Effect of buffering on AIMD throughput (50 points)

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In this assignment, we will calculate mathematically, under some assumptions, the effect of the router's buffer size (the queue size limit from last assignment) on the throughput of the AIMD strategy for congestion avoidance. Such calculations are practically important because they influence how small the router's buffer can be without compromising throughput.

1 Instructions

1. You can turn in the answer either as a LaTeX PDF or a scanned copy of handwritten sheets. If you do turn it in as a scanned copy, please make sure your handwriting is legible.
2. This assignment involves a fair amount of algebraic simplifications. Feel free to use a tool like Wolfram Alpha (https://www.wolframalpha.com/) if it helps you simplify expressions. However, please include the salient steps of the simplification process so that we know how you got from one step to the next.
3. RTT_{min} refers to the minimum round trip time, while RTT refers to the total round trip time including queueing delay.

2 Assumptions

Here is a list of assumptions we'll be making when computing AIMD throughput as a function of the buffer size B.

1. We will ignore the slow start phase of the AIMD protocol. In other words, we will only compute the protocol's throughput under the assumption that the protocol is always performing congestion avoidance.
2. We will assume a topology with one sender, one link, and one receiver. There is a single flow from the sender to the receiver, which is trying to use the single link (i.e., the bottleneck) to the fullest.
3. We will assume that the window size in congestion avoidance follows the sawtooth pattern: it climbs up from a particular lower window W_{low} to twice that value $2 * W_{low}$ through additive increase, then suffers a loss, falls back down to W_{low} through multiplicative decrease, and this process then repeats itself.
4. Although there is a packet loss at the end of every sawtooth period, it is OK to include these lost packets in your throughput calculation because the error in doing so is quite small because the frequency of lost packets is quite low.
5. Similarly, it is OK to assume that the window decrease from multiplicative decrease is instantaneous even though it takes around an RTT in practice. This is because the duration of a sawtooth period is much larger than one RTT.

3 Bufferless case ($B = 0$)

We will first calculate AIMD throughput when the buffer size B is zero because this is simpler and will give you good intuition for the more complicated case when B is non-zero.

1. (2 points) When does packet loss happen? If the window size at a particular time is W, the link's capacity is C, the minimum round-trip time is RTT_{min}, and the buffer size is B, what is the condition at which there is a packet loss? This condition will take the form of an inequality involving all four variables: W, B, C, and RTT_{min}.
2. (2 points) If B is zero, what is the window size W at which packet loss happens?
3. (2 points) Continuing with the case above, what is the window size right after the packet loss has been detected?
4. (2 points) Combining the above two answers and the fact that window size increases additively at the rate of 1 every RTT_{min}, what is the average window size during one period of the sawtooth?
5. (2 points) What is the average throughput based on the average window size?
6. (2 points) What is the link utilization based on the average throughput you just computed above?

4 Non-zero buffer ($B > 0$)

The average throughput and link utilization calculation here are more complicated than the bufferless case. In particular, there are 2 different regimes in each period of the sawtooth pattern. Each regime leads to a different average throughput calculation. The final average throughput is then a weighted average of the average throughputs in each of the regimes, weighted by the amount of time spent in each regime.

7. (2 points) Given a window size W, the minimum RTT of RTT_{min} and link capacity of C, what is the most general expression for throughput? Why?
8. (2 points) Recall that RTT_{min} and C are both constant, while W is variable. The two regimes correspond to two different ranges of values for W. Based on the expression above, fill in the blanks. In the first regime, throughput increases linearly with ———, while in the second regime, throughput is constant and equals ——-. Similarly, in the first regime, total round-trip latency[1] is constant and equals ——, while in the second regime it increases linearly with ——-.
9. (2 points) Identify the crossover point between the two regimes, i.e., when do you transition from the first regime to the second? This should be in the form of an equation that involves RTT_{min}, W, and C, but not B.
10. (2 points) What is the window size W at which packet loss happens with a non-zero buffer size of B?
11. (2 points) Based on the fact that the AIMD protocol performs additive increase (i.e., 1 packet every round-trip time), what is the amount of time it spends in the first regime within each period of the sawtooth pattern?
12. (5 points) Similarly, what is the amount of time it spends in the second regime within each period of the sawtooth pattern? This calculation is a bit more subtle because you have take into account the fact that the total RTT (including queueing delay) is no longer constant, but actually increasing itself. In other words, you are still doing additive increase where there is an increase of 1 every RTT, but the RTT is no longer constantly equal to RTT_{min}. If you can't solve exactly for this time, we'll give you partial points

[1] RTT_{min} + queueing delay

if you show us your reasoning, and you can move on to the next question by assuming an answer t_{second} for this question. However, you will only be able to get partial points for the remaining questions as well because fully solving them relies on having the correct answer for t_{second}. Also, if it helps with the algebraic simplification, you can assume $B >> 1$.

13. (3 points) Based on the time fractions above and the throughputs calculated for each regime before, what is the average throughput? You might find it easier to do the algebra if you substitute the variable $r = \frac{B}{C*RTT_{min}}$.

14. (1 point) What is the link utilization? You can sanity check the answer to your previous question by plugging in $B = 0$ in your expression for the link utilization and checking it against the link utilization you previously computed for the bufferless case.

5 Putting things together.

15. (2 points) Make a plot of the link utilization as a function of the buffer size using a plotting tool of your choice. You might find it easier to plot the link utilization as function of r, where $r = \frac{B}{C*RTT_{min}}$, which should give you the same shape as plotting aginst B itself because both C and RTT_{min} are constant.

16. (2 points) What is the buffer size that gives 100% utilization?

6 Generalizing this to other variants of AIMD.

To make this much more general, and reinforce the same concept, consider a case where the additive increase is by a general amount A, while the multiplicative decrease is by a general factor M. Standard AIMD as we know it so far sets A to 1 and M to 2, so that the window increases by 1 every RTT, while the window decreases by a factor of 2 during every loss.

17. (10 points) What is the throughput and link utilization of this generalized AIMD protocol as a function of C, RTT_{min}, B, A, and M?

18. (2 points) What is the throughput at zero buffer size? Does it go up or down as a function of A and M? Why?

19. (3 points) What is the buffer size to achieve 100% link utilization? Does it go up or down as a function of A and M? Why?