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§13.8 Divergence Theorem: $\iint_S \vec{F}\cdot d\vec{s} = \iiint_E (\vec{\nabla}\cdot\vec{F})\,dV$

① $\vec{F} = \langle x^2, -y, z\rangle$; E is solid cylinder $y^2+z^2 \le 9$ capped at $x=0$ & $x=2$.
Evaluate both sides of the Divergence Thm and verify that the results are the same.

② $\vec{F}(x,y,z) = \langle x^2yz, xy^2z, xyz^2\rangle$
S is the surface of the box enclosed by the planes, $x=0$, $x=a$, $y=0$, $y=b$, $z=0$, $z=c$. $(a,b,c>0)$.
Evaluate both sides of the Divergence Thm

13.7 Stokes Thm $\iint_S (\vec{\nabla}\times\vec{F})\cdot d\vec{S} = \int_C \vec{F}\cdot d\vec{r}$

③ Use Stokes Thm to evaluate $\int_C \vec{F}\cdot d\vec{r}$.
C is oriented counterclockwise as viewed from above.
$\vec{F} = \langle yz, 2xz, e^{xy}\rangle$, C is the circle $x^2+y^2=16$, $z=5$.

④ $F(x,y,z) = \langle y, z, x\rangle$.
S is the hemisphere $x^2+y^2+z^2=1$, $y\ge 0$
oriented in the direction of positive y axis.
Verify Stokes Thm by calculating both sides of equation

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(5) Evaluate $\iint_S (z + x^2 y)\, dS$

S is portion of cylinder $y^2 + z^2 = 1$ between $x = 0$ & $x = 3$ and in the 1st octant.

(6) Force Field $\vec{F}(x,y) = \langle x, x^3 + 3xy^2 \rangle$.
Particle travels from $(-2,0)$ to $(2,0)$ along x-axis & then ccw along the semicircle $y = \sqrt{4 - x^2}$ until reaching $(-2,0)$. Find the work done on this particle. Which theorem is appropriate here.

(7) Consider the region D enclosed by $y = a^2 - x^2$ $(a > 0)$ and the x-axis $(y = 0)$. Use a line integral to calculate the area of D.

(8) Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $a, b > 0$

Use an appropriate line integral to calculate the area of the ellipse.

(9) $\vec{F}(x,y) = \langle xy^2, x^2 y \rangle$
C: $\vec{r}(t) = \langle t + \sin(\frac{\pi}{2}t), t + \cos(\frac{\pi}{2}t) \rangle$, $0 \le t \le 1$
Use Fundamental Theorem for Line Integrals to evaluate $\int_C \vec{F} \cdot d\vec{r}$.