Lab 01 Problems/Solutions

1/30/17

1 The Tree Method

The God Problem: Is the following a logically correct inference? Either prove that it is using the tree method or else give a counterexample. To get started, first identify all of the atomic propositions and replace them with symbols (e.g. "God exists" can be replaced by E, "God can prevent evil" by P, etc.), and then translate each sentence into a compound proposition.

1. If God exists, then God is omnipotent.
2. If God exists, then God is omniscient.
3. If God exists, then God is benevolent.
4. If God can prevent evil, then if God knows that evil exists then God is not benevolent if God does not prevent evil.
5. If God is omnipotent then God can prevent evil.
6. If God is omniscient, then God knows that evil exists (if it does exist).
7. Evil does not exist if God prevents it.
8. Evil exists.

God Does not exist.

Of course, showing that the inference is correct does not prove that the conclusion is correct, since the premises can be false. The early philosophers, notably Rene Descartes, nonetheless made logical arguments to try and prove the existence of God. Ultimately, believing in the truth of the premises is a matter of faith, not logic.

SOLUTION:

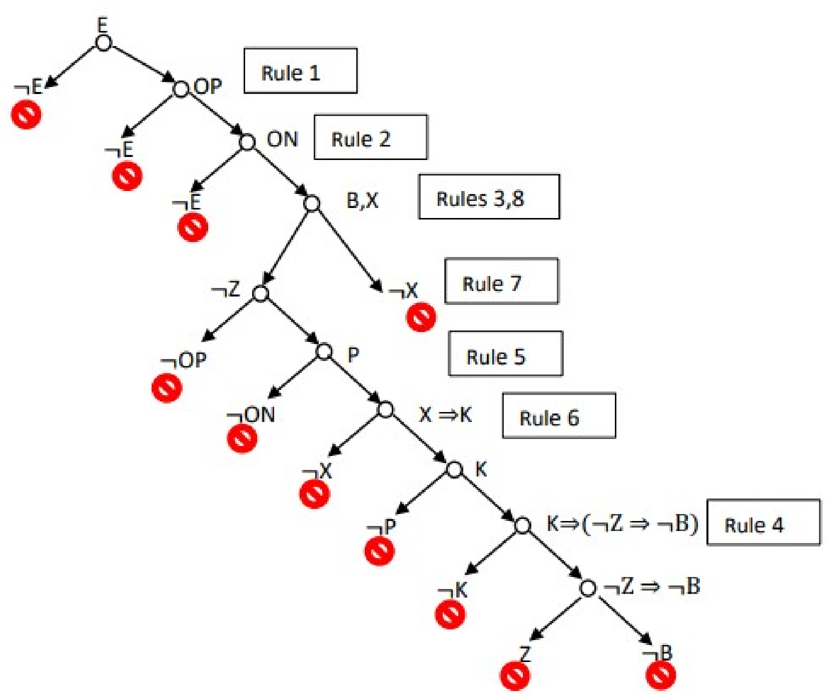
Notation:

- E — God Exists
- OP — God is omnipotent
- ON — God is omniscient
- B — God is benevolent
- P — God can prevent evil
- K — God knows evil exists
- X — Evil exists
- Z — God prevents evil

Premises:

1. $E \rightarrow OP$
2. $E \rightarrow ON$
3. $E \rightarrow B$
4. $P \rightarrow (K \rightarrow (\neg Z \rightarrow \neg B))$
5. $\neg P \rightarrow \neg OP$
6. $ON \rightarrow (X \rightarrow K)$
7. $Z \rightarrow \neg X$
8. X

Therefore $\neg E$



2 The Truth About Predicates and Quantifiers

Tautologies: For each statement below, show whether or not it is a tautology.

- $\exists x \exists y P(x,y) \rightarrow \exists y \exists x P(x,y)$

  SOLUTION: *Tautology.*

  Quantifiers of the same type are always commutative.

  $\exists x \exists y P(x,y) \equiv \exists x,y P(x,y) \equiv \exists y,x P(x,y) \equiv \exists y \exists x P(x,y)$

- $\forall x \exists y Q(x,y) \rightarrow \exists x \forall y Q(x,y)$

  SOLUTION: *Not Tautology.*

  Different quantifiers are not always commutative.

  It's easiest to understand with an example. If $Q(x,y)$ states that person y is working at time x, then the first statement is, "for all times there is a person that is working", while the second is, "there is a person that is working at all times", which is not neccessarily true given the previous statement.

- $\exists x \forall y R(x,y) \rightarrow \forall y \exists x R(x,y)$

  SOLUTION: *Tautology.*

  Sometimes different quantifiers are commutative.

  Using the previous example, even though it doesn't make sense in the real world, "there is some time that everyone is working" does indeed imply that "for all people, there is sometime that they are working"

Logical Equivalence: Prove the following equivalences.

- $\forall x(P(x) \rightarrow A) \equiv \exists x P(x) \rightarrow A$

  SOLUTION:

  $\forall x(P(x) \rightarrow A) \equiv \forall x(\neg P(x) \vee A)$ — Now we can distribute

  $\equiv \forall x \neg P(x) \vee \forall x A$ — The A term doesn't need a quantifier

  $\equiv \neg \exists x P(x) \vee A \equiv \exists x P(x) \rightarrow A$

- $\exists x(P(x) \rightarrow Q(x)) \equiv \forall x P(x) \rightarrow \exists x Q(x)$ *Optional*

  SOLUTION:

  $\exists x(P(x) \rightarrow Q(x)) \equiv \exists x(\neg P(x) \vee Q(x))$

  $\equiv \exists x \neg P(x) \vee \exists x Q(x)$

  $\equiv \neg \forall x P(x) \vee \exists x Q(x) \equiv \forall x P(x) \rightarrow \exists x Q(x)$