Homework #2
I pledge my honor that I have abided by the Stevens honor system.

1.3: 2, 6, 10a, 10b, 10c, 18, 20

2.) Show that ¬(¬p) ≡ p

p	¬p	***¬(¬p)***
T	F	***T***
F	T	***F***

Therefore, p≡¬(¬p).

6.) Use a truth table to verify De Morgan's First Law: ¬(p∧q) ≡ ¬p∨¬q

p	q	p∧q	***¬(p∧q)***	¬p	¬q	***¬p∨¬q***
T	T	T	***F***	F	F	***F***
T	F	F	***T***	F	T	***T***
F	T	F	***T***	T	F	***T***
F	F	F	***T***	T	T	***T***

10. Show each as a tautology.

a. [¬p∧(p∨q)]→q ≡ ¬[¬p∧(p∨q)]∨q ≡ T

p	q	¬p	p∨q	¬p∧(p∨q)	¬[¬p∧(p∨q)]	***¬[¬p∧(p∨q)]∨q***
F	F	T	F	F	T	***T***
F	T	T	T	T	F	***T***
T	F	F	T	F	T	***T***
T	T	F	T	F	T	***T***

	[¬p ∧ (p ∨ q)] → q	
≡	¬[¬p ∧ (p ∨ q)] ∨ q	implicit exchange
≡	p ∨ ¬(p ∨ q) ∨ q	double negation law
≡	p ∨ ¬p ∧¬q ∨ q	De Morgan's Law/associative
≡	p ∨ ¬p ∧ q ∨ ¬q	commutative property
≡	T ∧ T	Negation law
≡	T	Idempotent

b. [(p → q) ∧ (q → r)] → (p → r)

p	q	r	¬p	¬q	¬p∨¬q	¬q∨r	(¬p∨q)∧(¬q∨r)	¬[(¬p∨q)∧(¬q∨r)]	¬p∨r	***¬[(¬p∨q)∧(¬q∨r)]∨(¬p∨r)***
F	F	F	T	T	T	T	T	F	T	***T***
F	F	T	T	T	T	T	T	F	T	***T***
F	T	F	T	F	T	F	F	T	T	***T***
F	T	T	T	F	T	T	T	F	T	***T***
T	F	F	F	T	F	T	F	T	F	***T***
T	F	T	F	T	F	T	F	T	T	***T***
T	T	F	F	F	T	F	F	T	F	***T***
T	T	T	F	F	T	T	T	F	T	***T***

$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

$\equiv$	$[(\neg p \vee q) \wedge (\neg q \vee r)] \rightarrow (\neg p \vee r)$	implicit exchange
$\equiv$	$\neg[(\neg p \vee q) \wedge (\neg q \vee r)] \vee (\neg p \vee r)$	implicit exchange
$\equiv$	$\neg(\neg p \vee q) \vee \neg(\neg q \vee r) \vee (\neg p \vee r)$	De Morgan's Law
$\equiv$	$(p \wedge \neg q) \vee (q \wedge \neg r) \vee (\neg p \vee r)$	De Morgan's Law
$\equiv$	$p \wedge (\neg q \vee q) \wedge \neg r \vee (\neg p \vee r)$	Associative property
$\equiv$	$p \wedge T \vee \neg r \vee (\neg p \vee r)$	Negation Law
$\equiv$	$p \wedge \neg r \vee T \vee (\neg p \vee r)$	Commutative property
$\equiv$	$p \wedge \neg r \vee (\neg p \vee r) \vee T$	Commutative property
$\equiv$	$p \vee \neg p \wedge r \vee \neg r \vee T$	Commutative property
$\equiv$	$T \wedge T \vee T$	Negation Law
$\equiv$	T	Idempotent

c. $[p \wedge (p \rightarrow q)] \rightarrow q$

p	q	¬p	¬p∨q	p∧(¬p∨q)	¬(p∧(¬p∨q))	***¬[p∧(¬p∨q)]∨q***
T	T	F	T	T	F	***T***
T	F	F	F	F	T	***T***
F	T	T	T	F	T	***T***
F	F	T	T	F	T	***T***

$[p \wedge (p \rightarrow q)] \rightarrow q$

$\equiv$	$\neg[p \wedge (\neg p \vee q)] \vee q$	Implicit exchange
$\equiv$	$\neg[(p \wedge \neg p) \vee q] \vee q$	Associative property
$\equiv$	$\neg(p \wedge \neg p) \wedge \neg q \vee q$	De Morgan's Law
$\equiv$	$\neg p \vee p \wedge \neg q \vee q$	De Morgan's Law
$\equiv$	$T \wedge T$	Negation Law
$\equiv$	T	Idempotent

18.) Show that $p \rightarrow q$ and $\neg q \rightarrow \neg p$ are logically equivalent.

