Assignment 2 - Continuity and Branches of Complex Functions
Due February 5th

1. Let $f(z) = z^2 + 1$. Draw the image of the following objects by f (show all your work):

   a) (0,5 pts) The straight line $\{a + i \,|\, a \in \mathbb{R}\}$.

   b) (0,5 pts) The straight line $\{1 + ib \,|\, b \in \mathbb{R}\}$.

   c) (1 pt) The region $\{a + ib \,|\, a, b \geq 1\}$.

2. (2 pts) Write out all branches of $f(z) = z^{\frac{1}{4}}$.

3. Let $f : \mathbb{C} \setminus \{0\} \to \mathbb{C} \setminus \{0\}$ defined by $f(a + ib) = a^b + ib^a$

   a) (0,5pts) Show that f is continuous (you may use without proof the fact that the real function $F(x, y) = x^y$ is continuous for $x, y \neq 0$).

   b) (0,5pts) Show that we cannot pick any value for $f(0)$ that would make $f : \mathbb{C} \to \mathbb{C}$ continuous.