Antiderivative Problems

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0.1 $\int xsin(2x)dx$

$\int xsin(2x)dx \Rightarrow u = x,\ dv = sin2xdx\ \Rightarrow du = 1dx,\ v = -0.5cos(2x)$
$\int xsin(2x)dx = -x/2cos(2x) + sin(2x)/4 + c$

0.2 $\int xe^{x^2}dx$

$\int xe^{x^2}dx \Rightarrow u = x^2$
$\Rightarrow 0.5\int e^u du = 0.5e^{x^2} + c$

0.3 $\int xe^x dx$

$\int xe^x dx \Rightarrow u = x,\ dv = e^x dx \Rightarrow du = 1,\ v = e^x$
$\Rightarrow xe^x - \int e^x dx$
$= xe^x - e^x + c$

0.4 $\int e^{x^2}dx$

$f(x) = e^{x^2},\ f(0) = 1$
$f'(x) = 2xe^{x^2},\ f'(0) = 0$
$f''(x) = 2e^{x^2} + 4x^2e^{x^2},\ f''(0) = 2$
$f'''(x) = 12xe^{x^2} + 8x^3e^{x^2},\ f'''(0) = 0$
$f^{(4)}(x) = 12e^{x^2} + 48x^2e^{x^2} + 16x^4e^{x^2},\ f^{(4)}(0) = 12$
$\Rightarrow 1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6}$
In general: $e^{x^2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$
$\int e^{x^2} \approx \int 1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6}dx$
$= x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42}$

0.5 $\int(x\sqrt{1+x})dx$

$\int x\sqrt{1+x}dx \Rightarrow u = 1 + x,\ du = dx$
$\Rightarrow \int((u-1)\sqrt{u})du$
$= \int u^{3/2} - u^{1/2}du$
$= 2u^{5/2}/5 - 2u^{3/2}/3$
$= \frac{2}{5}(x-1)^{5/2} - \frac{2}{3}(x-1)^{3/2}$

0.6 $\int sec\theta\ d\theta$

$= \int \frac{sec^2\theta + sec\theta tan\theta}{sec\theta + tan\theta} d\theta \Rightarrow u = sec\theta + tan\theta,\ du = (sec^2\theta + sec\theta tan\theta)d\theta$
$= \int \frac{1}{u} du$
$= ln(sec\theta + tan\theta) + c$

0.7 $\int sec^2(\theta) d\theta$

=tan(x)+c

0.8 $\int sech^2\theta d\theta$

$= tanh\theta$

0.9 $\int \frac{x^2+2}{7-x^2}$

Using long division we find:
$\int (\frac{9}{7-x^2} - 1)dx$
Focusing on the first integral...
$9 \int \frac{1}{7-x^2} dx$
$= \frac{9}{7} \int \frac{1}{1-\frac{x^2}{7}} dx$
$\Rightarrow u = \frac{x}{\sqrt{7}},\ du = \frac{1}{\sqrt{7}}$
$\Rightarrow \frac{9}{\sqrt{7}} \int \frac{1}{1-u^2} du$
$= \frac{9}{\sqrt{7}} tanh^{-1} u$
$= \frac{9}{\sqrt{7}} tanh(\frac{x}{\sqrt{7}})$
$\Rightarrow \int (\frac{9}{7-x^2} - 1)dx = \frac{9}{\sqrt{7}} tanh(\frac{x}{\sqrt{7}}) - x + c$

0.10 $\int \frac{1}{ap-bp^2} dp$

Complete the square
$\int \frac{1}{\frac{a^2}{4b} - (\sqrt{b}p - \frac{a}{2\sqrt{b}})^2} dp \Rightarrow u = \sqrt{b}p - \frac{a}{2\sqrt{b}},\ du = \sqrt{b}dp$
$\Rightarrow \frac{1}{\sqrt{b}} \int \frac{1}{\frac{a^2}{4b} - u^2} du$
$= \frac{1}{\sqrt{b}} \int \frac{4b}{a^2(1-\frac{4bu^2}{a^2})} du$
$= \frac{4\sqrt{b}}{a^2} \int \frac{1}{1-\frac{4bu^2}{a^2}} du$
$s = \frac{2i\sqrt{b}u}{a},\ ds = \frac{2i\sqrt{b}}{a} du$
$\Rightarrow \frac{-2i}{a} \int \frac{1}{s^2+1} ds$
$= \frac{-2}{a} tanh^{-1}(1 - \frac{2bp}{a}) + c$