Problem Set 3:

1. Problem 1:
   1. Z-> N
   2. If and are onto, does it follow that is also onto? Prove or give counter example.

Assume g is NOT onto:

is not defined everywhere.

The domain of is .

would not have a complete domain

so is NOT onto \*\*\* contradiction

is onto.

* 1. , ,
     1. If h is surjective, then f must be surjective.

For some c ∈ C, there is some a ∈ A such that h(a) = c.

Let b = g(a). 🡪 b ∈ B and f(b)=f(g(a))=c

f(b)=c, therefore f is surjective.

* + 1. If h is surjective, g is surjective.

Let A={1}, B={1,2}, C={1}.

g(1) = 1, f(1)=f(2)=1.

g cannot map to 2, so g is NOT surjective.

* + 1. If h is injective, then f is injective.

Let A={1}, B={1,2}, C={1}

g(1)=1, f(1)=f(2)=1.

h is injective, but f is not.

* + 1. If h is injective, and f is total, then g is injective.

a,b ∈ A, a!=b

if h(a) != h(b),

🡪 f(g(a)) != f(g(b))

🡪g(a) != g(b)

therefore, g is injective.

1. Use well ordering principle to prove following argument:

Assume:

Let S be the set of n where P(n) is not true.

The well ordering principle states that S has to have a least element.

This element cannot be n = 0, because P(0) is true.

It cannot be n = 1, because of statement 2, any k in such that P(k) is true implies P(k+1) is also true. 🡪

This is a contradiction, therefore