Report 3
Interferometry

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Abstract

We do a systematic analysis of the data collected at ISTTOK utilising its interferometer, with the main goal of estimating the temporal evolution of the line-averaged electron density (LAD) in ISTTOK. Based on the signal's I/Q plot, we are able to extract how the phase shift varies as a function of time, which we can then relate to the LAD. The I/Q plot's offsets were removed by means of a circular fit and the phase shift [signal] had to be unwrapped and thoroughly scanned for possible fringe jumps, which were afterwards corrected manually. Only then could we obtain the LAD, which we also attempt to correlate to the plasma position.

Introduction

Interferometry is a well established technique utilised to measure a plasma's line-averaged electron density (LAD) from the phase shift of the waves transmitted through the plasma, relative to a reference signal. Naturally, we work with frequencies above the plasma frequency, so we don't have to worry about cut-offs (for the most part, at least). For O-modes, the phase shift resulting from the difference between the optical path, in vacuum, and the path through the plasma, can be related to the plasma density through the following expression:

$$\varphi = \frac{\lambda e^2}{4\pi c^2 \epsilon_0 m_e} \cdot \int_{z_1}^{z_2} n(z)dz \tag{1}$$

Which takes the simplified form seen below, when we are dealing with the line-averaged density, which is the case:

$$\varphi = \frac{\lambda e^2}{4\pi c^2 \epsilon_0 m_e} \cdot (z_2 - z_1) \cdot \overline{n(z)} \tag{2}$$

Where $\overline{n(z)}$ is the refered LAD. Here, the O-mode's frequency is selected so as to allow for a reliable determination of the phase. By this, we mean that we are looking for phase variations below 2π, but still measurable. At ISTTOK, we use phase-quadrature detection interferometry, which allows for separate measurements of the amplitude and phase through the use of an I/Q detector. Not going into too much detail, this allows us to get signals prportional to $A\cos(\varphi)$:

$$I = A\cos(\varphi) \tag{3}$$

$$Q = A\sin(\varphi) \tag{4}$$

Which we can then plot on the I/Q plane, such that:

$$A(n) = \sqrt{I^2(n) + Q^2(n)} \tag{5}$$

$$\varphi(n) = \arctan\left(\frac{Q(n)}{I(n)}\right) \tag{6}$$

We can now substitute φ from equation (6) in equation (2) and solve for $\overline{n(z)}$. This yields:

$$\overline{n(z)} = \frac{4\pi c^2 \epsilon_0 m_e}{\lambda e^2} \cdot \frac{\varphi}{(z_2 - z_1)} \tag{7}$$

Keep in mind that the interferometer at ISTTOK utilises a probing frequency of 100 GHz, which allows us to probe a maximum density of roughly 1.2×10^{20} m^{-3}. Note also that $\lambda = c/f$, with $f = 100$ GHz.

Methods and procedures

First and foremost, by constructing our I/Q plot, we will immediately realise the need to correct the offsets introduced by the electronic hardware in originating these signals. I will do this by performing a fit of the data points to a circle, followed by a re-centering of such circle to the origin. In practice, I just subtract the fitted circle's center's x coordinate to all I points and the y coordinate to all Q points. After this is done, I shall also only select points in the I/Q plane that are relatively close to the fitted circle. The reason being that we do not want points of low amplitude, since they are often problematic. These reduced amplitudes might be due to many factors (refraction of the beam, scattering due to turbulence, etc.), but we can never be 100% sure of their true origin (might be a mix of many factors). So, to avoid introducing more complexity into our analysis, we neglect points that stray too far away from the fitted circle. This also guarantees that the selected points have meaningful phases. After this is done, we can now compute the phase shifts through equation (6), utilising the selected I/Q points. Due to limitations of the arctangent function, though, we will need to unwrap the phase. Only then can we make use of equation (7) to obtain the line-averaged electron density.

Results and discussion

For this work, I utilised the data from shot number 36873, having retrieved the following signals:

- `'CENTRAL.OS9_ADC_VME_I8.IFOCS'`: Cos $\rightarrow$ I;
- `'CENTRAL.OS9_ADC_VME_I8.IFOSN'`: Sin $\rightarrow$ Q;
- `'MARTE_NODE_IVO3.DataCollection.Channel_081'`: Plasma position;
- `'POST.PROCESSED.IPLASMA'`: Plasma current.

