RANDOM VARIABLES

Loosely speaking, a random variable is a quantity that varies according to chance

ex/ Let X = the number coming up on a tossed die
possible values: 1, 2, 3, 4, 5, 6

ex Let X = the number of 4's that come up in 10 tosses of a die
possible values: 0, 1, 2, …, 10

ex/ Let X = # of Heads in 100 tosses of a fair coin
possible values: 0, 1, 2, …, 100

ex/ A game is played in which a die is tossed once. If numbers 1 or 2 come up, you win \$3, but if the numbers 3, 4, 5, or 6 come up, you lose \$2.
Let X = your gain from a play of this game
possible values: 3, −2

ex/ Toss a coin twice.
- Let X = # of H's — possible values = 0, 1, 2
- Let Y = # H's − # T's — possible values = −2, 0, 2

ex/ Let X = # tosses of a coin up to and including 1st Head $X = 1, 2, \ldots$

ex/ Let X = length of time until next bus arrives
possible values: $0 \le x \le 60$
CONTINUOUS RANDOM VARIABLE — takes on values over a continuous range

The first 5 examples are of DISCRETE RANDOM VARIABLES } FINITE OR COUNTABLY INFINITE # OF POSSIBLE VALUES

The sixth example is a CONTINUOUS RANDOM VARIABLE.

NOTATION: $X, Y, Z, \ldots$ uppercase: random variables

x: $x_1, x_2, \ldots$ the VALUES that the r.v.'s take on

PROBABILITY DISTRIBUTIONS

For now, we'll restrict our discussion of probability distributions to discrete random variables.

Defn: A probability distribution of a random variable is a list or table of all possible values the r.v. can take on together with their respective probabilities.

ex/ Toss a die once

Let X = the r.v. the # that comes up

The probability distribution of X is

x	$P(X=x)$
1	$P(X=1) = 1/6$
2	$P(X=2) = 1/6$
3	$P(X=3) = 1/6$
4	$P(X=4) = 1/6$
5	$P(X=5) = 1/6$
6	$P(X=6) = 1/6$
	1

ex/ Let X = # of accidents in 1 day at a factory

x	$P(x)$
0	.79
1	.17
2	.03
3	.01
	1.00

a) Find P(one or more accidents on a particular day):

$$= P(X \geq 1) = P(X=1 \text{ or } X=2 \text{ or } X=3)$$
$$= P(X=1) + P(X=2) + P(X=3)$$
$$= .17 + .03 + .01 \quad = .21$$

OR can do by:

$$P(\text{one or more accidents}) = 1 - P(\text{no accidents})$$
$$= 1 - .79 \quad = .21$$

b) Find P(the next two days will both have 0 accidents) =

= P(no accident tomrw AND no accident the day after)

= P(no accident tom'w)$\cdot$P(no accident the day after)

$$= (.79)(.79) \quad = .6241$$

Sometimes a function can be used to describe or generate a probability distribution.

ex/ $P(X=x) = \dfrac{3-x}{6}$ for $x = 0, 1, 2$

is the same as

X	P(X)
0	3/6
1	2/6
2	1/6
	1

Notation for probability distribution fcns (discrete)

probability mass function : pmf

$p(x_i)$ $\begin{cases} p(x_i) \geq 0 & \text{for all } i \\ \sum_{\text{all } i} p(x_i) = 1 \end{cases}$

or just

$p(x)$ $\begin{cases} p(x) \geq 0 & \text{all } x \\ \sum_{x} p(x) = 1 \end{cases}$

or

$P(X=k)$ $\begin{cases} P(X=k) \geq 0 & \text{all } k \\ \sum_{k} P(X=k) = 1 \end{cases}$

Recall that we said that a discrete r.v. can have either ~~a~~ discrete or countably infinite number of possible values.

ex| We are testing a large number of tubes, each having a 75% chance of testing positive and a 25% chance of testing negative.

Define the r.v. $X = \{$the # of tests until the first positive tube appears$\}$

$X = 1, 2, \ldots$

$$P(X=1) = \frac{3}{4}$$
$$P(X=2) = \frac{1}{4}\cdot\frac{3}{4}$$
$$P(X=3) = \frac{1}{4}\cdot\frac{1}{4}\cdot\frac{3}{4}$$
$$P(X=4) = \frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{3}{4}$$
$$\vdots$$
$$P(X=n) = \left(\frac{1}{4}\right)^{n-1}\frac{3}{4} \qquad n=1,2,\ldots$$

check: ① $P(X=n) \geq 0$ for all n

② $$\sum_{n=1}^{\infty} P(X=n) = \sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^{n-1}\frac{3}{4} = \frac{3}{4}\sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^{n-1}$$
$$= \frac{3}{4}\left(1+\frac{1}{4}+\left(\frac{1}{4}\right)^2+\left(\frac{1}{4}\right)^3+\ldots\right) = \frac{3}{4}\left(\frac{1}{1-\frac{1}{4}}\right) = 1$$

Find $Pr\{$# tests to first positive tube is even$\}$

$$= \sum_{n=1}^{\infty} Pr\{X=2n\} = \sum_{n=1}^{\infty}\frac{3}{4}\left(\frac{1}{4}\right)^{2n-1} = \frac{3}{16}+\frac{3}{256}+\ldots$$
$$= \frac{3}{16}\left(1+\left(\frac{1}{16}\right)+\left(\frac{1}{16}\right)^2+\left(\frac{1}{16}\right)^3+\ldots\right)$$
$$= \frac{3}{16}\left[\frac{1}{1-\frac{1}{16}}\right] = \frac{1}{5}$$