.

Assignment 1 - Complex numbers and functions - Solutions

1. Let $z = 2 - 2i$. Compute:

   a) $|z| = 2\sqrt{2}$

   b) $\arg z = -\frac{\pi}{4}$

   c) $z^3 = 16\sqrt{2}e^{-i\frac{3\pi}{4}} = -16 - 16i$

   d) cube roots of z are $\{\sqrt{2}e^{-i\frac{\pi}{12}}, \sqrt{2}e^{i\frac{7\pi}{12}}, \sqrt{2}e^{-i\frac{9\pi}{12}}\}$

2. Let $f : \mathbb{C} \to \mathbb{C}$ be defined by $f(a + ib) = b + ia$.

   a) $u(r, \theta) = r \sin\theta$, $v(r, \theta) = r \cos\theta$

   b) $f^{-1}(a + ib) = b + ia$. Since all points in $\mathbb{C}$ have one and only one pre-image, f has to be both one-to-one and onto.

   c) f is its own inverse

   d) $\frac{f(z)}{\bar{z}} = i$

3. Find two functions f_1, $f_2 : \mathbb{C} \to \mathbb{C}$ such that:

   a) Question is open-ended so many answers are possible, but the primitive square root $f_1(re^{i\theta}) = +\sqrt{r}e^{i\frac{\theta}{2}}$ is one-to-one but not onto

   b) Likewise many answers are possible, but the square function $f_2(re^{i\theta}) = r^2e^{i2\theta}$ is onto but not one-to-one