CS 135 Spring 2018: Problem Set 1 Solutions.

**Problem 1.** (10 points) Use the laws of propositional logic and the rules of inference for the following. Be sure to show every step of your proof.

1. Prove that the propositions and are equivalent.
2. Either prove that the following argument is valid, or else give a counterexample.

Premise 1: If Superman were able and willing to prevent evil, he would do so.

Premise 2: If Superman were unable to prevent evil, he would be impotent.

Premise 3: If Superman were unwilling to prevent evil, he would be malevolent.

Premise 4: Superman does not prevent evil.

Premise 5: If Superman exists, he is neither malevolent nor impotent.

Conclusion: Therefore, Superman does not exist.

First, to express the arguments in symbolic form, we define the following symbols:

The argument can now be represented as follows:

P1: P2: P3: P4: P5: C:

Here is a proof that the argument is valid:

1. Hypothesis, P1
2. Contrapositive, 1.
3. Hypothesis, P4.
4. Modus ponens, 2,3.
5. De Morgan’s Law, 4.
6. Identity Laws, 5. (applied twice) T is the value True.
7. Hypotheses P2,P3 are each True.
8. Modus ponens, 7 (applied twice).
9. Hypothesis, P5.
10. Contrapositive, 9.
11. De Morgan’s Law, 10.
12. Commutative Law, 11.
13. Modus Ponens, 8,12.

Note that in Step 7 we replaced T with equivalent propositions – hypotheses, each of which is assumed true.

**Problem 2.** (10 points)

It recently came to light that a certain cabal within the CS 135 CAs is plotting to make the problem sets ridiculously hard and demanding that the answers be written in ancient Egyptian hieroglyphics. This cabal will not be stopped unless you determine who is in the cabal and who is not.

Here are the facts: There are nine CAs: Ed, Cassidy, David, Jared, Jo, Katie, Ryan, Sam and Yifan. A membership roster has been found and appears below, but has been deviously encrypted in logic notation. The predicate indicates membership in the cabal; is True if and only if is in the cabal.

Translate each statement below into English and deduce who is in the cabal, explaining every step in the process.

Three distinct people are in the cabal. Note that this means the cabal has at least three members – but, there could be more. The proposition does not rule out the possibility that there are four, or more, members in the cabal.

Ed and Yifan are not both in the cabal. Equivalently, at least one of them is not in the cabal.

If either Sam or Cassidy is in the cabal then everyone is in the cabal.

If Ed is in the cabal then Yifan is in the cabal.

If Ryan is in the cabal then Sam is in the cabal.

If either Jared or Jo is in the cabal then Katie is not in the cabal.

If either Jared or Yifan is in the cabal then David is not in the cabal.

Next, we deduce cabal membership as follows:

D1. From 2 we infer that at least one person (Ed or Yifan) is not in the cabal.

D2. From D1, 3 we infer that neither Sam nor Cassidy is in the cabal (otherwise, everyone is in the cabal, contradicting D1).

D3. From D1, 4 we infer that Ed is not in the cabal and that Yifan is in the cabal.

D4. From D2, and the contrapositive of 5 we infer that Ryan is not in the cabal.

D5. From 7 and D3 we infer that David is not in the cabal.

So far, we have established that Yifan is in the cabal, and that Sam, Cassidy, Ryan, David and Ed are not in the cabal. From 1 we know that the cabal has at least 3 members. Therefore, of the remaining three people (Katie, Jared and Jo) at least 2 must be in the cabal.

D6. From 6 and the preceding remark, we infer that Katie is not in the cabal (if she were in the cabal, then neither Jared nor Jo would be members, leaving the cabal with only two members, in contradiction of 1).

D7. Jared and Jo are in the cabal, since the cabal has at least three members.

Therefore, Yifan, Jared and Jo are the members of the cabal!