$p \rightarrow q$

$\equiv$	$\neg p \vee q$	Implicit exchange
$\equiv$	$\neg p \vee \neg\neg q$	Commutative property
$\equiv$	$\neg\neg q \vee \neg p$	Commutative property
$\equiv$	$\neg q \rightarrow \neg p$	Implicit exchange

20.) Show that $\neg(p \oplus q) \equiv p \leftrightarrow q$

p	q	p⊕q	***¬(p⊕q)***	p→q	q→p	***(p→q)∧(q→p)***
F	F	F	***T***	T	T	***T***
F	T	T	***F***	T	F	***F***
T	F	T	***F***	F	T	***F***
T	T	F	***T***	T	T	***T***

1.4: 6, 8, 10, 12, 28

6.) Let $N(x)$ be “x has visited ND” , domain consists of students in school. Express in English.
a. $\exists x\ N(x)$: There is some student at school that has visited ND.
b. $\forall x\ N(x)$: Every student at school has visited ND.
c. $\neg\exists x\ N(x)$: No student at school has visited ND.
d. $\exists x\ \neg N(x)$: There is some student at school who has not visited ND.
e. $\neg\forall x\ N(x)$: Not all students at school have visited ND.
f. $\forall x\ \neg N(x)$: None of the students at school have visited ND.

8.)$R(x)$ is “x is a rabbit” and $H(x)$ is “x hops”, domain consists of all animals. Express in English.
a. $\forall x\ [R(x) \rightarrow H(x)]$: Every animal, that is a rabbit, hops.
b. $\forall x\ [R(x) \wedge H(x)]$: All animals are rabbits, and they hop.
c. $\exists x\ [R(x) \rightarrow H(x)]$: There is an animal that hops if it is a rabbit.
d. $\exists x\ [R(x) \wedge H(x)]$: There is an animal that is a rabbit and hops.

10.) $C(x)$ is “x has a cat”, $D(x)$ is “x has a dog”, $F(x)$ is “x has a ferret”, domain is students in your class.
a. A student in your class has a cat, a dog, and a ferret: $\exists x\ [C(x) \wedge D(x) \wedge F(x)]$
b. All students in your class have a cat, a dog, or a ferret: $\forall x\ [C(x) \vee D(x) \vee F(x)]$
c. Some student in your class has a cat, and a ferret, but not a dog: $\exists x\ [C(x) \wedge F(x) \wedge \neg D(x)]$
d. No student in your class has a cat, a dog, and a ferret: $\neg\exists x\ [C(x) \wedge D(x) \wedge F(x)]$
e. For each $C(x)$, $D(x)$, and $F(x)$, there is a student who has one: $\exists x\ C(x) \wedge \exists x\ D(x) \wedge \exists x\ F(x)$

12.) $Q(x)$ is “$x+1>2x$”. What are the truth-values, where domain is all ints?
a. $Q(0) =$ T
b. $Q(-1) =$ T
c. $Q(1) =$ F
d.$\exists x\ Q(x) =$ T
e.$\forall x\ Q(x) =$ F
f. $\exists x\ \neg Q(x) =$ T
g.$\forall x\ \neg Q(x) =$ F

28.) $Cor(x)$ is “x is in the correct place”, $Tool(x)$ is “x is a tool”, $EC(x)$ is “x is in excellent condition”.
a. Something is not in the correct place: $\exists x\ \neg Cor(x)$
b. All tools are in the correct place and are in excellent condition:$\forall x\ [Tool(x) \rightarrow Cor(x) \wedge EC(x)]$
c. Everything is in the correct place and are in excellent condition: $\forall x\ [Cor(x) \wedge EC(x)]$
d. Nothing is in the correct place and is in excellent condition: $\neg\exists x\ [Cor(x) \wedge EC(x)]$
e. One of your tools is not in the correct place, but it’s in excellent condition: $\exists x\ (Tool(x) \rightarrow \neg Cor(x) \wedge EC(x)$