Part 2:
Homework 6: 5.1: 4, 20

4. Let $P(n)$ be the statement that $1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ for the positive integer n.

a) What is the statement $P(1)$?
   $P(1)$ is the base case.
b) Show that $P(1)$ is true, completing the basis step of the proof.
   $1^3 = 1, (1 * (1 + 1)/2)^2 = (2 / 2)^2 = 1$
c) What is the inductive hypothesis?
   $$\left(\frac{(n+1)((n+1)+1)}{2}\right)^2$$
d) What do you need to prove the inductive step?
   $P(n) \rightarrow P(n+1)$
e) Complete the inductive step, identifying where you use the inductive hypothesis.

$(1^3 + 2^3 + \cdots + n^3) + (n+1)^3 = \left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3$ Given

$= \frac{(n^2+n)^2}{4} + (n+1)^3$ Algebra

$= \frac{n^4+2n^3+n^2}{4} + n^3 + 3n^2 + 3n + 1$ Algebra

$= n^4 + 2n^3 + n^2 + 4n^3 + 12n^2 + 12n + 4$
Simplification

$= n^4 + 6n^3 + 13n^2 + 12n + 4$ Simplification

$= \frac{(n+1)^2(n+2)^2}{4}$ Inductive
Hypothesis

f) Explain why these steps show that this formula is true whenever n is a positive integer.
   These steps show that this formula is true when n is a positive integer because you implement the base case and the inductive case, proving
   $$\frac{P(0) \qquad \forall n \in \mathbb{Z}^+ \; P(n) \rightarrow P(n+1)}{\forall n \in \mathbb{N}\,(P(n))}$$
   where $P(n)$ is true for all n.

20. Prove that $3^n < n!$ if n is an integer greater than 6.
Base Case: n = 7: $3^7 = 2187 < 7! = 5040$
Inductive hypothesis: $3^n < n!$ for $n \geq 7$
Inductive step: $3^{n+1} < (n+1)!$

1.	$3^{n+1} = 3 * (3^n)$	Simplification
2.	$(n+1)! = (n+1) * n!$	Simplification
3.	$(n+1) * 3^n < (n+1) * n!$	By Induction Hyp.
4.	$(n+1) * 3^n < (n+1)!$	By step 2
5.	$3 < (n+1)$	Because $n \geq 7$
6.	$3^{n+1} < (n+1) * 3^n$	By step 1, 5
7.	$3^{n+1} < (n+1)!$	By step 4, 6

Part 3:

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(define (expo n i)
  (cond
    [(eq? i 0) 1]
    [else (* n (expo n (- i 1) ) ) ] ) )
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Part 4:
Using the standard definition, prove that

$$n^i * n^j = n^{i+j}$$

for all natural numbers i, j, n.

Hints: The exponent function is defined recursively so we need to reason by induction. The equation involves three numbers, but the recursive calls are for the exponent, which suggests that the induction should be on i and/or on j. It turns out that you can do it by induction on i. What does that mean? It means to prove $\forall i \, P(i)$ where $P(i)$ is $\forall j, n \; (n\ i\ * n\ j\ = \ n\ i + j\ )$.

$P(j) = \ \forall i \ \in \ \mathbb{Z}_{\geq 0} : n^{i+j} = n^i * n^j$
$P(0)$ is true because
$n^{i+0} = n^i * 1 = n^i * n^0$

Base Case:
$P(1)$ is true because
$n^{i+1} = n^i * n = n^i * n^1$

Inductive Case:

$n^i * n^{k+1} = n^i * (n^k * n)$	Integer Power
$= \left(n^i * n^k\right) * n$	Associative
$= n^{i+k} * n$	Induction Hypothesis
$= n^{i+k+1}$	Integer Power

Therefore,
$P(k) \rightarrow P(k+1)$ and $\forall i, j \in \mathbb{Z}_{\geq 0} : n^{i+j} = n^i * n^j$