Required Online Textbook

Discrete Mathematics

Evaluate
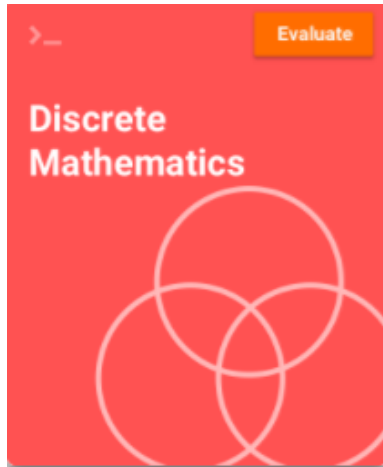
1. Go to learn.zybooks.com
2. Create your account
3. Enter code

   STEVENSCS135BhattSpring2018

4. Subscribe

   Subscription is $58 (expires May 30, 2018)

Set up your subscription TODAY!

The first assignment is due before Monday's class.

Previous Lecture

Definition: A proposition is a declarative statement that is either True or False (but not both).

- 4 + 3 = 7
- 1 + 1 = 3
- Give me an A!
- X + 1 = 2
- Humans are mortal
- This proposition is false.

- We will use letters to denote propositions:
- H : Humans are mortal
- A : I will get an A in CS135.

Today's Lecture

Building compound propositions using logical connectives

Negation $\quad\quad \neg G \quad\quad$ "not G"

Conjunction $\quad (G \wedge H) \quad$ "G and H" $\quad\quad$ True when both G, H are true

Disjunction $\quad (G \vee H) \quad$ "G or H" $\quad$ True when one (or both) of G, H is True

Implication $\quad (G \Rightarrow H) \quad$ "If G then H"

Truth Tables

Logical Equivalence

Logical Connectives: Negation

Compound propositions are built from simpler ones using connectives.

If G is a proposition then $\neg G$ is a proposition (the negation of G).

The truth values of G and $\neg G$ are opposites of each other.

Example: G: I am grumpy.

$\neg G$: not(I am grumpy) = I am not grumpy.

Example: H: I am hungry. $\neg H$: I am not hungry

What is $\neg(\neg H)$?

$\neg(\neg H)$: not(I am not hungry) = I am not (not hungry) = I am hungry = H.

$\neg(\neg X)=X$: **Two negatives make a positive**

Logical Connectives: Conjunction

If G and H are propositions then ($G \land H$) is a proposition

($G \land H$) is called the *conjunction* of G and H.

($G \land H$) is True when both G, H are True, and False if one (or both) of G, H is False.

Example: G: I am grumpy

                  H: I am hungry

                  ($G \land H$): I am grumpy and I am hungry.

If I am not grumpy then the proposition ($G \land H$) is False

If I am neither grumpy nor hungry then ($G \land H$) is False

If I am hungry and I am grumpy then ($G \land H$) is True

Logical Connectives: Disjunction

If G and H are propositions then $(G \lor H)$ is a proposition

$(G \lor H)$ is called the *disjunction* of G and H.

$(G \lor H)$ is True when at least one of G, H is True, and False when both G, H are False.

Example: G: I am grumpy

H: I am hungry

$(G \lor H)$: I am grumpy or I am hungry.

If I am grumpy then the proposition $(G \lor H)$ is True

If I am both grumpy and hungry then $(G \lor H)$ is True

If I am neither hungry nor grumpy then $(G \lor H)$ is False

Inclusive vs. Exclusive OR : Disjunction is Inclusive!

Truth Tables: Basic Connectives

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Logical Connectives: Implication

If G and H are propositions then ($G \Rightarrow H$) is a proposition

($G \Rightarrow H$) is read as "If G then H" or as "G implies H"

Examples

      If I am grumpy then I am hungry

      If you complete the reading assignment then you will get full credit

      If <antecedent> then <consequent>

When is an implication True and when is it False?

Logical Connectives: Implication

The statement "If I am grumpy then I am hungry"
is FALSE in case I am grumpy but not hungry.

"If you complete the reading assignment then you will get full credit" is FALSE in case you complete the reading assignment but do not get full credit.

If you do not complete the reading assignment, the statement is TRUE – regardless of whether you get full credit or not!

If I am not grumpy the first implication is TRUE – whether I am hungry or not!

$(G \Rightarrow H)$ is False when G is True and H is False; otherwise it is True.

The Truth Table for Implications

If 1+1 = 3 then 2 > 10.

When the antecedent is false the implication is …?   **TRUE!**

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

When the antecedent is false, the implication is true whether or not the consequent is true!

If pigs can whistle then I am king. T

If 1 = 2 then (Moriarty escapes and Moriarty is captured). T

If the moon shines white then the moon is made of white cheddar. F

Truth Tables Summary

TABLE 1 The Truth Table for the Negation of a Proposition.

p	¬p
T	F
F	T

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

TABLE 5 The Truth Table for the Conditional Statement p → q.

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Logical Tautologies

A proposition that is always True is called a tautology.

Examples:

    True

    2 = 1+1

    It is raining or it is not raining

    Holmes will catch Moriarty or Holmes will not catch Moriarty

    $Q \lor \neg Q$

Logical Contradiction

A proposition that is always False is called a contradiction.

Examples:

> False
>
> 1 > 1
>
> It is raining and it is not raining
>
> Holmes will catch Moriarty and Holmes will not catch Moriarty
>
> $Q \wedge \neg Q$

Logical Equivalence

Two propositions P and Q that have the same truth table are *logically equivalent*

$$P \equiv Q$$

How do we prove that two propositions are equivalent?

$\neg(p \wedge q) \equiv (\neg p \vee \neg q)$?

Example:

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Logical Equivalence

How do we prove that two propositions are NOT equivalent?

Give a counterexample: an assignment of truth values to the atoms that make one proposition true and the other one false.

Example: $P: (x \lor y) \Rightarrow w$ *vs.* $Q: (x \land y) \Rightarrow w$

Counterexample: $x = True, \quad y = False, \quad w = False$

$P: (T \lor F) \Rightarrow F \qquad Q: (T \land F) \Rightarrow F$

FALSE   **TRUE**

Pitfalls to Watch For!

"If I attend lectures then I will do well in CS135

OR

If I do my homeworks then I will do well in CS135"

If I attend lectures **and** do my homeworks then I will do well in CS135

Are these two propositions logically equivalent?

Next Lecture: Logical Equivalences

Table 1.5.1: Laws of propositional logic.

Idempotent laws:	$p \lor p \equiv p$	$p \land p \equiv p$
Associative laws:	$(p \lor q) \lor r \equiv p \lor (q \lor r)$	$(p \land q) \land r \equiv p \land (q \land r)$
Commutative laws:	$p \lor q \equiv q \lor p$	$p \land q \equiv q \land p$
Distributive laws:	$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$	$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
Identity laws:	$p \lor F \equiv p$	$p \land T \equiv p$
Domination laws:	$p \land F \equiv F$	$p \lor T \equiv T$
Double negation law:	$\neg\neg p \equiv p$	
Complement laws:	$p \land \neg p \equiv F$ $\neg T \equiv F$	$p \lor \neg p \equiv T$ $\neg F \equiv T$
De Morgan's laws:	$\neg(p \lor q) \equiv \neg p \land \neg q$	$\neg(p \land q) \equiv \neg p \lor \neg q$
Absorption laws:	$p \lor (p \land q) \equiv p$	$p \land (p \lor q) \equiv p$
Conditional identities:	$p \rightarrow q \equiv \neg p \lor q$	$p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)$