Differential Equations

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0.1 $\frac{dx}{dt} = 3x,\ x(0) = 1.0$

$\int \frac{1}{x} dx = \int 3dt$
$ln(x) = 3t + c$
$x = Ae^{3t}$
$\Rightarrow 1 = A$
$x = e^{3t}$

0.2 $\frac{dx}{dt} = 3tx,\ x(0) = 1.0$

$\int \frac{1}{x} dx = \int 3tdt$
$ln(x) = \frac{3}{2}t^2 + c$
$x = Ae^{\frac{3}{2}t^2}$
$x = e^{\frac{3}{2}t^2}$

0.3 $\frac{dx}{dt} = \frac{1}{10}x - \frac{3}{1000}x^2,\ x(0) = 4$

$\frac{dx}{dt} = \frac{1}{10}x - \frac{3}{1000}x^2$
$\frac{dx}{dt} - \frac{1}{10}x = -\frac{3}{1000}x^2$
$-\frac{1}{x^2} \cdot \frac{dx}{dt} + \frac{1}{10x} = \frac{3}{1000}$
$\Rightarrow u = \frac{1}{x},\ \frac{du}{dt} = -\frac{1}{x^2}$
$\Rightarrow \frac{du}{dt} + \frac{1}{10}u = \frac{3}{1000}$
$\Rightarrow \mu = e^{\int \frac{1}{10} dt} = e^{\frac{t}{10}}$
$\Rightarrow \frac{d}{dt}(e^{\frac{t}{10}}u) = \frac{3}{1000}e^{\frac{t}{10}}$
$e^{\frac{t}{10}}u = \frac{3}{1000}e^{\frac{t}{10}} + c$
$u = \frac{3e^{\frac{t}{10}} + 100c}{100e^{\frac{t}{10}}}$
$x = \frac{100e^{\frac{t}{10}}}{3e^{\frac{t}{10}} + A}$
$4 = \frac{100}{3+A}$
$3 + A = 25$
$A = 22$
$x = \frac{100e^{\frac{t}{10}}}{3e^{\frac{t}{10}} + 22}$

0.4 $\frac{dx}{dt} = \frac{1}{10}x - \frac{3}{1000}x^2,\ x(0) = 400$

$400 = \frac{100}{3+A}$
$3 + A = \frac{1}{4}$
$A = -\frac{11}{4}$
$x = \frac{100e^{\frac{t}{10}}}{3e^{\frac{t}{10}} - \frac{11}{4}}$