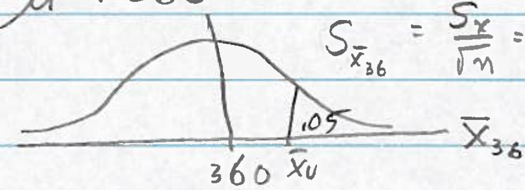
HW #10: SOLUTIONS

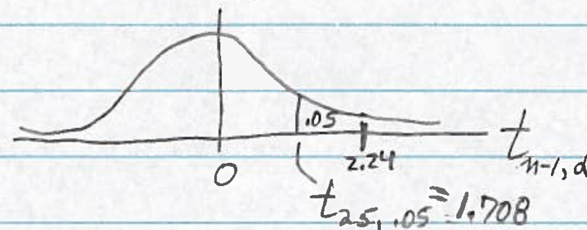
(1.) $n = 26$ $\quad \bar{x}_{26} = 370.69$ $\quad S_x = 24.36$ $\quad \alpha = .05$

$H_0: \mu \le 360$ (seconds) $\quad$ or $H_0: \mu = 360$

$H_1: \mu > 360$

$$S_{\bar{x}_{26}} = \frac{S_x}{\sqrt{n}} = \frac{24.36}{\sqrt{26}} = 4.78$$





CRITICAL REGION: $t_{sample} > 1.708$

$$t_{sample} = \frac{\bar{x}_{26} - \mu}{s_x/\sqrt{n}} = \frac{370.69 - 360}{4.78} = 2.24$$ which is IN the critical region

$$\left(1.708 = \frac{\bar{x}_U - 360}{4.78} \Rightarrow \bar{x}_U = 368.16 \right.$$

$\left. \text{so C.R. for } \bar{x}_{26} \text{ is } \bar{x}_{sample} > 368.16 \right)$

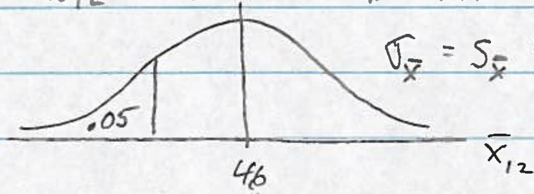
So: REJECT H_0. The data do indicate that the prior belief is incorrect, and the mean is now greater than 6 minutes

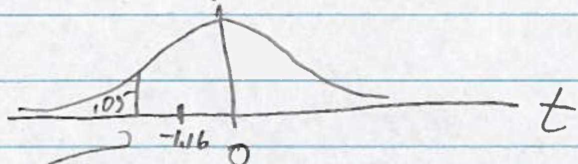
2. $H_0: \mu \geq 46$ OR $H_0: \mu = 46$

$H_1: \mu < 46$

$n = 12 \quad \bar{x}_{12} = 42 \quad S_x = 11.9 \quad \alpha = .05$

$\sigma_{\bar{x}} = S_{\bar{x}} = \frac{S_x}{\sqrt{n}} = \frac{11.9}{\sqrt{12}} = 3.43$





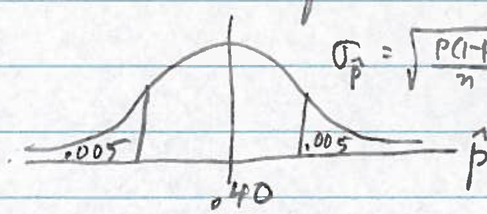
$t_{11,.05} = -1.796$

C.R.: $t_{sample} < -1.796$ / our $t_{sample} = \frac{\bar{x} - \mu}{S_x/\sqrt{n}} = \frac{42-46}{3.43} = -1.16$

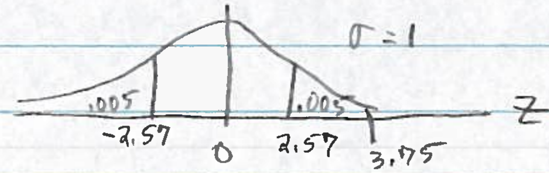
NOT IN C.R. SO ACCEPT H_0: The sample data do not indicate that the true mean is less than 46 KWH per year

3. $H_0: p = .40 \quad H_1: p \neq .40 \quad \alpha = .01 \quad n = 150 \quad \hat{p} = \frac{82}{150} = .55$

