

Assignment 4 - Cauchy-Riemann equations and Harmonic functions
Due February 19th

1. (1 pt) Knowing that $f(x + iy) = u(x, y) + iv(x, y)$ is entire, $f(0) = 0$ and $u(x, y) = 2x^2 - 2y^2 + x$, find $v(x, y)$.
2. (1 pt) Knowing that $f(re^{i\theta}) = u(r, \theta) + iv(r, \theta)$ is entire, $f(0) = 0$ and $u(r, \theta) = r \sin \theta$, find $v(r, \theta)$.
3. Check if the following functions are harmonic and whether they could be the real and imaginary parts of an analytic function:
 - a) (1 pt) $u(x, y) = xy$, $v(x, y) = x^2 + y^2$
 - b) (1 pt) $u(x, y) = xy$, $v(x, y) = x^2 - y^2$
4. Let $f(x, y)$ and $g(x, y)$ be real-valued harmonic functions.
 - a) (0.5 pt) Show that $f + g$ will be harmonic
 - b) (0.5 pt) Show that fg will not necessarily be harmonic