CS 135 Spring 2018: Problem Set 4.

Problem 1. (10 points) Recall that a set A is *countably infinite* when there is a bijection from the set $\mathbb{N}$ to A. Prove each of the following statements:

a. If A, B are countably infinite then so is $A \cup B$.
b. Every infinite subset of a countably infinite set is countably infinite.
c. If A, B are countably infinite then so is $A \times B$.
d. The set $\mathbb{Q}$ of rational numbers is countable. (Hint: represent each rational number as a subset of $\mathbb{Z} \times (\mathbb{N} - \{0\})$, making sure that no rational number is represented twice in your subset).

Problem 2. (10 points) Find the smallest relation (one with fewest elements) that contains the relation $\{(1,2), (1,4), (3,3), (4,1)\}$ that is:

a. Reflexive and transitive
b. Symmetric and transitive
c. Reflexive, symmetric and transitive