Homework 1 Mathematical Prerequisites
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I pledge my honor that I have abided by the Stevens Honor System.

Sets:

1: f(x) = 0 ⟶ {0, 10, 20, 30, …}

2: a) A ∪ B ⟶ {0, 2, 4, 6, 8, 10, 12, …}
b) A ∩ B ⟶ {0, 10, 20, 30, …}
c) A \ B ⟶ {Ø}

Functions:

1. $f(x) = \ x + 2$
   - i. $\mathbb{N} \rightarrow \mathbb{N}$ Injective, 0 not mapped to.
   - ii. $\mathbb{N} \rightarrow \mathbb{Z}_5$ Surjective, x=0, x=5 map to 2.
   - iii. $\mathbb{N} \rightarrow \mathbb{Z}_{10}$ Surjective, x=0, x=10 map to 2.
   - iv. $\mathbb{Z} \rightarrow \mathbb{Z}$ bijective
2. $f(x) = 2x$
   - i. $\mathbb{N} \rightarrow \mathbb{N}$ Injective, 1 not mapped to.
   - ii. $\mathbb{N} \rightarrow \mathbb{Z}_5$ Surjective, x=0, x=5 map to 0.
   - iii. $\mathbb{N} \rightarrow \mathbb{Z}_{10}$ Surjective, x=0, x=5 map to 0.
   - iv. $\mathbb{Z} \rightarrow \mathbb{Z}$ Injective, 1 not mapped to.
3. $f(x) = 3x$
   - i. $\mathbb{N} \rightarrow \mathbb{N}$ Injective, 1 not mapped to.
   - ii. $\mathbb{N} \rightarrow \mathbb{Z}_5$ Surjective, x=0, x=5 map to 0.
   - iii. $\mathbb{N} \rightarrow \mathbb{Z}_{10}$ Surjective, x=0, x=10 map to 0.
   - iv. $\mathbb{Z} \rightarrow \mathbb{Z}$ Injective, 1 not mapped to.

Boolean Logic

1: A↔(B∧C)

A	**B**	**C**	**(B∧C)**	**A↔(B∧C)**
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	F
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

2. $(A \vee B) \rightarrow C$

A	**B**	**C**	**$(A \vee B)$**	**$(A \vee B) \rightarrow C$**
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	F	T

3. $A \rightarrow (B \oplus C)$

A	**B**	**C**	**$(B \oplus C)$**	**$A \rightarrow (B \oplus C)$**
T	T	T	F	F
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	F	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

Strings and Languages:

1. $\sum$ = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, -, *, /, (,) }
2. "2+34/" ; "((01-*/()" ; "03/0"
3. a. No number's first digit is 0.
   b. Does not end in an operator.
   c. All open parentheses have a closing parenthesis.
   d. No set of parentheses is empty.
   e. No double operators.
   f. No division by 0.

Proofs:

1. Proof by Contrapositive:
   a) Use truth tables to prove $(A \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg A)$

A	B	$\neg A$	$\neg B$	$A \rightarrow B$	$\neg B \rightarrow \neg A$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	T	T

b) State contrapositive of the following: "If $n^2 \in \mathbb{N}$ is odd, then n is also odd."

If n is not odd, then $n^2 \in \mathbb{N}$ is not odd.

c) Use arithmetic to prove the contrapositive statement.

$n^2 = n * n$

$n * n$ is not odd if n is not odd

Therefore, if n is not odd, then n^2 is also not odd.

2. Proof by Induction: Given $n \in \mathbb{N}$, prove that $2^n \geq n^2$, for all $n \geq 4$.

Base Case:

$n = 4$

$2^4 = 16 = 4^2$

Inductive Hypothesis:

$2^n \geq n^2$

Inductive Case:

$2^{n+1} = 2 * 2^n$

$2 * 2^n \geq 2n^2$ Ind. Hyp.

$2n^2 = n^2 + n^2$ arith.

$n^2 + n^2 \geq n^2 + 2n + 1$ arith.

$n^2 + 2n + 1 \geq (n+1)^2$

Therefore:

$2^{n+1} \geq (n+1)^2$