Homework 10: 9.3: 4a, 6, 8, 9.4: 4, 16, 18, 9.5: 2, 16, 42, 44
I pledge my honor that I have abided by the Stevens Honor System.
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9.3:

4a. List the ordered pairs in the relation on {1, 2, 3, 4} corresponding to the matrix.

[1 1 0 1]
[1 0 1 0]
[0 1 1 1]
[1 0 1 1]

$$\{(1,1),(1,2),(1,4),(2,1),(2,3),(3,2),(3,3),(3,4),(4,1),(4,3),(4,4)\}$$

6. How can the matrix representing a relation R on a set A be used to determine whether the relation is asymmetric?

- Center diagonal is all 0's
- Every value must be different from its transposed position.

8. Determine whether the relations represented by the matrices in exercise 4 are reflexive, irreflexive, symmetric, antisymmetric, and/or transitive.

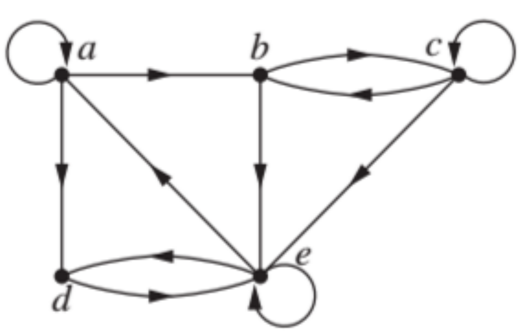
- 4a: symmetric, transitive
- 4b: reflexive, antisymmetric
- 4c: irreflexive, symmetric, transitive

9.4:

4. How can the directed graph representing the reflexive closure of a relation on a finite set be constructed from the directed graph of the relation?

The reflexive closure of a relation is just the relation unioned with any reflexive pairs not already in the relation. The directed graph would just have the self-loops added to the relation's original directed graph.

16. Determine if these sequences are paths in the directed graph.



a) a, b, c, e	TRUE
b) b, e, c, b, e	FALSE
c) a, a, b, e, d, e	TRUE
d) b, c, e, d, a, a, b	FALSE
e) b, c, e, b, e. d, e, d	FALSE
f) a, a, b, b, c, c, b, e, d	FALSE

18. Determine whether there is a path that starts at the first vertex, and ends at the second vertex.

a) a, b	True: a, b	f) c, d	True: c, e, d
b) b, a	True: b, e, a	g) d, d	True: d, e, d
c) b, b	True: b, c, b	h) e, a	True: e, a
d) a, e	True: a, b, e	i) e, c	True: e, a, b, c
e) b, d	True: b, e, d		

9.5:

2. Which of these relations on the set of all people are equivalence relations? What properties do the others lack?

a) $\{(a, b) |\ a \text{ and } b \text{ are the same age.}\}$

True

b) $\{(a, b) |\ a \text{ and } b \text{ have the same parents.}\}$

True

c) $\{(a, b) |\ a \text{ and } b \text{ share a common parent.}\}$

False—not transitive

d) $\{(a, b) |\ a \text{ and } b \text{ have met.}\}$

False—not transitive

e) $\{(a, b) |\ a \text{ and } b \text{ speak a common language.}\}$

False—not transitive

16. Let R be the relation on the set of ordered pairs of positive integers such that $((\boldsymbol{a}, \boldsymbol{b}), (\boldsymbol{c}, \boldsymbol{d})) \in \boldsymbol{R}$ if and only if $\boldsymbol{ad} = \boldsymbol{bc}$. Show that R is an equivalence relation.

- R is reflexive:
  $((a, b), (a, b)) \in R \rightarrow ab = ab$
- R is symmetric:
  $((a, b), (c, d)) \in R \rightarrow ad = bc$
  $cb = da \rightarrow ((c, d), (a, b)) \in R$
- R is transitive:
  $((a, b), (c, d)) \in R \,\&\, ((c, d), (e, f)) \in R \rightarrow ad = bc \,\&\, cf = de$
  $af = \frac{adcf}{dc} = \frac{bcde}{dc} = be \rightarrow ((a, b), (e, f)) \in R$

42. Which of these collections of subsets are partitions of $\{-3, -2, -1, 0, 1, 2, 3\}$?

a) $\{-3, -1, 1, 3\}, \{-2, 0, 2\}$ True
b) $\{-3, -2, -1, 0\}, \{0, 1, 2, 3\}$ False
c) $\{-3, 3\}, \{-2, 2\}, \{-1, 1\}, \{0\}$ True
d) $\{-3, -2, 2, 3\}, \{-1, 1\}$ False

44. Which of these collections of subsets are partitions of the set of integers?

a) Set of even integers & the set of odd integers
   True
b) Set of positive integers & the set of even integers
   False (no 0)
c) Set of integers divisible by 3, the set of integers with a remainder of 1 after being divided by 3, and the set of integers with a remainder of 2 after being divided by 3.
   True
d) Set of integers less than -100, set of integers with an absolute value not exceeding 100, and integers greater than 100.
   True
e) Set of integers not divisible by 3, the set of even integers, and the set of integers leaving a remainder of 3 when divided by 6.
   False (2 is in the first 2 sets)