 199 = 10 \cdot 19 + 9$

22. Find $a \operatorname{div} m$ and $a \bmod m$ when

a) $a = -111$, $m = 99$

$-111 \operatorname{div} 99 = -2 \rightarrow 111/99$ is between 1 and 2

$-111 \bmod 99 = 87 \rightarrow -111 = -2 \cdot 99 + 87$

b) $a = -9999$, $m = 101$

$-9999 \operatorname{div} 101 = -99 \rightarrow 9999/101 = 99$

$-9999 \bmod 101 = 0 \rightarrow -9999 = 99 \cdot 101 + 0$

c) $a = 10299$, $m = 999$

$10299 \operatorname{div} 999 = 10 \rightarrow 10299/999$ is between 10 and 11

$10299 \bmod 999 = 309 \rightarrow 10299 = 10 \cdot 999 + 309$

d) $a = 123456$, $m = 1001$

$123456 \operatorname{div} 1001 = 123 \rightarrow 123456/1001$ is between 123 and 124

$123456 \bmod 1001 = 333 \rightarrow 123456 = 123 \cdot 1001 + 333$

24. Find the integer a such that

a) $a \equiv 43 \pmod{23}$ and $-22 \leq a \leq 0$

$a = -3 \qquad 23 \mid (43 - (a)) \rightarrow 23 \mid 46$

b) $a \equiv 17 \pmod{29}$ and $-14 \leq a \leq 14$

$a = -12 \qquad 29 \mid (17 - (-12)) \rightarrow 29 \mid 29$

c) $a \equiv -11 \pmod{21}$ and $90 \leq a \leq 110$

$a = 94 \qquad 21 \mid (94 - (-11)) \rightarrow 21 \mid 105$

28. Decide whether each is equivalent to 3 modulo 7. $\rightarrow 7 \mid a - 3$

a) 37 F $\quad 7 \mid (37 - 3) = 7 \mid 34$

b) 66 T $\quad 7 \mid (66 - 3) = 7 \mid 63$

c) 17 F $\quad 7 \mid (3 + (-17)) = 7 \mid 20$

d) -67 T $\quad 7 \mid (3 + (+67)) = 7 \mid 70$

4.2: 2a. Convert the decimal expansion of each of these integers to a binary expansion.

a) 321

101000001_2

2 | 321	1
2 | 160	0
2 | 80	0
2 | 40	0
2 | 20	0
2 | 10	0
2 | 5	0
2 | 2	1
2 | 1	0
0	1

4a. Convert binary expansion to decimal.

a) $(11011)_2$

1 1 0 1 1 $\quad 1+2+8+16 = 27_{10}$

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$

6a. Convert binary expansion to octal

a) $(11110111)_2$

011 110 111

367_8

4.3 4. Find the Prime Factorization of each

a) $39 = 3 * 13$

b) $81 = 3 * 3 * 3 * 3$

c) $101 = 1 * 101$

d) $143 = 11 * 13$

e) $289 = 17 * 17$

f) $899 = 29 * 31$

14. Which positive integers less than 12 are relatively prime to 12?

1, 5, 7, 11

prime Factors of 12: 2, 3

what numbers don't have them as factors?

16a. Are they pairwise relatively prime?

a) 21, 34, 55 Yes

(21, 34)

(21, 55) 21: 3, 7

(34, 55) 34: 2, 17

55: 5, 11

28. Find $\gcd(1000, 625)$ and $\operatorname{lcm}(1000, 625)$ and verify that $\gcd(1000, 625) \cdot \operatorname{lcm}(1000, 625) = 1000 \cdot 625$

$1000 = 2^3 \cdot 5^3$, $625 = 5^4$

$\gcd(1000, 625) = 5^3 = 125$

$\operatorname{lcm}(1000, 625) = 2^3 \cdot 5^4 = 5000$

$1000 \cdot 625 = 625{,}000$

$125 \cdot 5000 = 625{,}000$ ✓