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Assignment 7 - Solutions

1. a) $\log(-1) = \log(e^{i\pi}) = \{\ln(1)+i(\pi+2n\pi)|n \in \mathbb{Z}\} = \{(2n+1)\pi i|n \in \mathbb{Z}\}$

   b) $(-1)^{x+iy} = e^{\log(-1)(x+iy)} = \{e^{(2n+1)\pi i(x+iy)}|n \in \mathbb{Z}\}$
   $= \{e^{-(2n+1)\pi y}e^{i(2n+1)\pi x}|n \in \mathbb{Z}\}$
   $= \{e^{-(2n+1)\pi y}(\cos((2n+1)\pi x) + i\sin((2n+1)\pi x)|n \in \mathbb{Z}\}$

   c) $\log(i) = \log(e^{\frac{i\pi}{2}}) = \{\ln(1) + i(\frac{\pi}{2} + 2n\pi)|n \in \mathbb{Z}\} = \{(2n + \frac{1}{2})\pi i|n \in \mathbb{Z}\}$

   d) $(i)^{x+iy} = e^{\log(i)(x+iy)} = \{e^{(2n+\frac{1}{2})\pi i(x+iy)}|n \in \mathbb{Z}\}$
   $= \{e^{-(2n+\frac{1}{2})\pi y}e^{i(2n+\frac{1}{2})\pi x}|n \in \mathbb{Z}\}$
   $= \{e^{-(2n+\frac{1}{2})\pi y}(\cos((2n + \frac{1}{2})\pi x) + i\sin((2n + \frac{1}{2})\pi x)|n \in \mathbb{Z}\}$

2. a) $z^{a+b} = e^{\log(z)(a+b)} = e^{a\log(z)+b\log(z)} = e^{a\log(z)}e^{b\log(z)} = z^a z^b$

   b) $z^{a-b} = e^{\log(z)(a-b)} = e^{a\log(z)-b\log(z)} = \frac{e^{a\log(z)}}{e^{b\log(z)}} = \frac{z^a}{z^b}$

Remark: As both properties had previously been proven for the function exp, we were allowed to use them in the proofs above.