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55 minutes to complete. Closed book. No cooperation. No electronic communication. Calculators are allowed but all your solutions should be easily verifiable without one.

There are 33 points in this paper. To get a full mark, you need to score 30 points or more. Please write your answers on these question sheets in the space provided. If you run out of space, use extra paper.

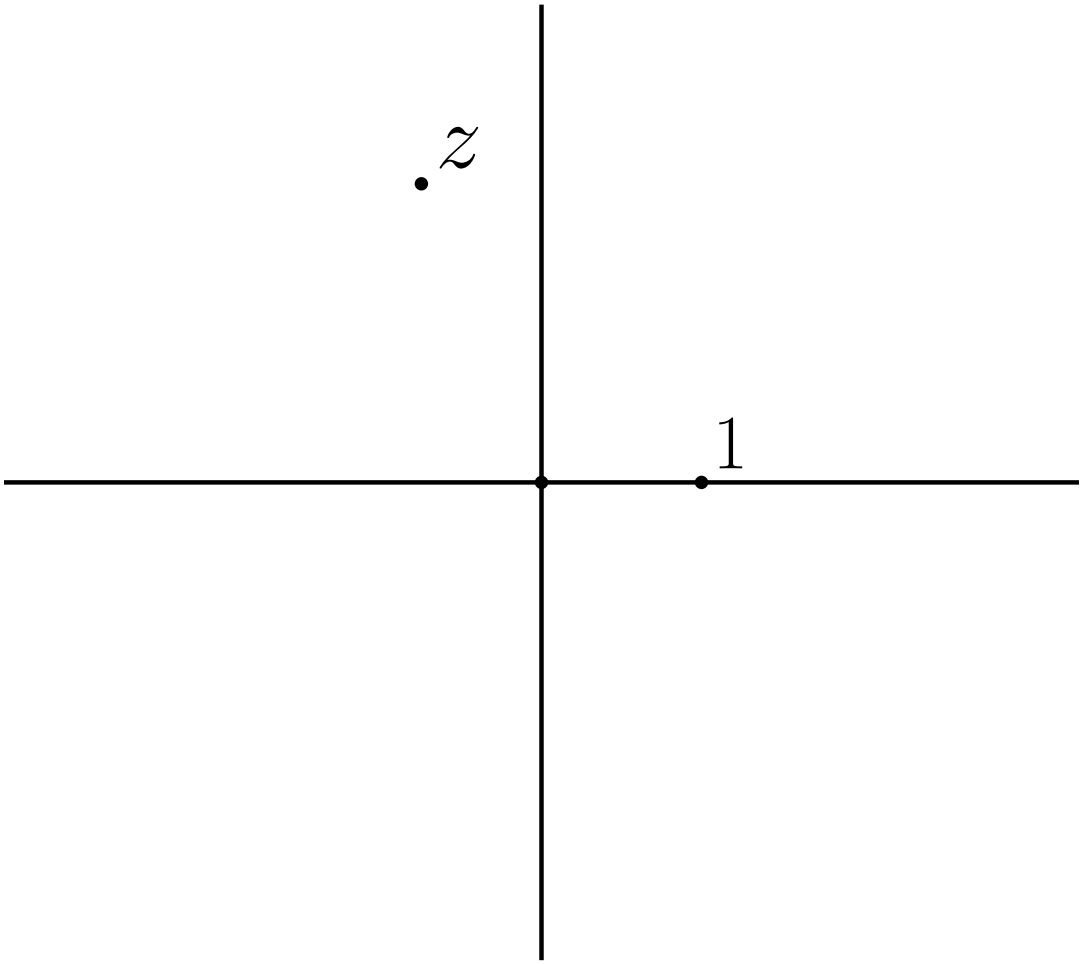
Name:

Pledge:

Please write your name and pledge before turning the page.

A	B1	B2	B3	B4	B5	B6	C1	C2	C3	$\sum$

Part A. [6 points] For the complex number z shown in the figure below, depict the following in the figure:
$\bar{z}$, $-z$, iz, $\frac{1}{z}$, z^2, both square roots of z.



Part B. In this part, only provide answers. Each question is worth **2 points**.

(B1) [2pt] Find $\arg\left((\cos 2 + i \sin 2)(\cos 8 - i \sin 8)\right)$. (Answer only.)

(B2) [2pt] Find center and radius of the circle $|z - 2 + 3i| = 16$. (Answer only.)

(B3) [2pt] Which of the following five complex functions are one-to-one? (Answer only.)

$f(z) = z^{-2}$, $g(z) = z^{-1}$, $h(z) = 1$, $j(z) = z$, $k(z) = z^2$.

(B4) [2pt] Which of the following five subsets of $\mathbb{C}$ are open? (Answer only.)

$A = \{3\}$, $B = \{z \in \mathbb{C} : |z - 3| < 3\}$, $C = \{z \in \mathbb{C} : |z - 3| \leq 3\}$,

$D = \{z \in \mathbb{C} : |z - 3| = 3\}$, $E = \{z \in \mathbb{C} : |z - 3| > 3\}$.

(B5) [2pt] Let $\sqrt{\ }$ be the principal square root function. Give an example of a complex number z such that $\sqrt{z^2} \neq z$. (Answer only.)

(B6) [2pt] Find the derivative $((z^2 + z^{-1} + i)^{2015})'$. (Answer only.)

Part C. In this part, show your work and provide explanations. Each question is worth **5 points**.

(C1) [5pt] Find and sketch image of the region $\{x+iy : 0 < x < 2, -1 < y < 0\}$ under the mapping $f(z) = z^2$.

(C2) [5pt] Find and sketch image of the region $\{x+iy : 0 < x < 2, -1 < y < 0\}$ under the mapping $f(z) = z^{-1}$.

(C3) [5pt] Find a real number A such that the function

$$f(z) = f(x+iy) = -y^3 + Ax^2y + i(Axy^2 - x^3)$$

is complex differentiable everywhere on $\mathbb{C}$.
(Question for 1 extra point: express f as a function of z.)