Homework 2 Problem 3

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a

$\frac{df}{dx} \approx \frac{\sqrt{x+h}-\sqrt{x}}{h}$
$\Rightarrow \frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$
$\Rightarrow \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$
$\Rightarrow \frac{1}{\sqrt{x+h}+\sqrt{x}}$

From the images below, we see that the conjugate derivate approximation is more stable than the regular derivative approximation used. At 2^{-53} the absolute error is 0.25. The standard derivative approximation $\frac{\sqrt{4+2^{-53}}-\sqrt{4}}{2^-53}$ is evaluated as $\frac{2.0-2}{2^{-53}} = 0$, the absolute error then becomes $|0 - 0.25| = 0.25$.
When we use the conjugate approximation we don't have this issue. $\frac{1}{\sqrt{4+2^{-53}}+\sqrt{4}} = \frac{1}{2.0+2} = 0.25$ We are able to use smaller and smaller h values of base 2, with no eventual added error (at least in the range of $[0.5, 2^{-100}]$

b

Using the 4th order taylor series expansion $x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$ and the derivative approximation $f'(x) = \frac{f(x+h)-f(x)}{h}$ we get the absolute error graph shown below. We see that as h trends towards 0, the absolute error becomes 0. Computationally the difference between the initial sin(x) expression and the taylor series expansion is zero.

