Katie Prescott
Homework 3
I pledge my honor that I have abided by the Stevens Honor System.

Construction of Context Free Languages

1) Define a context free grammar that accepts any valid mathematical expression over the alphabet {0, 1, 2, 3, 4, 5, 6, 7, 8, 8, +}.

V := {E, N, D}
Σ := {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +}
R := { $E \rightarrow N$
$E \rightarrow E + E$
$N \rightarrow D$
$N \rightarrow ND$
$D \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$
}
S := E

2) Define a context free grammar that accepts any palindrome over the alphabet {0, 1}.

V := {P}
Σ := {0, 1}
R := { $P \rightarrow 0P0$
$P \rightarrow 1P1$
$P \rightarrow 0$
$P \rightarrow 1$
$P \rightarrow \varepsilon$
}
S := P

Construction of Pushdown Automata

1) Define a pushdown automaton which accepts any string over the alphabet {0, 1} for which there are the same number of "0"s as "1"s.

Q := {st, q_1, q_2, fin}
Σ := {0, 1} Γ := {0, 1, $}
q_0 := st F := {st, fin}
δ :=

Input	0				1				ε			
Stack	0	1	$	ε	0	1	$	ε	0	1	$	ε
st												(q_1, $)
q_1		(q_1, ε)		(q_1, 0)	(q_1, ε)			(q_2, 1)			(fin, ε)	
q_2		(q_1, ε)		(q_1, 0)				(q_2, 1)				
fin												

2) Let w, z be strings over the alphabet {0, 1, $}. Define a pushdown automaton which accepts any string $w\$z$, where w^{-1} is a subsequence of z, and both w and z do not contain "$".

Ex: "**110**$**0**0**1**00**1**"
Ex: "**0101**$0**10**0**1**11**0**101"

Input	0				1				$				ε			
Stack	0	1	$	ε	0	1	$	ε	0	1	$	ε	0	1	$	ε
st																(q_1, $)
q_1				(q_1, 0)				(q_1, 1)				(q_2, ε)				
q_2	(q_2, ε)			(q_2, ε)		(q_2, ε)		(q_2, ε)							(fin, ε)	
fin																

Pumping Lemma

1) Let $\Sigma = \{0, 1\}$. Prove that the set $\{ w^2 \mid w \in \Sigma^* \}$ is not context-free.

w = $0^p 10^p 1$

00…00 0 1 0 00…001

\ u / \v^p/ \x/ \y^p/ \ z /

$|vyx| \leq p$

$|vyx| = 2p + 1$

Therefore, $\{ w^2 \mid w \in \Sigma^* \}$ is not context free.