MA 222 B

SPRING 2016

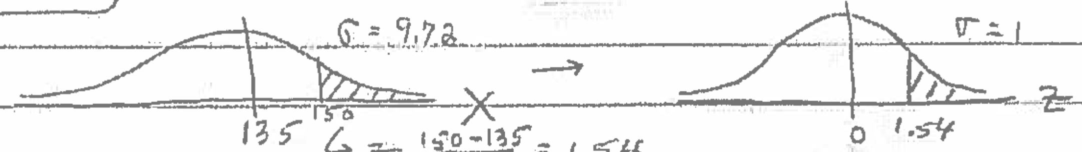
EXAM #3 : SOLUTIONS

(1.) X = The number of students who decide to attend the college

$X \sim bin(n = 450, p = .30)$ // use $X \sim N(np, \sqrt{npq})$

Find $P(X > 150)$ // or $X \sim N(135, 9.72)$

ok to also use $P(150 \le X \le 450)$ (same answer)



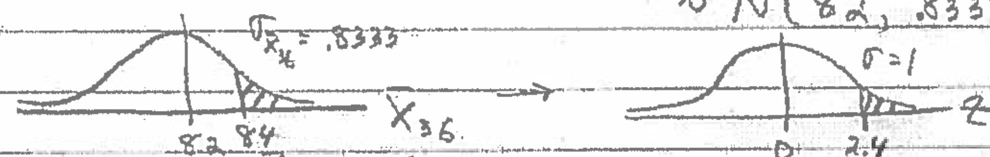
$z = \frac{150-135}{9.72} = 1.54$

so: $P(X > 150) \equiv P(Z \geq 1.54) = .5 - .4382 = .0618$

(2.) X = final grades of students $\sim N(\mu = 82, \sigma_x = 5)$

Find: $P(\bar{X}_{36} > 84)$ where $\bar{X}_{36} \sim N(82, \frac{5}{\sqrt{36}})$

$\sim N(82, .8333)$



$z = \frac{84-82}{.8333} = 2.4$

so: $P(\bar{X}_{36} > 84) \equiv P(Z \geq 2.4) = .5 - .4938 = .0062$

(3.) Marginal dist. for X is: $P(X=0) = \frac{1}{6}$ $P(X=1) = \frac{1}{4}$ $P(X=2) = \frac{7}{12}$

Find $V(X)$, using $V(X) = E(X^2) - [E(X)]^2$

where $E(X) = 0(\frac{1}{6}) + 1(\frac{1}{4}) + 2(\frac{7}{12}) = \frac{17}{12}$

and $E(X^2) = 0^2(\frac{1}{6}) + 1^2(\frac{1}{4}) + 2^2(\frac{7}{12}) = \frac{31}{12}$

$\therefore V(X) = E(X^2) - \boxed{E(X)}^2 = \frac{31}{12} - (\frac{17}{12})^2 = 2.58 - 2 = .58$

(4) One way to do this is to make a list of all possible outcomes and their probabilities:

W win \$1 with prob. $\frac{18}{38} = .473$

LW win \$0 with prob. $\frac{20}{38} \cdot \frac{18}{38} = .24$

LL lose \$2 with prob. $\frac{20}{38} \cdot \frac{20}{38} = .277$

$\therefore E(\text{winnings}) = (1)(.473) + 0(.24) + (-2)(.277) = -\$.08$

(5) Let Y = the number of names selected by both Person A and Person B

Define:

$$X_i = \begin{cases} 1 & \text{if the } i^{th} \text{ name is selected by both people} \\ 0 & \text{if the } i^{th} \text{ name is NOT selected by both people} \end{cases}$$

where: $P(X_i = 1) = P(i^{th}$ name is selected by Person A AND selected by Person B$)$

where $P(i^{th}$ name selected by Person A$) = 1 - P($NOT selected by person A$)$

$= 1 - (\frac{9}{10})(\frac{8}{9})(\frac{7}{8}) = .3$

and $\therefore P($selected by Person B$)$ is ALSO $.3$

$\therefore P(X_i = 1) = (.3)(.3) = .09$

$\therefore E(X_i) = 1(.09) + 0(.91) = .09$

and since

$$Y = X_1 + X_2 + \dots + X_{10}$$

$$E(Y) = E(X_1 + X_2 + \dots + X_{10})$$

$$= E(X_1) + E(X_2) + \dots + E(X_{10})$$

$$= 10(.09)$$

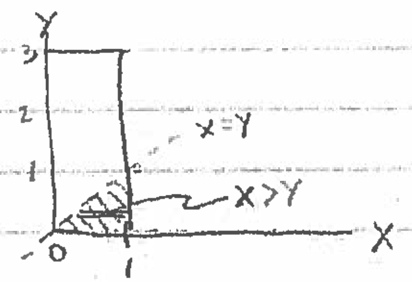
$= .9$ names on average will be selected by both Person A and Person B

EXTRA CREDIT:

$X \sim U(0,1) \Rightarrow f_X(x) = 1 \quad 0 \le x \le 1$

$Y \sim U(0,3) \Rightarrow g_Y(y) = \frac{1}{3} \quad 0 \le y \le 3$

FIND $P(X > Y)$



JOINT DIST. OF X and Y is $h_{X,Y}(x,y) = f_X(x) \cdot g_Y(y)$

since X and Y are independent

so $h_{X,Y}(x,y) = 1 \cdot \frac{1}{3} = \frac{1}{3}$ for $0 \le x \le 1$ and $0 \le y \le 3$

and $P(X>Y) = \int_0^1 \int_y^1 \frac{1}{3}\, dx\, dy = \int_0^1 \left[\frac{x}{3} \Big|_y^1 \right] dy$

$= \int_0^1 \left(\frac{1}{3} - \frac{y}{3} \right) dy = \left(\frac{y}{3} - \frac{y^2}{6} \right) \Big|_0^1 = \frac{1}{6}$

NOTE: in this case, since $h_{X,Y}(x,y)$ is a flat plane of height $= \frac{1}{3}$ above the x-y axis, you can also compute the volume under the plane by multiplying the area of the triangle times the height:

$\left[\frac{1}{2}(1)(1) \right] \cdot \frac{1}{3} = \frac{1}{6}$