MA222 A EXAM #2 : SOLUTIONS SPRING 2016

(1.) $E(X) = \int_0^2 \frac{x}{2}\, x\, dx = 4/3$

$P(X > 4/3) = \int_{4/3}^2 \frac{x}{2}\, dx = \frac{5}{9}$

(2.) With 4 accidents per 100 hours, we can use $\lambda = .04 \frac{\text{accidents}}{\text{hour}}$ (or could use $\lambda = 4 \frac{\text{accidents}}{\text{100 hours}}$, and use a different t)

a) P(at least one accident in the next 50 hours)

= 1 − P(no accidents in the next 50 hours)

$= 1 - \frac{e^{-(.04)(50)}[(.04)(50)]^0}{0!} = 1 - e^{-2} = 1 - .135 = .865$

b) P(time to 1st accident is less than 20 hours)

= P(at least 1 accident in 20 hours)

= 1 − P(no accidents in 20 hours)

$= 1 - \frac{e^{-(.04)(20)}[(.04)(20)]^0}{0!} = 1 - e^{-.8} = 1 - .45 = .55$

OR: use exponential: T = time to 1st accident

$P(T < 20) = \int_0^{20} .04e^{-.04t}\, dt = -e^{-.04t}\Big|_{t=0}^{t=20} = 1 - e^{-.8} = .55$

(3.) Let X = the number of patrons showing up (out of 20 reservations)

$X \sim \text{bin}(n=20, p=.9)$

$P(X > 18) = P(X=19) + P(X=20)$

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$= \binom{20}{19}(.9)^{19}(.1)^1 + \binom{20}{20}(.9)^{20}(.1)^0$

$= .27 + .12 = .39$

(4.) P(failure in year 1) = .4 = .4

P(" " " " 2) = (.6)(.4) = .24

P(" " " " 3) = $(.6^2)(.4)$ = .144

P(" " " " 4) = $(.6^3)(.4)$ = .0864

E(payout) = .4(4000) + .24(3000) + .144(2000) + (.0864)(1000) = $\boxed{\$2694}$

(5.) $X \sim geo(p)$ since $E(X) = \frac{1}{p} = 12.5$, we have $p = .08$

P(6TH person is first one with high BP)

= P(first 5 do NOT have high BP AND 6TH person DOES)

= $(.92)^5(.08)$ = .053