Homework #3: 1.5: 4, 8(a-d), 12(a-f), 1.6: 4, 8 24, 1.7: 10

I pledge my honor that I have abided by the Stevens honor system.

1.5:

4.) Let P(x, y) be the statement “Student x has taken class y,” where x consists of students in your class and y consists of all computer science classes.

a. ∃x∃y P(x, y):

* There is a student in your class who has taken a computer science class.

b. ∃x∀y P(x, y):

* There is a student in your class who has taken all computer science classes.

c. ∀x∃y P(x, y):

* Every student in your class has taken a computer science class.

d. ∃y∀x P(x, y):

* Every student in your class had to take the same computer science class.

e. ∀y∃x P(x, y):

* For all computer science classes, a student in your class has taken it.

f. ∀x∀y P(x, y):

* Every student in your class has taken every computer science class.

8.) Let Q(x, y) be the statement “student x has been a contestant on quiz show y.”

a. There is a student at your school who has been a contestant on a quiz show:

* ∃x∃y Q(x, y)

b. No student at your school has ever been a contestant on a quiz show:

* ¬∃x∃y Q(x, y)

c. There is a student at your school who has been a contestant on Jeopardy and Wheel of Fortune.

* ∃x (Q(x, Jeopardy)∧Q(x, Wheel of Fortune))

d. Every quiz show has had a student from your school as a contestant.

* ∀y∃x Q(x, y)

12. Let I(x) be the statement “x has an internet connection” and C(x, y) be the statement “x and y have chatted over the Internet,” where the domain for x and y consists of students in your class.

a. Jerry does not have an Internet connection.

* ¬I(Jerry)

b. Rachel has not chatted with Chelsea.

* ¬C(Rachel, Chelsea)

c. Jan and Sharon have never chatted online.

* ¬C(Jan, Sharon)

d. No one in the class has chatted with Bob.

* ¬∃x C(x, Bob)

e. Sanjay has chatted with everyone except Joseph.

* ∀y: y ≠ Joseph, C(Sanjay, y) ∧ ¬C(Sanjay, Joseph)

f. Someone in your class does not have an Internet connection.

* ∃x I(x)

1.6:

4. What rule of inference was used in each?

a. Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

* Simplification: p∧q→q

b. It is either hotter that 100° today or the pollution is dangerous. It is less than 100° outside today. Therefore, the pollution is dangerous.

* Disjunctive Syllogism: ((p∨q)∧¬p)→q

c. Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

* Modus Ponens: (p∧(p→q))→q

d. Steve will work at a computer company this summer. Therefore, this summer, Steve will work at a computer company or he will be a beach bum.

* Addition: p→(p∨q)

e. If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

* Hypothetical Syllogism: ((p→q)∧(q→r))→(p→r)

8. What rules of inference are used in this argument? “No man is an island. Manhattan is an island. Therefore, Manhattan is not a man.”

* Modus Tollens

24. Identify the error or errors in this argument that supposedly shows that if ∀x (P(x) ∨ Q(x)) is true then ∀x P(x) ∨ ∀x Q(x) is true.

1. ∀x (P(x) ∨ Q(x)) Premise
2. P(c) ∨ Q(c) Universal instantiation from (1)
3. P(c) Simplification from (2)
4. ∀x P(x) Universal generalization from (3)
5. Q(c) Simplification from (2)
6. ∀x Q(x) Universal generalization from (5)
7. ∀x (P(x) ∨ ∀x Q(x)) Conjunction from (4) and (6)
   * Step 3 and 5 → not simplification, its addition

1.7:

10. Use a direct proof to show that the product of two rational numbers is rational.

**STEP REASON**

1. r and s are rational numbers Premise

2. ∃(a, b, c, d) ∈ Z Definition of rational numbers

3. r = a/b, b ≠ 0 Substitute a/b for r

4. s = c/d, d ≠ 0 Substitute c/d for s

5. r\*s = (a\*c)/(b\*d) Multiply (3) and (4)

6. x = r\*s Assign x to r\*s

7. x is rational b\*d ≠ 0, and a\*c ∈ Z, therefore, rational