CS 135 Spring 2018: Problem Set 3.

Problem 1. (10 points)

a. Give explicit formulas for functions from the set $\mathbb{Z}$ of integers to the set $\mathbb{N}$ that are:
   i. One-to-one but not onto
   ii. Onto but not one-to-one
   iii. One-to-one and onto
   iv. Neither one-to-one nor onto
b. If functions f and $f \circ g$ are both onto, does it follow that g is onto? Prove or give a counterexample.
c. Let A, B, and C be sets, and let $f: B \rightarrow C \text{ and } g: A \rightarrow B$ be functions. Let $h: A \rightarrow C$ be the composition, $f \; o \; g$, that is, $h(x) = f(g(x))$ for $x \in A$. Prove, or give a counterexample, for each of the following claims:
   i. If h is surjective, then f must be surjective.
   ii. If h is surjective, then g must be surjective.
   iii. If h is injective, then f must be injective.
   iv. If h is injective and f is total[1], then g must be injective.

Problem 2. (10 points)

Use the well-ordering principle to prove the validity of the following logical argument in which $P(x)$ is a predicate defined over the set of natural numbers, i.e., $x \in \mathbb{N}$.

$$\begin{array}{l} P(0) \\ \underline{\forall k \in \mathbb{N}: (P(k) \Rightarrow P(k+1))} \\ \therefore \forall n \in \mathbb{N}: P(n) \end{array}$$

[1] See the definition of a total function in the textbook.