10/16/2015 Midterm Math331 Student (PRINT)________________

1, Given a sample of 15 nonnegative observations: 90, 87, 57, 64, 76, 77, 56, 81, 83, 68, 97, 76, 89, 85, 78, assume they are from exponential distribution with density function $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$ and the parameter $\lambda > 0$.

(i). Find the moment estimator of λ.

(ii). Find the MLE of λ.

Solution:

(i)$E(X) = \int_0^\infty x\lambda e^{-\lambda x}dx = -\int_0^\infty xde^{-\lambda x} = -xe^{-\lambda x}\Big|_0^\infty + \int_0^\infty e^{-\lambda x}dx = \frac{e^{-\lambda x}}{-\lambda}\Big|_0^\infty = \frac{1}{\lambda}$

So we have $\hat{\lambda} = \frac{1}{\bar{x}} = 77.6$

(ii)$L = \prod_{i=1}^n f(x_i) = \lambda^n e^{-\lambda\sum_{i=1}^n x_i} \Rightarrow logL = nlog\lambda - \lambda\sum_{i=1}^n x_i \Rightarrow \frac{\partial logL}{\partial\lambda} = \frac{n}{\lambda} - \sum_{i=1}^n x_i$

Let $\frac{\partial logL}{\partial\lambda} = 0$, then $\hat{\lambda} = \frac{1}{\bar{x}}$, and $\frac{\partial^2 logL}{\partial\lambda^2} = -\frac{n}{\lambda^2} < 0$

So $\hat{\lambda} = \frac{1}{\bar{x}} = 77.6$

2, Given a sample of 10 observations: 96.5, 101.7, 100.3, 100.1, 100.0, 102.1, 98.6, 100.2, 99.3, 100.4.

(i), Assume they are from normal distribution $X \sim N(\mu, 4)$, then (a)compute the confidence interval of μ with confidence level 0.95, (b)test $H_0: \mu = 98$ vs $H_0: \mu \neq 98$ at the significant level $\alpha = 0.05$.(hint: $z_{0.975} = 1.96$, pnorm(3.04)=0.9988)

Solution:

(a)mean(x)=99.92. Since $\sigma = 2$ is known, then we use z-test for mean μ

$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$, then $P\left(\left|\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| \leq Z_{0.975}\right) = 95\%$,

|μ-99.92|<=1.24, so [98.68,101.16] is the CI of μ with confidence level 95%.

(b) $z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{99.92-98}{2/sqrt(10)} = 3.04$

$P(|Z| \geq z) = 2 * (1 - pnorm(3.04)) = 0.0024 < 0.05$. Reject H_0

(ii), Assume they are from normal distribution $X \sim N(\mu, \sigma^2)$ with σ^2 unknown, then (a) compute the confidence interval of μ with confidence level 0.95, (b)test $H_0: \mu = 101$ vs $H_0: \mu \neq 101$ at the significant level $\alpha = 0.05$.(hint: $t_{0.975}(9) = 2.262, t_{0.975}(10) = 2.228$, pt(2.173,9)=0.971,pt(2.173,10)=0.973)

Solution:

(a)mean(x)=99.92, sd(x)=1.572. Since σ is unknown, then we use t-test for mean μ

$T = \frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t(n-1)$, then $P\left(\left|\frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}}\right| \leq t_{0.975}(9)\right) = 95\%$,

|μ-99.92|<=1.123, so [98.797,101.043] is the CI of μ with confidence level 95%.

(b) $t = \frac{\bar{X}-\mu}{s/\sqrt{n}} = \frac{99.92-101}{1.572/sqrt(10)} = 2.173$

$P(|T| \geq t) = 2 * (1 - pt(2.173,9)) = 0.058 > 0.05$. Not reject H_0

3, Given a sample of 15 observations: 1, 1, 0, 0, 0, 1, 0, 0, 1, 1, 0, 1, 1, 0, 1, they are from $B(1, p)$.

(i). Find point estimator of p.

(ii). Compute the confidence interval of p with confidence level 0.95.

(iii). Test $H_0: p = 0.5$ vs $H_0: p \neq 0.5$ at the significant level $\alpha = 0.05$.

(hint: $z_{0.975} = 1.96$, pnorm(0.2585)=0.602, pnorm(0.07898)=0.5315, pnorm(0.2498)=0.5986)

(i). $p = E(X) = 0.5333$

(ii). var(X)=2.6667, mean(X)=0.5333,

According to central limit theorem,

$Z = \frac{\bar{X}-p}{\sqrt{S^2/n}} = \frac{\bar{X}-p}{\sqrt{\bar{X}(1-\bar{X})/n}} \sim N(0,1)$, then $p\left(\left|\frac{\bar{X}-\mu}{\sqrt{\bar{X}(1-\bar{X})/n}}\right| \leq Z_{0.975}\right) = 95\%$,

|μ-0.5333|<=0.2524, so [0.2809,0.7857] is the CI of μ with confidence level 95%.

(iii) $z = \frac{\bar{X}-\mu}{\sqrt{S^2/n}} = \frac{0.5333-0.5}{\sqrt{0.5333*(1-0.5333)/15}} = 0.2585$

$P(|Z| \geq z) = 2 * (1 - pnorm(0.2585)) = 0.796 > 0.05$. Not reject H_0