1 Propositional Logic

1.1 "Not", "And"

An argument is *valid* if its conclusion is true whenever all its premises are. In other words if there are no counterexamples.

Premise:	Min is not both home and on board	Not (A and B)
Premise:	She's home	A
Conclusion:	She's not on board	Not B

Check for counterexamples.

The second premise is true: A is true
The conclusion is false: B is true
Then (A and B) is true, so first premise is false
Thus there are no counterexamples

Or check validity by constructing a truth table.

A	B	Not	(	A and B	)	A	Not B
t	t	f	(	t	)	t	f
t	f	t	(	f	)	t	t
f	t	t	(	f	)	f	f
f	f	t	(	f	)	f	t

There is no row with the premises are all true and the conclusion false.

1.2 "Or"

Premise:	Min is home or Min is on board	A or B
Premise:	Henry is home or Min is home	C or A
Premise:	Min is not home	Not A
Conclusion:	?	?

Search for a valid conclusion; it must be true when all premises are.

For the third premise to be true, A must be false
For first premise to be true, B must be true
Likewise C must be true

Conclusion: Min is on board and Henry is home B and C

1.3 Exercise

Is the following argument valid?

> Min is home or on board. So is Henry. Neither is on board, so they are both home.

1.4 Exercise

Write Moriarty's argument symbolically and check it.

> MORIARTY: "She can't be home, because we know she's home or on board, and I've just learned that she's on board."
> THIN: "Do we know she doesn't *live* on board?"
> MORIARTY: "Oops."

In everyday language "or" can be either inclusive or exclusive. In propositional logic it is always inclusive: A or B is true if either A is true or B is true, or both are true. Moriarty's argument is invalid under the inclusive interpretation of "or".

A	B	A or B
t	t	t
t	f	t
f	t	t
f	f	f

1.5 Soundness

An argument is *sound* if it is valid and its premises are true in real life.

1.6 "If"

We agree that the following argument is valid.

$$\begin{array}{cc} \text{If Min works on board that tub, she is underpaid} & \text{If } A,\ B \\ \text{She works on board that tub} & A \\ \hline \text{She is underpaid} & B \end{array}$$

So if A is true, and B is false, then (If A,B) must be false. Thus we know one line in the truth table for If A, B.

A	B	If A, B
t	t	?
t	f	f
f	t	?
f	f	?

1.7 Denial, Conjunction, Disjunction

Terminology and notation.

A and B	$A \wedge B$	Conjunction
A or B	$A \vee B$	Disjunction
Not A	$\neg A$	Denial or negation

English has many ways of expressing these operations.

Min is home and on board	$A \wedge B$
Min is home, so is Henry	$A \wedge C$
Min is home, but Henry's on board	$A \wedge D$

English uses "but" and "both" to indicate grouping. Logical notation uses parentheses.

Min is not home but on board	$\neg A \wedge B$
Min is not both home and on board	$\neg(A \wedge B)$
Min is not both home and on board, she's at work	$\neg(A \wedge B) \wedge W$

Disjunction always means inclusive inclusive "or".

Min and Hen are home, or Min's on board	$(A \wedge C) \vee B$
Either Min's home and Hen's on board or Min's on board	$(A \wedge D) \vee B$

Exclusive "or":

Min is home or on board, but not both	$(A \vee B) \wedge \neg(A \wedge B)$

1.8 Conditionals

We write "If A,C" symbolically as "$A \to C$".

If Min is home, then Hen is	If A then C	$A \to C$
If Min is home, Hen is	If A, C	$A \to C$
Hen is home if Min is	C if A	$A \to C$
Min is home only if Hen is	C only if A	$A \to C$
Only if Hen is home is Min home	Only if C, A	$A \to C$

We interpret all the above sentences as saying that it cannot be the case that Min is home and Hen is not. Thus $A \to C$ has the same truth table as $\neg(A \wedge \neg C)$.

A	C	$A \to C$	$\neg$ (	A	$\wedge$	$\neg$	C)
t	t	t	t	t	f	f	t
t	f	f	f	t	t	t	f
f	t	t	t	f	f	f	t
f	f	t	t	f	f	t	f

We saw line two of this truth table before. The other two lines may seem strange, because in ordinary conversation we would not use the sentences above when we know for a fact that Min is not home or that Hen is.

1.9 Counterfactual Conditionals

1. If it is raining, then he is inside.
2. If it were raining, then he would be inside.

The first sentence is a truth functional conditional. The second is a so-called counterfactual conditional. Even it we know whether or not it's raining and whether or not he is inside, it's still not clear whether the second sentence is true. In other words the counterfactual conditional is not a truth function.