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.4)(.6)}{150}} = .04$



$\sigma = 1$



C.R. is $Z_{sample} < -2.57$ or > 2.57 $\quad Z_{sample} = \frac{.55 - .40}{.04} = 3.75$

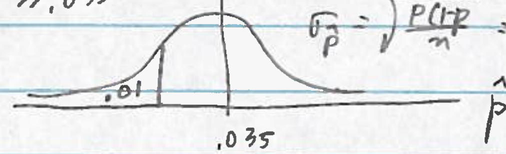
is IN CRITICAL REGION

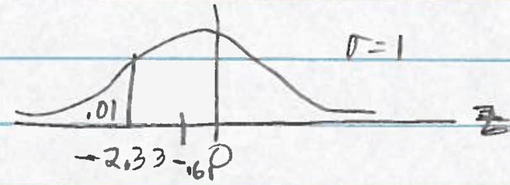
So REJ H_0: The data do indicate that the percentage of Type A blood donations is no longer 40%

4. $H_0: p = .035 \quad H_1: p < .035 \quad \alpha = .01 \quad n = 500 \quad \hat{p} = \frac{15}{500} = .03$

or $H_0: p \geq .035$

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.035(.965)}{500}} = .0082$





C.R. is $z_{sample} < -2.33$ // OUR $z_{sample} = \frac{.03 - .035}{.0082} = -.61$

z_{sample} is NOT in REJECTION REGION, so: ACCEPT H_0: The robots have not demonstrated superior performance (lower error rate)

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5. $H_0: \mu_1 - \mu_2 = 0$

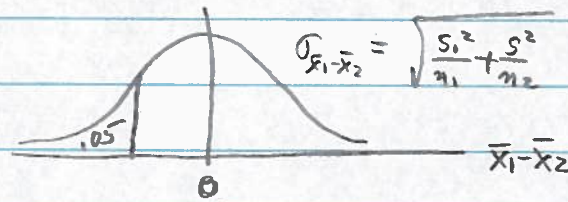
$H_1: \mu_1 - \mu_2 < 0$

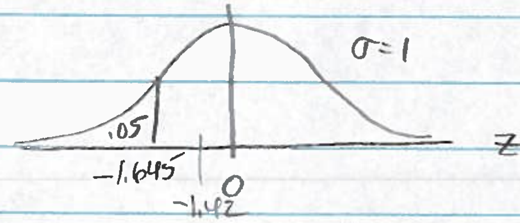
(μ_1 = mean calories / loaf for new process)

(μ_2 = " " " for old process)

ONE-SIDED TEST: $\alpha = .05$

$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{213^2}{50} + \frac{238^2}{30}} = 52.8$





critical region is $z_{sample} < -1.645$

OUR z_{sample} is $\frac{(1255 - 1330) - 0}{52.8} = -1.42$ NOT IN REJ. REGION

SO. ACCEPT H_0: The data do not suggest that the mean number of calories per loaf is lower for the new leavening process.

(6.) μ_1 = population mean of abrasive wear for Material 1
μ_2 = " " " " " 2

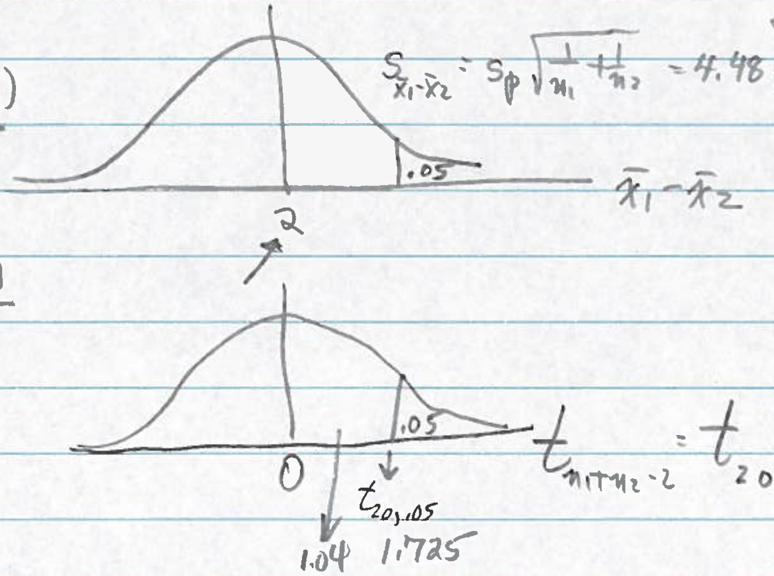
$H_0: \mu_1 - \mu_2 = 2$ (use small-sample test)
$H_1: \mu_1 - \mu_2 > 2$

