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MA 234
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Homework 4

1. Knowing $f(x+iy) = u(x,y) + iv(x,y)$ is entire, $f(0)=0$ & $u(x,y) = 2x^2 - 2y^2 + x$, find $v(x,y)$

$u_x = 4x+1 = v_y \rightarrow v = \int 4x+1 \, dy + C(x) = 4xy + y + C(x)$

$u_y = -4y = -v_x \rightarrow v = \int 4y \, dx + C(y) = 4yx + C(y)$

$C(x)$ drops out,

$\boxed{v(x,y) = 4xy + y}$

2. Knowing $f(x+iy) = u(r,\theta) + iv(r,\theta)$ is entire, $f(0)=0$, & $u(r,\theta) = r\sin\theta$, find $v(r,\theta)$

$u_r = \sin\theta = \frac{1}{r} v_\theta \rightarrow v = \int r\sin\theta \, d\theta = -r\cos\theta + C(r)$

$u_\theta = r\cos\theta = -r v_r \rightarrow v = \int -\cos\theta \, dr = -r\cos\theta + C\theta$

$-r\cos\theta + C(r) = -r\cos\theta + C(\theta)$

$C(r)$ & $C(\theta)$ drop out

$\boxed{v(r,\theta) = -r\cos\theta}$

3. Check if following functions are harmonic and whether they could be the real & imaginary parts of an analytic function:

a. $u(x,y) = xy$, $v(x,y) = x^2+y^2$

$u_{xx} + u_{yy} = 0$ ✓ $\quad v_{xx} + v_{yy} = 0$ ✗

$u_x = y \quad u_y = x \quad v_x = 2x \quad v_y = 2y$

$u_{xx} = 0 \quad u_{yy} = 0 \quad v_{xx} = 2 \quad v_{yy} = 2$

u is harmonic
v is NOt

$u_{xx} = 0 = v_{yx} \quad u_{yx} = 0 \neq -2 = -v_{xx}$

$u_{xy} = 0 \neq 2 = v_{yy} \quad u_{yy} = 0 = -v_{xy}$

can't be the real & imaginary parts of an analytic function

b. $u(x,y) = xy$, $v(x,y) = x^2 - y^2$

$u_x = y \quad u_y = x \quad v_x = 2x \quad v_y = -2y$

$u_{xx} = 0 \quad u_{yy} = 0 \quad v_{xx} = 2 \quad v_{yy} = -2$

$u_{xx} + u_{yy} = 0 + 0 = 0$ ✓ $\quad v_{xx} + v_{yy} = 2 - 2 = 0$ ✓

u & v are harmonic

$u_{xx} = 0 = v_{yx} \quad u_{yx} = 0 \neq -2 = -v_{xx}$

$u_{xy} = 0 \neq -2 = v_{yy} \quad u_{yy} = 0 = -v_{xy}$

can't be real & imaginary parts of an analytic function

4. Let $f(x,y)$ & $g(x,y)$ be real-valued harmonic functions.

a. Show $f+g$ will be harmonic

$f_{xx} + f_{yy} = 0$, $g_{xx} + g_{yy} = 0$

$(f+g)' = f' + g'$

$(f'+g')' = f'' + g''$

$(f+g)_{xx} + (f+g)_{yy} = 0$

$(f_{xx} + g_{xx}) + (f_{yy} + g_{yy}) = f_{xx} + f_{yy} + g_{xx} + g_{yy} = 0$

$\boxed{f+g \text{ is harmonic}}$

b. Show fg will not necessarily be harmonic

$f_{xx} + f_{yy} = 0$ $g_{xx} + g_{yy} = 0$

$(fg)' = f'g + fg'$

$(f'g + fg')' = f''g + f'g' + f'g' + fg''$

$(fg)_{xx} + (fg)_{yy} = f_{xx}g + 2f_xg_x + fg_{xx} + f_{yy}g + 2f_yg_y + fg_{yy}$

$= g(f_{xx} + f_{yy}) + 2(f_xg_x + f_yg_y) + f(g_{xx} + g_{yy})$

$= 2(f_xg_x + f_yg_y)$

$\boxed{\text{Not necessarily zero.} - \text{Not necessarily harmonic}}$

I pledge my honor that I have abided by the Stevens honor System.

Kaitlyn [signature]