Previous Lecture

Logical Equivalence:	Laws of Propositional Logic
Logical Arguments:	Rules of Inference

Laws of Propositional Logic

Idempotent laws:	$p \vee p \equiv p$	$p \wedge p \equiv p$
Associative laws:	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws:	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Distributive laws:	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity laws:	$p \vee F \equiv p$	$p \wedge T \equiv p$
Domination laws:	$p \wedge F \equiv F$	$p \vee T \equiv T$
Double negation law:	$\neg\neg p \equiv p$	
Complement laws:	$p \wedge \neg p \equiv F$ $\neg T \equiv F$	$p \vee \neg p \equiv T$ $\neg F \equiv T$
De Morgan's laws:	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Conditional identities:	$p \rightarrow q \equiv \neg p \vee q$	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Applying the Laws of Propositional Logic

$$P \equiv (L \Rightarrow W) \vee (H \Rightarrow W)$$

$\equiv (\neg L \vee W) \vee (\neg H \vee W)$ Conditional Identity Law (applied twice)

$\equiv (\neg L \vee W) \vee W \vee \neg H$ Commutative Law

$\equiv \neg L \vee (W \vee W) \vee \neg H$ Associative Law

$\equiv \neg L \vee W \vee \neg H$ Idempotent Law

$\equiv \neg L \vee \neg H \vee W$ Commutative Law

$\equiv \neg (L \wedge H) \vee W$ De Morgan's Law

$\equiv (L \wedge H) \Rightarrow W$ Conditional Identity

Rules of Inference

p $p \to q$ $\therefore q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	Modus tollens
p $\therefore p \vee q$	Addition
$p \wedge q$ $\therefore p$	Simplification

p q $\therefore p \wedge q$	Conjunction
$p \to q$ $q \to r$ $\therefore p \to r$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	Disjunctive syllogism
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	Resolution

Proof of Validity

A. $S \Rightarrow G$

B. $\neg S \Rightarrow E$

C. $\neg G \Rightarrow \neg E$

$\therefore G$

1. $\neg G \Rightarrow \neg E$ hypothesis, C
2. $E \Rightarrow G$ contrapositive, 1
3. $\neg S \Rightarrow E$ hypothesis, B
4. $\neg S \Rightarrow G$ hypothetical syllogism, 3,2
5. $S \Rightarrow G$ hypothesis, A
6. $G \vee \neg S$ conditional identity, 5
7. $G \vee S$ conditional identity, 4
8. G resolution, 7

Another Example

Moriarty will escape unless Holmes acts
We shall rely on Watson only if Holmes does not act
If Holmes does not act, Moriarty will escape unless we rely on Watson

$\neg H \Rightarrow M$

$W \Rightarrow \neg H$

$\neg H \Rightarrow (\neg W \Rightarrow M)$

Proof of Validity

A. $\neg H \Rightarrow M$

B. $W \Rightarrow \neg H$

C. $\neg H \Rightarrow (\neg W \Rightarrow M)$

1. $\neg H \Rightarrow M$ hypothesis, A
2. $H \vee M$ conditional identity, 1
3. $H \vee M \vee W$ addition, 2
4. $\neg H \Rightarrow (W \vee M)$ conditional identity, commutative law, 3
5. $\neg H \Rightarrow (\neg W \Rightarrow M)$ conditional identity, 4

Hypothesis B was never used in the proof!

Is there a more straightforward method to prove validity?

What if the argument is invalid? How do we find a counterexample?

A different kind of example

Is the following argument valid?

$(H \vee W) \Rightarrow M$

$\therefore (M \vee H)$

Yet another example

Is the following argument valid?

Babies are illogical.
Nobody who can manage a crocodile is despised.
<u>Illogical persons are despised.</u>
∴ Babies cannot manage crocodiles.

$B \Rightarrow I$
$M \Rightarrow \neg D$
$I \Rightarrow D$
$B \Rightarrow \neg M$

One more example!

Is the following argument valid?

If there is life on Mars, then the experts are wrong and the government is lying.
If the government is lying, then the experts are right or there is no life on Mars.
<u>The government is lying.</u>
∴ There is life on Mars

$M \Rightarrow W \wedge L$
$L \Rightarrow \neg W \vee \neg M$
L
$\therefore\ M$

The Tree Method

1. Write down each premise
2. Negate the conclusion
3. Look for a counterexample
   a) If all leaves are blocked off: no counterexample exists
   b) Else, if some leaf is not blocked off but all compound propositions are checked off: counterexamples exist
   c) If there an unchecked compound proposition, expand it at every leaf below and check off the proposition
   d) Close off every leaf whose path to the top contains a contradiction

Is an inference valid?

1. Use the rules of inference, starting with the premise and try to derive the conclusion.

2. Build a truth table.

3. Apply the tree method to search for counterexamples.