MA 222 B EXAM #2: SOLUTIONS

SPRING 2016

(1.) FOUR EQUIVALENT WAYS TO SOLVE:

POISSON ARRIVALS $\Rightarrow$ EXPONENTIAL INTERARRIVAL TIMES

1. using $\lambda = .05 \frac{\text{arrivals}}{\text{sec}}$ and X = # arrivals in time t (seconds)

$P(X=0 \text{ in } 60 \text{ seconds}) = \frac{e^{-(.05)(60)}[(.05)(60)]^0}{0!} = .049 \text{ or } .05$

2. $\lambda = 3$ arrivals/minute X = # arrivals in time t

$P(X=0 \text{ in } 1 \text{ minute}) = \frac{e^{-3}3^0}{0!} = .049 \text{ or } .05$

3. In terms of exponential dist. and using seconds as time unit

T = time between arrivals $E(T) = 20 \text{ sec} \Rightarrow \lambda = \frac{1}{20} = .05$

so $P(T > \underset{\text{sec.}}{60}) = \int_{60}^{\infty} .05e^{-.05t}\,dt = 0 - [-e^{-.05(60)}] = .049 \text{ or } .05$

4. In terms of exponential dist and minutes as time unit

T = time betw. arrivals $E(T) = \frac{1}{3}$ minute $\Rightarrow \lambda = 3 \frac{\text{arrivals}}{\text{min.}}$

$P(T > 1 \text{ minute}) = \int_1^{\infty} 3e^{-3t}\,dt = 0 - [-e^{-3(1)}] = .049 \text{ or } .05$

(2.) $P(\text{at least 9 complete in a group}) = \binom{10}{9}(.8)^9(.2)^1 + \binom{10}{10}(.8)^{10}(.2)^0$

$= .376$

so P(that this happens for one group BUT NOT the other)

$= (.376)(.624) + (.624)(.376) = .469$

(3.) $f(x) = 2x;\ 0 \le x \le 1$ $E(X) = \int_0^1 x\,2x\,dx = \frac{2}{3}$ $E(X^2) = \int_0^1 x^2\,2x\,dx = \frac{1}{2}$

So: $V(X) = E(X^2) - [E(X)]^2 = (\frac{1}{2}) - (\frac{2}{3})^2 = \frac{1}{18} = .055$

$\therefore$ standard deviation of $X = \sqrt{.055} = .235$

$\therefore$ half a standard deviation $= .118$

$\therefore$ we seek

$P(\frac{2}{3} - .118 < X < \frac{2}{3} + .118) = P(.67 - .118 < X < .67 + .118)$

$= P(.55 < X < .79) = \int_{.55}^{.79} 2x\,dx = x^2 \Big|_{.55}^{.79} = .624 - .303$

$= .32$

SPRING 2016
MA222B
EXAM 2

(4.) P(all 3 dice are different) $= \frac{6}{6}\cdot\frac{5}{6}\cdot\frac{4}{6} = \frac{120}{216} = \frac{5}{9}$

Let X = # trials until all 3 dice are different

Then $X \sim geo\left(p = \frac{5}{9}\right)$ and $E(X) = \frac{1}{p} = \frac{9}{5}$ or 1.8

(5.) T = printer lifetime (time to failure) $\sim \exp(\lambda = .5)$

$\therefore$ P(fail in 1^{ST} year) $= \int_0^1 .5e^{-.5t}\,dt = -e^{-.5t}\Big|_0^1 = 1 - e^{-.5} \doteq .394$

P(fail in 2^{nd} year) $= \int_1^2 .5e^{-.5t}\,dt = -e^{-.5t}\Big|_1^2 = -e^{-1} + \underset{.606}{e^{-.5}} = .239$

$\therefore$ E(REFUND) $= 200(.394) + (100)(.239) \doteq$ \$102.70

(6.) Let X = the r.v.: the number of claims filed by a policyholder

Then $X \sim$ Poisson(λ)

where $P(X=2) = 3P(X=4)$

$$\frac{e^{-\lambda}\lambda^2}{2!} = 3\cdot\frac{e^{-\lambda}\lambda^4}{4!}$$

$$\frac{1}{2!} = \frac{3\lambda^2}{4!} \Rightarrow \lambda^2 = 4 \Rightarrow \lambda = 2$$