EXAM #3: SOLUTIONS

(1.) a) NO, not independent, since

$P(X=x_i)P(Y=y_j) \neq P(X=x_i, Y=y_j)$

for all (i,j)

for example: $P(X=6, Y=1) = \frac{1}{4} \neq P(X=6)P(Y=1) = \frac{3}{8}\cdot\frac{5}{8} = \frac{15}{64}$

X \ Y	0	1	P(X)
6	1/8	1/4	3/8
9	1/4	1/4	1/2
12	0	1/8	1/8
P(Y):	3/8	5/8	

b)

Z	P(Z)
6	$P(X=6, Y=0) = 1/8$
7	$P(X=6, Y=1) = 1/4$
9	$P(X=9, Y=0) = 1/4$
10	$P(X=9, Y=1) = 1/4$
12	$P(X=12, Y=0) = 0$
13	$P(X=12, Y=1) = \frac{1}{8}$
	1

(2.) E(gain from one play) $= (.6)(\$-3) + (.4)(\$4) = \$-.20$

If X_i = gain from play i $\quad i = 1, 2, \ldots, 100$

And X = total gain on 100 plays

Then $X = X_1 + X_2 + \ldots + X_{100}$

$\therefore \quad E(X) = E(X_1 + X_2 + \ldots + X_{100}) = E(X_1) + E(X_2) + \ldots + E(X_{100})$

$= 100(\$-.20) = -\$20$

(3.) Let X = # of people out of 100 wearing eyeglasses

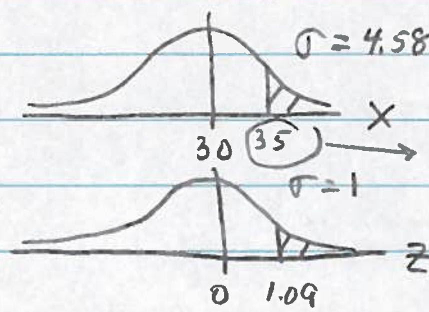
$X \sim bin(n=100, p=.30)$

We seek $P(X>35)$ (35 is 35% of 100)

Use Normal Approx. to Binomial (since $np>5$ and $n(1-p)>5$)

$X \sim N(\mu = np = 30, \sigma = \sqrt{np(1-p)} = \sqrt{100(.3)(.7)})$

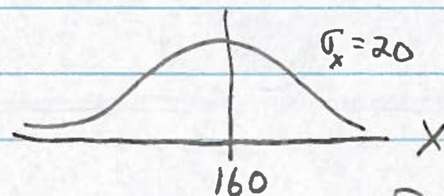
so $X \sim N(\mu = 30, \sigma = 4.58)$ and we need $P(X>35)$



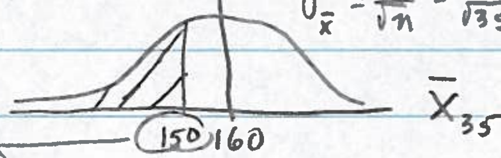
$z = \frac{x-\mu}{\sigma} = \frac{35-30}{4.58} = 1.09$

So $P(X>35) = P(Z>1.09) = .5000 - .3621 = .1379$

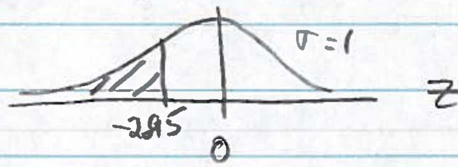
(4.)



$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{20}{\sqrt{35}} = 3.38$



$z = \frac{150-160}{3.38}$



$\therefore P(\bar{X}_{35} < 150) \equiv P(Z < -2.95) = .5000 - .4984 = .0016$

5. a) 2 ways: really similar

1) find the marginal for x and then compute $P(X<1)$

$$g(x) = \int f(x,y)\,dy = \int_0^1 \frac{1}{3}x^3 y\,dy = \frac{1}{3}x^3 \frac{y^2}{2}\Big|_{y=0}^{y=1}$$
$$= \frac{1}{6}x^3 \quad \text{for } 0 \le x \le 2$$

Then: $P(X<1) = \int_0^1 g(x)\,dx = \int_0^1 \frac{1}{6}x^3\,dx = \frac{x^4}{24}\Big|_{x=0}^{x=1} = \frac{1}{24}$

OR: 2) Integrate $f(x,y)$ directly:

$$P(X<1) = \int_0^1\int_0^1 \frac{1}{3}x^3 y\,dx\,dy = \int_0^1 \left(\frac{1}{3}\frac{x^4}{4}\right)\Big|_{x=0}^{x=1} y\,dy$$
$$= \int_0^1 \frac{1}{12}y\,dy = \frac{y^2}{24}\Big|_{y=0}^{1} = \frac{1}{24}$$

b) Now you need the marginal probability distribution of X:

$$E(X^3) = \int_0^2 x^3 g(x)\,dx = \int_0^2 x^3 \frac{1}{6}x^3\,dx$$
$$= \int_0^2 \frac{x^6}{6}\,dx = \frac{x^7}{42}\Big|_{x=0}^{x=2} = \frac{128}{42} = 3.05$$