$$S_{\bar{x}_1-\bar{x}_2} = S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 4.48\sqrt{\frac{1}{12}+\frac{1}{10}} = 1.92$$

$$S_p = \sqrt{\frac{s_1^2(n_1-1)+s_2^2(n_2-1)}{n_1+n_2-2}}$$

$$= \sqrt{\frac{16(12-1)+25(10-1)}{12+10-2}}$$

$$= 4.48$$



C.R.: $t_{sample} > 1.725$

$$t_{sample} = \frac{(85-81)-2}{1.92} = 1.04$$ NOT IN REJ. REGION

ACCEPT H_0.
There is no evidence that the mean abrasive wear of material 1 exceeds that of material 2 by more than 2 units.

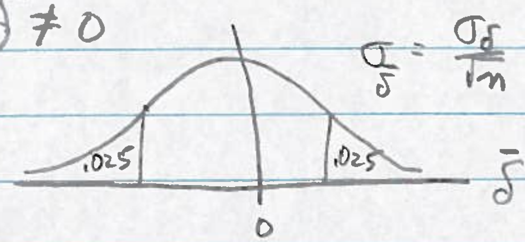
7. Let δ = mean population difference in androgen levels betw. before & after injection

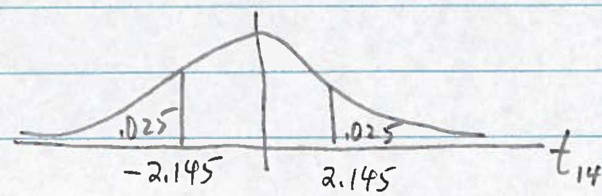
$H_0: \delta = 0$

$\delta \neq 0$

$\sigma_{\bar{\delta}} = \frac{\sigma_d}{\sqrt{n}} = \frac{18.474}{\sqrt{15}} = 4.77$ ~ from δ_i samples

$\bar{\delta} = 9.848$





So: Rejection region is $t_{sample} > 2.145$ or < -2.145

Our $t_{sample} = \frac{9.848 - 0}{4.77} = 2.06$ NOT IN REJ REGION

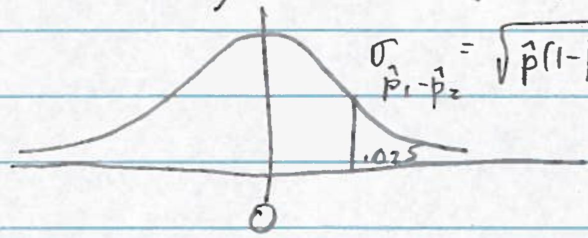
So the sample data do not indicate a significant difference in androgen levels after the injection.

ACCEPT Ho

(8) $H_0: p_1 = p_2$ $\quad H_1: p_1 > p_2$ $\quad \Big\} \quad H_0: p_1 - p_2 = 0$ $\quad H_1: p_1 - p_2 > 0$

$\hat{p}_1 = \frac{120}{200} = .60$

$\hat{p}_2 = \frac{240}{500} = .48$

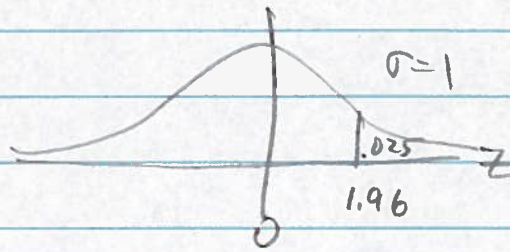


$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{(.514)(.486)\left[\frac{1}{200} + \frac{1}{500}\right]}$

$= .0418$

where $\hat{p}$ = pooled estimate of p

$= \frac{120 + 240}{200 + 500} = .514$



C.R. $\quad z_{sample} > 1.96$

our $z_{sample} = \frac{(.60 - .48) - 0}{.0418} = 2.87$ IS IN REJ REG.

so REJECT H_0

The data do indicate that the population proportion of voters in Town A in favor of the proposal is greater than the proportion in Town B in favor of the proposal.