1.10 Biconditionals

A if and only if C $\quad A \longleftrightarrow C$

A	C	$A \longleftrightarrow C$
t	t	t
t	f	f
f	t	f
f	f	t

Two sentences which have the same truth values in all cases are *logically equivalent*. $A \longleftrightarrow C$ is logically equivalent to $(A \rightarrow C) \wedge (C \rightarrow A)$.

$A \rightarrow C$ is logically equivalent to $\neg(A \wedge \neg C)$, which is logically equivalent to $\neg A \vee C$.

1.11 Rules of Valuation

The truth tables for $\neg, \wedge, \vee, \rightarrow, \longleftrightarrow$ tell us how to compute the truth values of sentences.

Also for $A \wedge B \wedge C \cdots \wedge Z$ and likewise for $\vee$ and $\longleftrightarrow$

1.12 Oddities of "If"

$A \rightarrow C$ is logically equivalent to $\neg(A \wedge \neg C)$, which is logically equivalent to $\neg A \vee C$, but they these equivalences do not seem to hold up in everyday language.

Socrates is dead	B
If George sits down, then Socrates is dead	$A \to B$

It's not the case that if I break my leg today, I'll ski tomorrow	$\neg(A \to B)$
I'll break my leg today	A

If they withdraw if we advance, we'll win	$(A \to B) \to C$
We won't advance	$\neg A$
We'll win	C

1.13 Rules of Formation

Sentences are constructed as follows.

0. Capital letters with or without subscripts are sentences. We will call them just "letters".
1. If α is a sentence, so is $\neg\alpha$.
2. If $\alpha_1, \ldots, \alpha_n$ are sentences, so is $(\alpha_1 \wedge \alpha_2 \wedge \cdots \wedge \alpha_n)$, $n \geq 2$.
3. Likewise for $\vee$.
4. If α and β are sentences, so is $(\alpha \to \beta)$.
5. Likewise for $\longleftrightarrow$.

1.14 Consistency

A set of sentences is consistent or satisfiable if there is a case, i.e., a choice of "true" or "false" for each capital letter, such that every sentence in the set is true.

The set A, $\neg A$ is clearly not consistent, because A and $\neg A$ cannot both be true at the same time.

A set of sentences is not consistent is called inconsistent or contradictory or not satisfiable.

Recall that an argument is valid if there is no case in which the premises are all true and the conclusion false. That is the same as saying there is no case in which the premises are all true and the negation of the conclusion is also true. And *that* is the same as saying that the premises and the negation of the conclusion are inconsistent.

1.15 Tautologies

A tautology is a sentence which is true for all choices of "true" of "false" for the capital letters involved.

$A \vee \neg A$ is a tautology.

The argument

$$\begin{array}{c} P_1 \\ P_2 \\ \hline C \end{array}$$

is valid if and only if $(P_1 \wedge P_2) \to C$ is a tautology.

Tautology, consistency and validity of arguments are all related.

Homework

1. Interpret "E", "J" and "M" as meaning that the earth is the third planet from the sun, that Jupiter is, and that Mars is. Work out the truth values (in real life) of the following sentences.

   (a) $(M \wedge J) \vee E$ (b) $M \wedge (J \vee E)$
   (c) $\neg(M \vee J \vee E)$ (d) $\neg M \vee J \vee E$
   (e) $\neg(\neg M \vee \neg J \vee \neg E)$ (f) $\neg(\neg M \wedge \neg J \wedge \neg E)$

2. "Thin is guilty," observed Watson,"because (i) either Holmes is right and the vile Moriarty is guilty or he is wrong and the scurrilous Thin did the job; but (ii) those scoundrels are either both guilty or both innocent; and, as usual, (iii) Holmes is right."

   (a) Write down the argument symbolically. Use "T, M" for "Thin is guilty, Moriarty is guilty", "$\neg T, \neg M$" for innocence, and "H" for "Holmes is right."

   (b) Is the argument valid?

   (c) Are the premises and the conclusion consistent, i.e., is there a case in which they are all true?

   Solve (b) and (c) efficiently by looking for counterexamples and then solve them by truth tables.

3. Show that the following argument is valid. First argue efficiently; then use truth tables.

$$\begin{array}{l} A \vee B \\ A \to C \\ B \to C \\ \hline C \end{array}$$

4. Convert the following arguments to symbolic notation, and then find counterexamples if there are any.

   (a) Moriarty: "If Min is home, so is Henry." Thin: "Indeed, and if Min is home, Henry isn't." Moriarty: "Ah, I see. Min's not home."

   (b) "Min's home if Henry is, but he isn't, so she isn't."

   (c) "It's false that if Min is home, she's on board. Then if she's home, she's not on board."

   (d) "It's false that if Min is home, she's on board, because if she's home, she's not on board."

   (e) "Look, we know that Min is on board if Henry is home. Then she has to be on board if she's home, because Henry's home if she is."