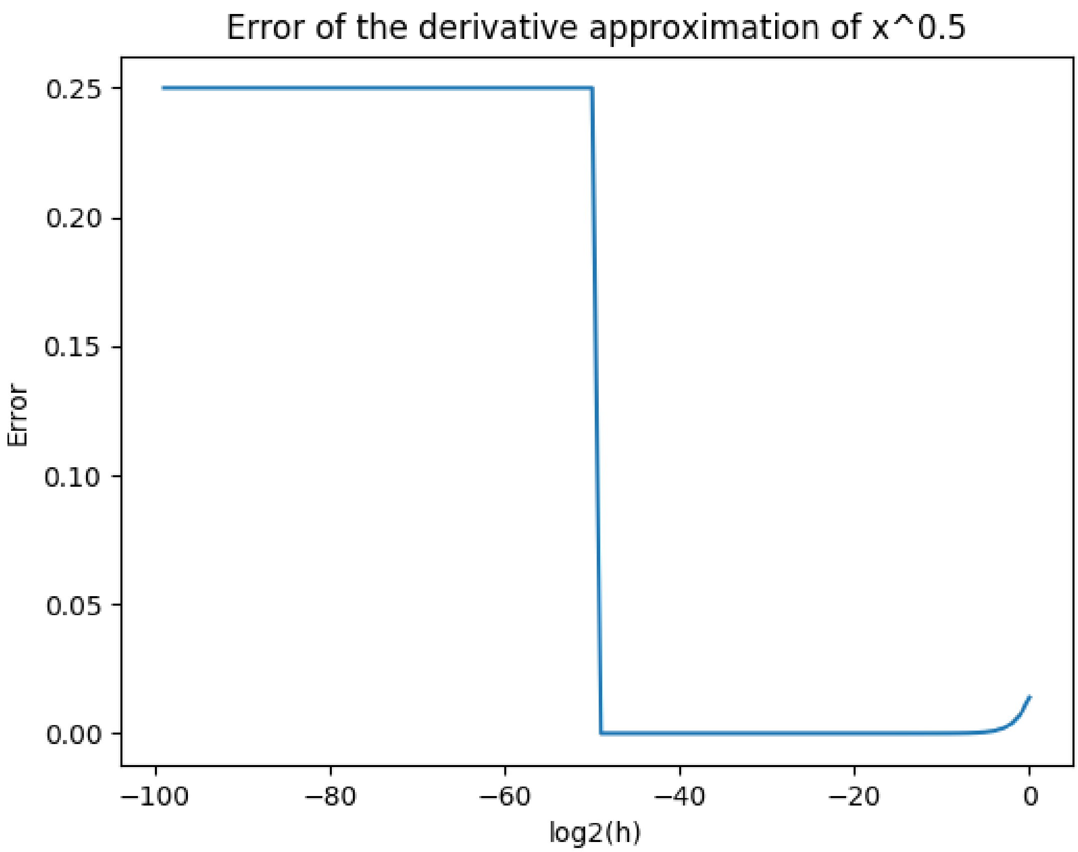

Figure 1: Absolute Error using regular derivative approximation

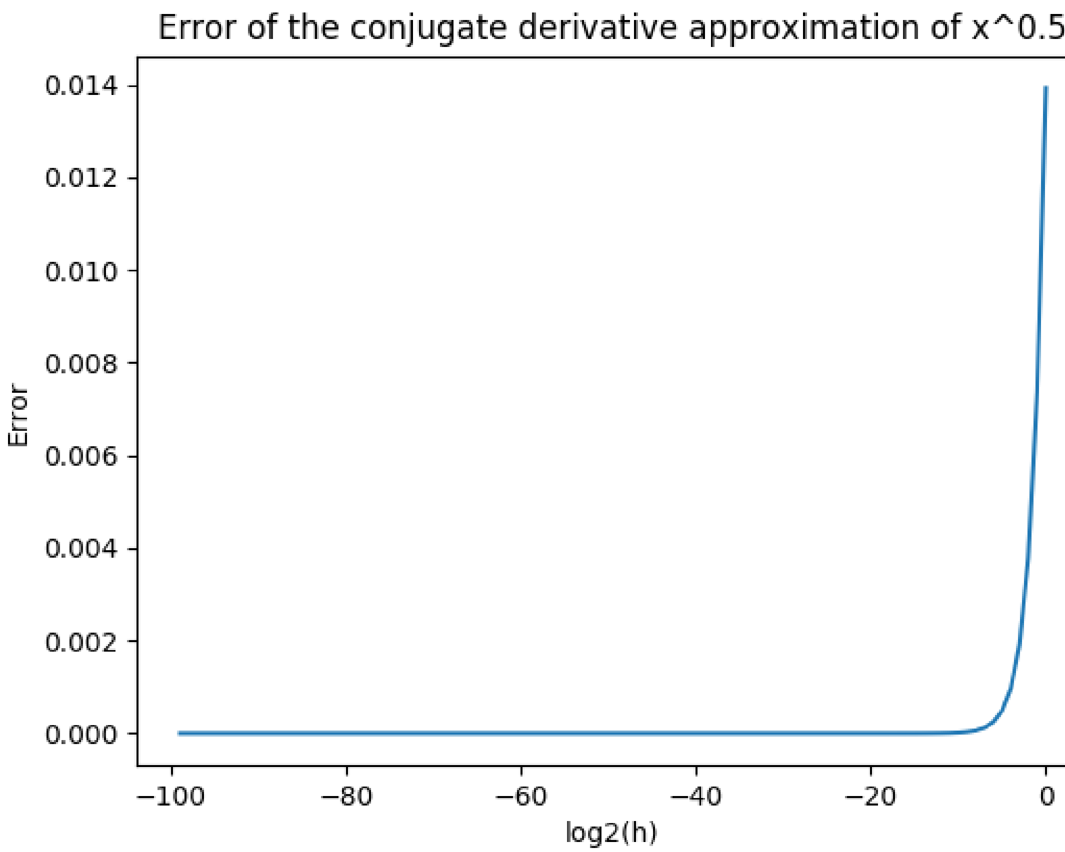

Figure 2: Absolute Error using the conjugate derivative approximation

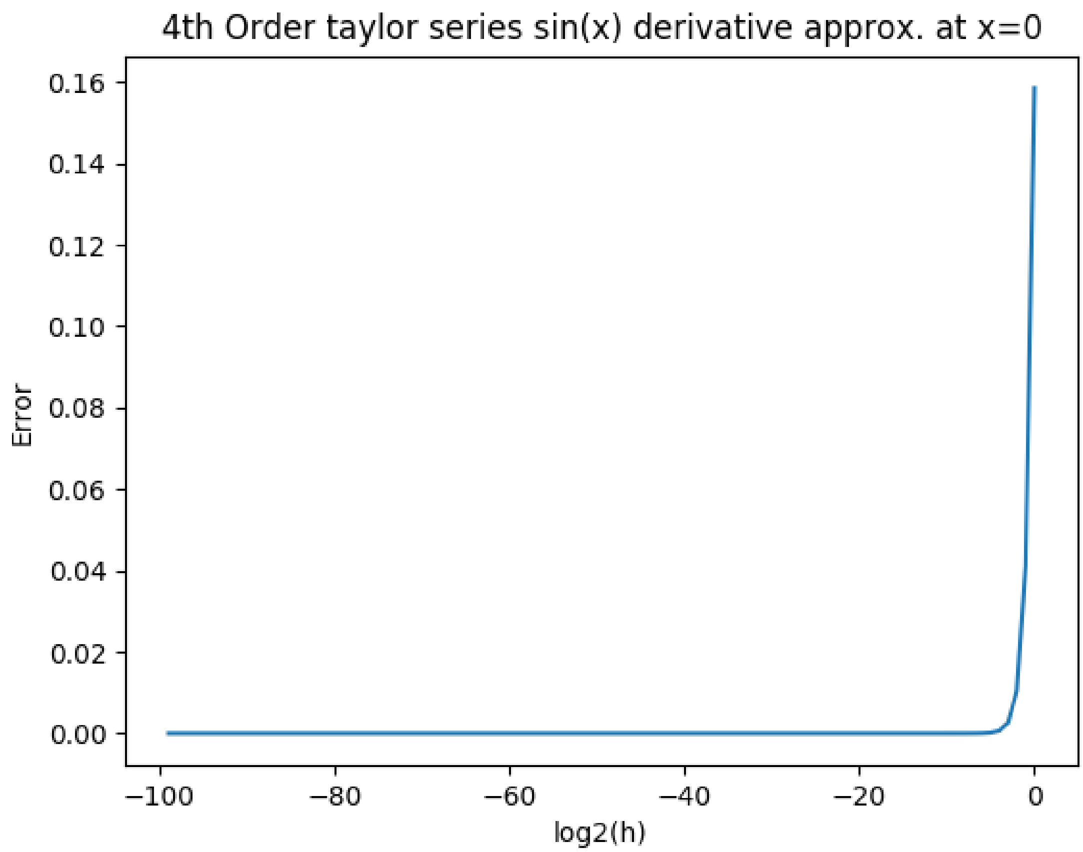

Figure 3: 4th Order sinx derivative approx. at x=0