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Assignment 12 - Laurent Series, Zeroes and Poles
Due May 5th

1. a) (0.5 pts) Let $f(z) = \cos(\frac{1}{z})$. Find the Laurent series of f centered at $z_0 = 0$.

   b) (1 pt) Let $g(z) = \frac{1}{\cos(\frac{1}{z})}$. Find all the singularities of g and state if they are isolated or not.

   Hint: Note that, since cos is entire and $\frac{1}{z}$ is analytic unless $z = 0$, $g(z)$ will be analytic as long as $\cos(\frac{1}{z})$ exists and is not equal to 0.

   c) (1 pt) Find two sequences, $\{z_n\}$ and $\{w_n\}$ such that $\lim_{n\to\infty} z_n = \lim_{n\to\infty} w_n = 0$ but $\lim_{n\to\infty} |f(z_n)| = 0$ and $\lim_{n\to\infty} |f(w_n)| = \infty$ for all $n \in \mathbb{N}$.

   Hint: $\{z_n\}$ can be picked to be real and $\{w_n\}$ imaginary

   d) (0.5 pts) How do a) and c) verify Riemann's theorem and the theorem of classification of singularities?

2. a) (0.5 pts) Let $f(z) = z + \frac{1}{z}$. Show that f has a pole of order 1 at 0 and zeroes of order 1 at i and $-i$.

   b) (1 pt) Let $g(z) = \frac{1}{z+\frac{1}{z}}$. Find all the singularities of g and state if they are removable, poles (giving the order) or essential.

   Hint: You do not need to compute any integrals for a) and b).

   c) (0.5 pts) Let $h(z) = \begin{cases} g(z) & z \neq 0 \\ 0 & z = 0 \end{cases}$. Show that h is analytic in $D_1(0)$.