EXPECTED VALUE OF A RANDOM VARIABLE

Defn: The EXPECTED VALUE OF A R.V. X (denoted by $E(X)$) IS THE LONG-RUN AVERAGE VALUE OF THAT R.V.

$$\boxed{E(X) = \sum_{i=1}^{n} x_i \, P(X = x_i)}$$

FORMULA FOR EXPECTED VALUE OF THE R.V. X

A motivating example:

ex/ a game, where X = winnings on a single play

X	P(X)
+2	.5
−1	.5

In 1,000 plays, we "expect" 500 wins and 500 losses, since P(win) = P(lose) = .5.

So we win \$2 500 times ⟶ +\$1,000
and we lose \$1 500 times ⟶ −\$500

for a total winnings of \$500. Then the AVERAGE WINNINGS PER PLAY are $\frac{\$500}{1000 \text{ plays}} = \$.50$

OR, EASIER WITH FORMULA:

$$E(X) = (+2)(.5) + (-1)(.5) = \$.50$$

ex/ An investment gives returns X with the following probability distribution:

X	−100	0	+250	+400
P(X)	.2	.3	.25	.25

$$E(X) = (-100).2 + 0(.3) + (250)(.25) + (400)(.25) = \$142.50$$

ex/ You're selling your home next month, and you think there's a 25% chance you'll get \$80,000 for it, a 40% chance you'll get \$100,000, and a 35% chance that you'll get \$125,000. What is the expected value of your sale price?

Let X = sale price.

Then

X	80,000	100,000	125,000
P(X)	.25	.40	.35

$$E(X) = .25(80,000) + .40(100,000) + .35(125,000)$$
$$= \$103,750$$

ex/ $P(X=x) = \dfrac{3-x}{6}$; $x = 0,1,2$

$$E(X) = \sum_{x=0}^{2} x\left(\frac{3-x}{6}\right) = 0 + 1\left(\frac{3-1}{6}\right) + 2\left(\frac{3-2}{6}\right) = \frac{2}{6} + \frac{2}{6} = \frac{2}{3}$$

X	P(X)
0	1/2
1	1/3
2	1/6

$$E(X) = 0\left(\tfrac{1}{2}\right) + 1\left(\tfrac{1}{3}\right) + 2\left(\tfrac{1}{6}\right) = \frac{2}{3}$$

VARIANCE OF A RANDOM VARIABLE

: a quantitative measure of variability or dispersion

Df: Let X be a r.v.
The variance of X, denoted $V(X)$ or σ_x^2 is:

$$V(X) \triangleq E\{[X-E(X)]^2\}$$

and the standard deviation of X is the positive square root of $V(X)$ $(= \sigma_x)$
in same units as X

Why not use $E[X-E(X)]$ as our measure of dispersion

Because $E[X-E(X)] = E(X) - E(\mu) = \mu - \mu = 0$, always

Theorem (useful in computation)

$$V(X) = E(X^2) - [E(X)]^2$$

Proof:

$$V(X) = E\{[X-E(X)]^2\}$$
$$= E[X^2 - 2X E(X) + [E(X)]^2]$$
$$= E(X^2) - 2E(X)E(X) + [E(X)]^2$$
$$= E(X^2) - [E(X)]^2$$

remember that $E(X)$ is a constant

ex/

X	P(x)
1	.2
2	.5
3	.3

$E(X) = 2.1$

① From the definition:

X	X − E(X)	$[X-E(X)]^2$
1	−1.1	1.21
2	−.1	.01
3	.9	.81

$$E[(X-E(X))^2] = 1.21(.2) + .01(.5) + .81(.3) = \underline{\underline{.49 = V(X)}}$$

② Short-cut method

X	X^2
1	1
2	4
3	9

$$E(X^2) = 1(.2) + 4(.5) + 9(.3) = 4.9$$

$$\therefore V(X) = E(X^2) - [E(X)]^2 = 4.9 - (2.1)^2 = \underline{\underline{.49}}$$