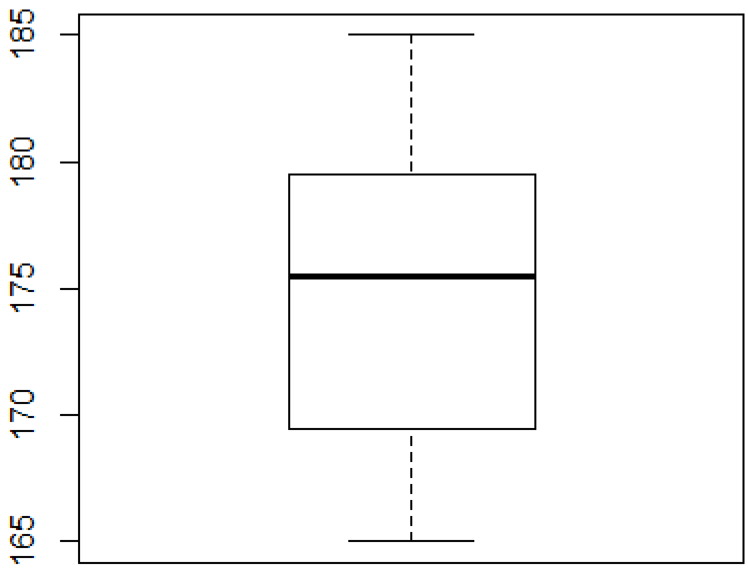
9/18/2015 Quiz01 Math331 Student (PRINT)______________________

According to the historical data, the average body length of male graduates in New Jersey is 174cm. Recently, we got the following sample of male graduates in Stevens Institute of Technology,

170, 172, 177, 168, 169, 184, 180, 175, 165, 176, 178, 181, 174, 179, 166, 185.

1.Provide 5-number summary of the sample and construct the box plot. --15pts.

Solution:
Min: 165 Q1: 169.8 Median: 175.5 Q3: 179.2 Max: 185



2.Evaluate sample mean, sample variance and sample standard deviation. Then tell the tail skewness of the sample. ---15pts

Solution:
Sample mean: 174.94 sample variance: 38.20 sample standard deviation: 6.18
Left skewed.

3.Assume $X_1, \dots, X_n$ is a simple and random sample from the population $X \sim U(0, 2\theta + 1)$ the uniform distribution with probability density $f(x) = \frac{1}{2\theta+1}$ for $x \in [0, 2\theta + 1]$. Find the moment estimator $\hat{\theta}$. ------------------------------20pts

Solution:

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx = \int_{0}^{2\theta+1} \frac{x}{2\theta + 1} dx = \frac{x^2}{4\theta + 2} |_0^{2\theta+1} = \frac{2\theta + 1}{2}$$

For a SRS $X_1, \dots, X_n$ of X, setting $\frac{2\theta+1}{2} = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, then we solve the moment estimator $\hat{\theta} = \bar{X} - \frac{1}{2}$

4.Assume $X_1, \dots, X_n$ is a simple and random sample from the population $X \sim G(p)$ with the probability function $f(x) = p(1-p)^{x-1}, for\ x = 1,2, \dots, and\ p \in (0,1)$. Using maximum likelihood estimation to estimate moment estimator $\hat{p}$.--20pts

Solution:
The likelihood function is:

$$L(p,x) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} p(1-p)^{x_i - 1} = p^n (1-p)^{\sum_{i=1}^{n} x_i - n}$$

$$logL(p,x) = nlogp + (\sum_{i=1}^{n} x_i - n)\log(1-p)$$

Set $\frac{\partial logL(p,x)}{\partial p} = 0$, we get $\frac{n}{p} - \frac{\sum_{i=1}^{n} x_i - n}{1-p} = 0$, which is solved by $p = \frac{n}{\sum_{i=1}^{n} x_i} = \frac{1}{\bar{x}}$.
Therefore, the MLE of p is $\hat{p} = \frac{n}{\sum_{i=1}^{n} x_i} = \frac{1}{\bar{X}}$.

170, 172, 177, 168, 169, 184, 180, 175, 165, 176, 178, 181, 174, 179, 166, 185.

5.If $X_1, \dots, X_n$ is a simple and random sample from the population $X \sim N(\mu, \sigma^2)$, $\bar{X}$ is the average of the sample, then $T = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sigma^2} \sim \chi_{n-1}^2$. Suppose $\sigma^2 = 38$, according to the data above, compute $\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{\sigma^2}$, and then compute $P(T > \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{\sigma^2})$.(Hint: $pchisq(x,y) = P(\chi_y^2 \le x)$, pchisq(15,15)=0.5486, pchisq(15,16)=0.4754, pchisq(10,15)=0.1803, pchisq(10,16)=0.1334) ---15pts

Solution:
We can compute that $\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{\sigma^2} = \frac{573}{38} = 15$, since $T = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sigma^2} \sim \chi_{n-1}^2$, n=16, then we have

$$P\left(T > \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{\sigma^2}\right) = P(T > 15) = 1 - P(T \le 15) = 1 - \mathrm{pchisq}(15,15) = 1 - 0.5486 = 0.4514$$

6.If $X_1, \dots, X_n$ is a simple and random sample from the population $X \sim N(\mu, \sigma^2)$, $\bar{X}$ is the average of the sample, $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is the variance of the sample, then $T = \frac{\bar{X} - \mu}{\sqrt{S^2/n}} \sim t_{n-1}$. Suppose $\mu = 173.39$, according to the data above, compute $\frac{\bar{x} - \mu}{\sqrt{s^2/n}}$, and then compute $P(T > \frac{\bar{x} - \mu}{\sqrt{s^2/n}})$.(Hint: pt(x,y)=P($t_y \le x$), pt(0.96,15)=0.8239, pt(0.96,16)=0.8243, pt(1,15)=0.8334, pt(1,16)=0.8339)------------------------------- ---15pts

Solution:
We can compute that $\frac{\bar{x} - \mu}{\sqrt{s^2/n}} = \frac{1.55}{1.55} = 1$, since $T = \frac{\bar{x} - \mu}{\sqrt{s^2/n}} \sim t_{n-1}$, n=16, then we have

$$P\left(T > \frac{\bar{x} - \mu}{\sqrt{s^2/n}}\right) = P(T > 1) = 1 - P(T \le 1) = 1 - \ \mathrm{pt}(1,15) = 1 - 0.8334 = 0.1666$$