MA-227 Review Session (Answers) 10/30/2016

(1) Final Answer is 36π

For surface integral there are three separate pieces,

S_1: $x=2$ $\rightarrow \iint_{S_1} \vec{F}\cdot d\vec{S} = 36\pi$

S_2: $x=0$ $\rightarrow \iint_{S_2} \vec{F}\cdot d\vec{S} = 0$

S_3: cylinder $\rightarrow \iint_{S_3} \vec{F}\cdot d\vec{S} = 0$

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(2) Answer $= \frac{3}{4} a^2 b^2 c^2$

Surface integral has 6 separate pieces

Surfaces at $x=0$, $y=0$, $z=0$ yield $\iint_S \vec{F}\cdot d\vec{S} = 0$.

Surfaces at $x=a$, $y=b$, $z=c$

$\rightarrow \iint_S \vec{F}\cdot d\vec{S} = \frac{1}{4} a^2 b^2 c^2$ $(\times 3)$.

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(3) Answer $= 80\pi$. using $\iint_S (\vec{\nabla}\times\vec{F})\cdot d\vec{S}$

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(4) Answer $= -\pi$.

For surface integral, use the surface

S_2: Disk $x^2+z^2 \le 1$, $y=0$.

$\vec{\nabla}\times\vec{F} = \langle -1,-1,-1\rangle$; $\hat{n} = \hat{k} = \langle 0,0,1\rangle$

$$\iint_{S_2} (\vec{\nabla}\times\vec{F})\cdot d\vec{S} = \iint_{S_2} (\vec{\nabla}\times\vec{F})\cdot\hat{n}\, dS = \iint_{S_2} (-1)\, dS = -\text{Area}(S_2) = \boxed{-\pi}$$

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(5) Answer $= \boxed{12}$

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(6) Evaluate $\iint_D \left(\frac{\partial}{\partial x}(x^3 + 3xy^2) - \frac{\partial}{\partial y}(x) \right) dA$ By Green's Thm $= \oint_C \vec{F} \circ d\vec{r} =$ Work

Answer $= \boxed{12\pi}$

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(7) $\text{Area}(D) = \frac{1}{2} \oint_C (-y\,dx + x\,dy) = \boxed{\frac{2}{3} a^3}$

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(8) $\vec{r}(t) = \langle a\cos(t), b\sin(t) \rangle \quad 0 \le t \le 2\pi$

$\text{Area}(D) = \frac{1}{2} \oint_C (-y\,dx + x\,dy) = \oint_C \vec{F} \circ d\vec{r}$

$= \frac{1}{2} \int_0^{2\pi} \langle -b\sin t, a\cos t \rangle \circ \langle -a\sin t, b\cos t \rangle dt$ where $\vec{F}(x,y) = \langle -\frac{y}{2}, \frac{x}{2} \rangle$

$= \frac{1}{2} \int_0^{2\pi} ab(\sin^2 t + \cos^2 t)\,dt = \frac{1}{2} ab \cdot 2\pi = \boxed{\pi ab}$

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(9) $\vec{F}(x,y) = \langle xy^2, x^2 y \rangle = \vec{\nabla} f(x,y)$ for $f(x,y) = \frac{x^2 y^2}{2}$.

$\left.\begin{array}{l} \vec{r}(0) = \langle 0+0, 0+1 \rangle = \langle 0,1 \rangle \\ \vec{r}(1) = \langle 1+1, 1+0 \rangle = \langle 2,1 \rangle \end{array}\right\}$ End points of C

$\int_C \vec{F} \circ d\vec{r} = \int_C (\vec{\nabla} f) \circ d\vec{r} = f(\vec{r}(1)) - f(\vec{r}(0)) = f(2,1) - f(0,1)$

$= 2 - 0 = \boxed{2}$