Homework #3: 1.5: 4, 8(a-d), 12(a-f), 1.6: 4, 8 24, 1.7: 10

I pledge my honor that I have abided by the Stevens honor system.

1.5:
4.) Let P(x, y) be the statement “Student x has taken class y,” where x consists of students in your class and y consists of all computer science classes.
a. $\exists x \exists y\ P(x, y)$:
- There is a student in your class who has taken a computer science class.

b. $\exists x \forall y\ P(x, y)$:
- There is a student in your class who has taken all computer science classes.

c. $\forall x \exists y\ P(x, y)$:
- Every student in your class has taken a computer science class.

d. $\exists y \forall x\ P(x, y)$:
- Every student in your class had to take the same computer science class.

e. $\forall y \exists x\ P(x, y)$:
- For all computer science classes, a student in your class has taken it.

f. $\forall x \forall y\ P(x, y)$:
- Every student in your class has taken every computer science class.

8.) Let Q(x, y) be the statement “student x has been a contestant on quiz show y.”
a. There is a student at your school who has been a contestant on a quiz show:
- $\exists x \exists y\ Q(x, y)$

b. No student at your school has ever been a contestant on a quiz show:
- $\neg \exists x \exists y\ Q(x, y)$

c. There is a student at your school who has been a contestant on Jeopardy and Wheel of Fortune.
- $\exists x\ (Q(x, \text{Jeopardy}) \wedge Q(x, \text{Wheel of Fortune}))$

d. Every quiz show has had a student from your school as a contestant.
- $\forall y \exists x\ Q(x, y)$

12. Let I(x) be the statement “x has an internet connection” and C(x, y) be the statement “x and y have chatted over the Internet,” where the domain for x and y consists of students in your class.
a. Jerry does not have an Internet connection.
- $\neg I(\text{Jerry})$

b. Rachel has not chatted with Chelsea.
- $\neg C(\text{Rachel}, \text{Chelsea})$

c. Jan and Sharon have never chatted online.
- $\neg C(\text{Jan}, \text{Sharon})$

d. No one in the class has chatted with Bob.
- $\neg \exists x\ C(x, \text{Bob})$

e. Sanjay has chatted with everyone except Joseph.
- $\forall y$: $y \neq \text{Joseph}$, $C(\text{Sanjay}, y) \wedge \neg C(\text{Sanjay}, \text{Joseph})$

f. Someone in your class does not have an Internet connection.
- $\exists x\ I(x)$

1.6:

4. What rule of inference was used in each?

a. Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

- Simplification: $p \wedge q \rightarrow q$

b. It is either hotter that 100° today or the pollution is dangerous. It is less than 100° outside today. Therefore, the pollution is dangerous.

- Disjunctive Syllogism: $((p \vee q) \wedge \neg p) \rightarrow q$

c. Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

- Modus Ponens: $(p \wedge (p \rightarrow q)) \rightarrow q$

d. Steve will work at a computer company this summer. Therefore, this summer, Steve will work at a computer company or he will be a beach bum.

- Addition: $p \rightarrow (p \vee q)$

e. If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

- Hypothetical Syllogism: $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

8. What rules of inference are used in this argument? "No man is an island. Manhattan is an island. Therefore, Manhattan is not a man."

- Modus Tollens

24. Identify the error or errors in this argument that supposedly shows that if $\forall x\,(P(x) \vee Q(x))$ is true then $\forall x\,P(x) \vee \forall x\,Q(x)$ is true.

| | |
|---|---|
| 1. $\forall x\,(P(x) \vee Q(x))$ | Premise |
| 2. $P(c) \vee Q(c)$ | Universal instantiation from (1) |
| 3. $P(c)$ | Simplification from (2) |
| 4. $\forall x\,P(x)$ | Universal generalization from (3) |
| 5. $Q(c)$ | Simplification from (2) |
| 6. $\forall x\,Q(x)$ | Universal generalization from (5) |
| 7. $\forall x\,(P(x) \vee \forall x\,Q(x))$ | Conjunction from (4) and (6) |

- Step 3 and 5 → not simplification, its addition

1.7:

10. Use a direct proof to show that the product of two rational numbers is rational.

| **STEP** | **REASON** |
|---|---|
| 1. r and s are rational numbers | Premise |
| 2. $\exists (a, b, c, d) \in Z$ | Definition of rational numbers |
| 3. $r = a/b,\ b \neq 0$ | Substitute a/b for r |
| 4. $s = c/d,\ d \neq 0$ | Substitute c/d for s |
| 5. $r*s = (a*c)/(b*d)$ | Multiply (3) and (4) |
| 6. $x = r*s$ | Assign x to r*s |
| 7. x is rational | $b*d \neq 0$, and $a*c \in Z$, therefore, rational |