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Assignment 5 - Solutions

1. a) $\sum_{n=2}^{\infty} \frac{(1+i)^n}{(2+i)^n} = \frac{(1+i)^2}{(2+i)^2} \sum_{n=0}^{\infty} \frac{(1+i)^n}{(2+i)^n} = \frac{(1+i)^2}{(2+i)^2} \frac{1}{1-\frac{(1+i)^n}{(2+i)^n}}$

   b) $\sum_{n=0}^{\infty} \frac{(1+i)^{n+2}}{(2+i)^n} = (1+i)^2 \sum_{n=0}^{\infty} \frac{(1+i)^n}{(2+i)^n} = (1+i)^2 \frac{1}{1-\frac{(1+i)^n}{(2+i)^n}}$

   c) $\sum_{n=2}^{\infty} \frac{(1+i)^n}{(2+i)^{n+2}} = \frac{1}{(2+i)^2} \sum_{n=2}^{\infty} \frac{(1+i)^n}{(2+i)^n} = \frac{(1+i)^2}{(2+i)^4} \sum_{n=0}^{\infty} \frac{(1+i)^n}{(2+i)^n} = \frac{(1+i)^2}{(2+i)^4} \frac{1}{1-\frac{(1+i)^n}{(2+i)^n}}$

2. a) Using the limit ratio test:

$$\lim_{n\to\infty} \frac{|z_{n+1}|}{|z_n|} = \lim_{n\to\infty} \frac{(n+1)^2|z|^{n+1}}{n^2|z|^n} = |z| \lim_{n\to\infty} \frac{(n+1)^2}{n^2} = |z| < 1 \Rightarrow \text{ convergent}$$

   b) Using the absolute convergence test:

$$\sum_{n=1}^{\infty} |z_n| = \sum_{n=1}^{\infty} \frac{1}{n^2} \text{ converges by integral test} \Rightarrow \text{ convergent}$$

   c) Using the limit ratio test:

$$\lim_{n\to\infty} \frac{|z_{n+1}|}{|z_n|} = \lim_{n\to\infty} \frac{|1+i|^{n+1}}{(n+1)!} \frac{n!}{|1+i|^n} = \lim_{n\to\infty} \frac{|1+i|}{n+1} = 0 < 1 \Rightarrow \text{ convergent}$$

   d) First notice that if $n = 2k$ we have $\frac{i^n}{n} = \frac{(-1)^k}{2k}$ and if $n = 2k+1$ we have $\frac{i^n}{n} = \frac{(-1)^k i}{2k+1}$ so we have $\sum_{n=1}^{\infty} \frac{i^n}{n} = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k} + i \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$. However, both $\sum_{k=0}^{\infty} \frac{(-1)^k}{2k}$ and $\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$ converge by the alternating series test, so our series converges.

3. a) Using the limit ratio test:

$$\lim_{n\to\infty} \frac{|z_{n+1}|}{|z_n|} = \lim_{n\to\infty} \frac{(n+1)^2|z|^{n+1}}{n^2|z|^n} = |z| \lim_{n\to\infty} \frac{(n+1)^2}{n^2} = |z| > 1 \Rightarrow \text{ divergent}$$

   b) (0.5 pts) $\sum_{n=1}^{\infty} \frac{i^n}{\cos n}$ Using the divergence test:

$$\lim_{n\to\infty} |z_n| = \lim_{n\to\infty} \frac{1}{\cos n}, \text{ DNE} \Rightarrow \text{ divergent}$$