- Bufferless case

1. $W > C \cdot RTT_{min} + B$

2. $C RTT_{min}$

Window increases by $\frac{1}{W}$ on every ACK. It gradually increases towards BDP. When it surpasses BDP by a small amount, packet loss will be detected and multiplicative decrease gets triggered. Therefore, we set $C RTT_{min}$ to be the time at which packet loss happens.

3. $\frac{1}{2} C RTT_{min}$

4. $\frac{1}{2}\left[C RTT_{min}\left(1+\frac{1}{2}\right)\right] = \frac{3}{4} C RTT_{min}$

5. $\text{throughput} = \frac{W}{RTT_{min}} = \frac{3}{4} C$

6. $\text{utilization} = \text{throughput} / \text{capacity} = \frac{3}{4}$

- Non-zero Buffer

7. $$\text{throughput} = \begin{cases} \frac{W}{RTT_{min}}, & W \le C \cdot RTT_{min} \\ C, & C \cdot RTT_{min} < W \le C RTT_{min} + B \end{cases}$$

When $W \le C \cdot RTT_{min}$, the throughput is $\frac{W}{RTT_{min}}$ which is derived in class. In the other case, the queue starts to fill up, and throughput is the same as capacity C.

8. W, C, RTT_{min}, W

9. $W \le C \cdot RTT_{min}$ (first) $\qquad W > C \cdot RTT_{min}$ (second)

$\uparrow$ BDP $\qquad \uparrow$ Starts to fill up queue.

10 W increases by $\frac{1}{W}$ on every ACK. Pack loss happens when W surpasses $C \cdot RTT_{min} + B$.

$\Rightarrow W = C \cdot RTT_{min} + B$.

11 $C \cdot RTT_{min} - \frac{1}{2}(C \cdot RTT_{min} + B) = \left(\frac{1}{2} C \cdot RTT_{min} - \frac{B}{2}\right)$ units of time

12 We know that in the second phase. W increases from $C \cdot RTT_{min}$ to $C \cdot RTT_{min} + B$. W increases by $\frac{1}{W}$ on every ACK.

t_{second} is basically the time required for W to increase from B to $BDP + B$. With large W, we can approximate this sum as integral

$$t_{second} = \int_0^B RTT_{min} + \frac{x}{C}\, dx$$

$$= \left[RTT x + \frac{x^2}{2C} \right]_0^B = RTT_{min} \cdot B + \frac{B^2}{2C}$$

13 $t_{first} + t_{second} = \frac{1-r}{2} C RTT_{min}^2 + B RTT_{min} + \frac{B^2}{2C}$

average window size during $t_{first} = \frac{1}{2}\left[BDP + \frac{1+r}{2} BDP \right]$

$$= \frac{3+r}{4} BDP$$

average throughput during $t_{first} = \frac{3+r}{4} \frac{BDP}{RTT_{min}}$

$$\Rightarrow \text{average throughput} = \frac{3+r}{4} C \frac{\frac{1-r}{2} C RTT_{min}^2}{\frac{1-r}{2} C RTT_{min}^2 + B RTT_{min} + \frac{B^2}{2C}}$$

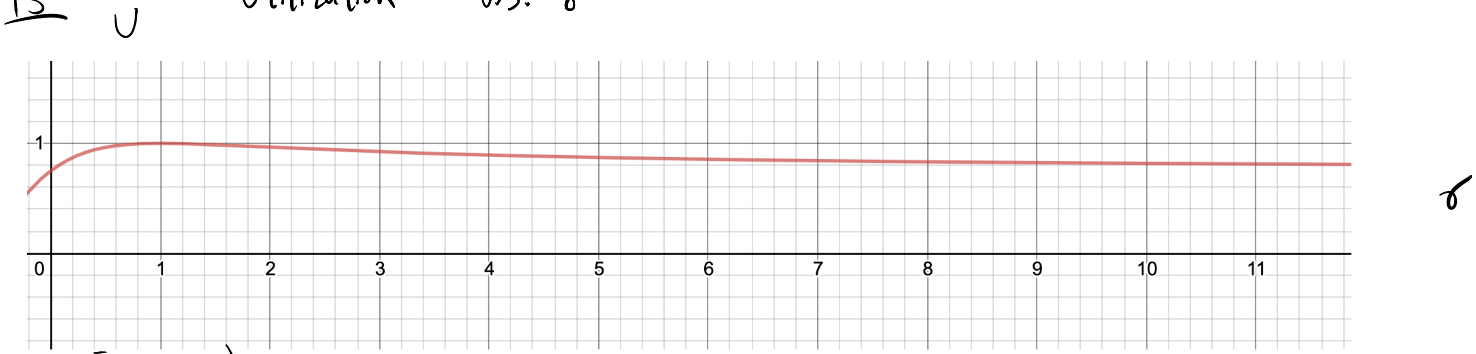
$$+ C \frac{B RTT_{min} + \frac{B^2}{2C}}{\frac{1-r}{2} C RTT_{min}^2 + B RTT_{min} + \frac{B^2}{2C}}$$

$$= C \frac{4 C^2 R^2 r^2 + 8 C^2 R^2 r - C^2 (r^2 + 2r - 3) R^2}{4\left[C^2 R^2 r^2 + 2r C^2 R^2 - C^2 R^2 (r-1) \right]}$$

$$= C \frac{3r^2 + 6r + 3}{4r^2 + 4r + 4}$$

14 utilization $= \text{throughput} / C = \frac{3r^2 + 6r + 3}{4r^2 + 4r + 4}$

15 Utilization v.s. γ

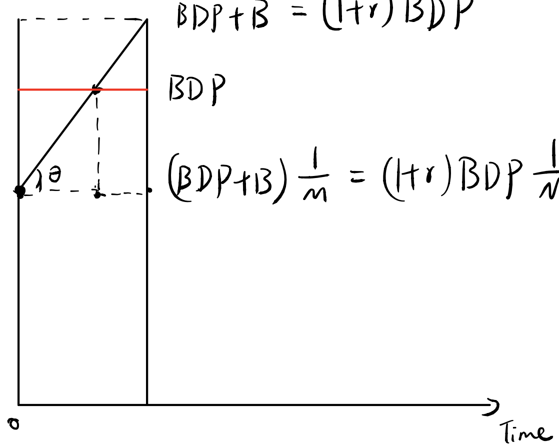


$\gamma \in [0, \infty)$

16 $\gamma = 1 \Rightarrow \text{Utilization} = \frac{12}{12} = 100\%$

$\Rightarrow B = C \cdot RTT_{min}$

17



$BDP + B = (1+\gamma) BDP$

BDP

$(BDP+B)\frac{1}{M} = (1+\gamma) BDP \frac{1}{M}$

$\tan\theta = A$

$$t_{first} = \left(BDP - (1+\gamma) BDP \frac{1}{M}\right)\frac{1}{A} = \frac{M-1-\gamma}{AM} C RTT_{min}^2$$

$$t_{second} = \left[B RTT_{min} + \frac{2B^2}{C}\right]\frac{1}{A}$$

average window size during $t_{first} = \frac{1}{2}\left[BDP + (1+\gamma) BDP \frac{1}{M}\right] = \frac{M+1+\gamma}{2M} BDP$

$$\Rightarrow \text{average throughput} = \frac{M+1+r}{2M} C \cdot \frac{\frac{M-1-r}{AM} CRTT_{min}^2}{\frac{M-1-r}{AM} CRTT_{min}^2 + \left(BRTT_{min} + \frac{2B^2}{C}\right)}$$

$$+ C \frac{\left(BRTT_{min} + \frac{2B^2}{C}\right)\frac{1}{A}}{\frac{M-1-r}{AM} CRTT_{min}^2 + \left(BRTT_{min} + \frac{2B^2}{C}\right)\frac{1}{A}}$$

utilization = throughput / C

18. $U(0) = \frac{M^2 - 1}{2M(M-1)} = \frac{M+1}{2M} = \frac{1}{2} + \frac{1}{2M}$

With $B = 0$, utilization goes down as a function of M.

since it's independent of A and U goes down as M increases

With large M, the window size decreases sharply after surpassing $C \cdot RTT_{min}$, And small window size means underutilization.

19. $U(r) = 1 \Rightarrow M^2(2r+1) - (r+1)^2 = 2(M-1)M(r+1)$

$\Rightarrow r = M-1 \quad \Rightarrow B = (M-1)\,BDP$

It goes up as a function of M.

W_{max} in this case would be $C \cdot RTT_{min} + B$. Similar to the analysis above in Q18, we need to address the issue of having a very small window size after multiplicative decrease, in order to increase utilization. We can achieve that by increasing size of buffer.

Numerically,

$$BDP + B = (M-1+1)\,BDP = M\,BDP \xrightarrow{\text{decrease}} \frac{1}{M} M\,BDP = BDP$$

A