The plasma position and current will be useful later on. The former will be used to investigate whether a correlation can be established between the plasma position and the LAD, whereas the latter will prove convenient for identifying fringe jumps, a possible undesired consequence of the unwrapping procedure. Moving forwards, below I present the obtained I/Q plots.

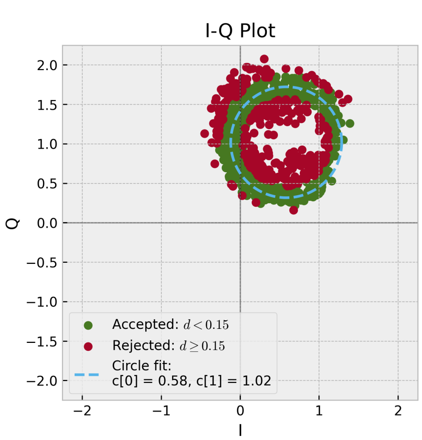

Figure 1: Raw data - I/Q plot.

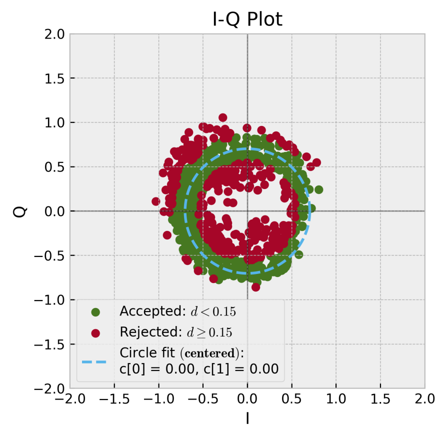

Figure 2: Centered I/Q plot.

As discussed earlier, I removed the offsets by fitting our data to a circle. This was done utilising *scikit-guess*'s `nsphere_fit`[1] function. Note that in going from figure 1 to figure 2, I merely shifted all the points by the fitted circle's center's coordinates. Ultimately, I ended up accepting points whose distance to the fitted circle was < 0.15 (in a.u.). This value was chosen arbitrarily at first, but it proved to yield good results, so I kept it. We can now compute the phases through equation (6). This resulted in figure 3, for the wrapped phase:

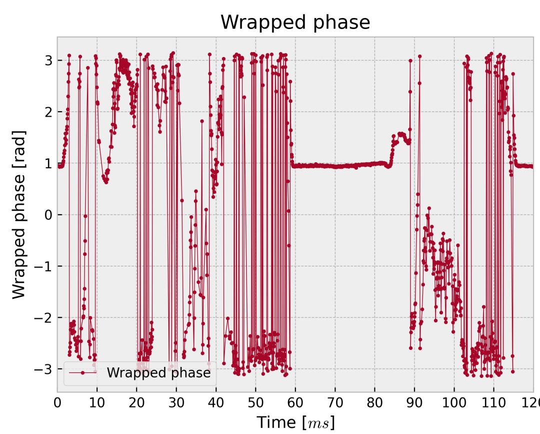

Figure 3: Wrapped phase as a function of time.

I only plot until 120 ms since nothing happens after that. The calculations of the phase were done through *numpy*'s `arctan2`[2], which returns a value in the interval $[-\pi, \pi]$. Due to this, we now need to unwrap the phase. This will be done utilising *numpy*'s `unwrap`[3] function. By default, this function unwraps a radian phase p such that adjacent differences are never greater than π by adding $2k\pi$ for some integer k. The obtained results are presented below, in figure 4:

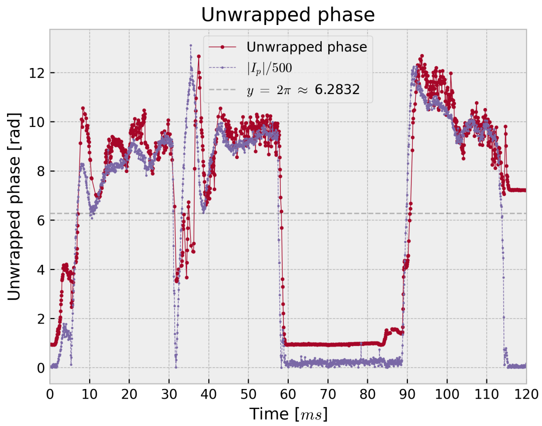

Figure 4: Unwrapped phase as a function of time.

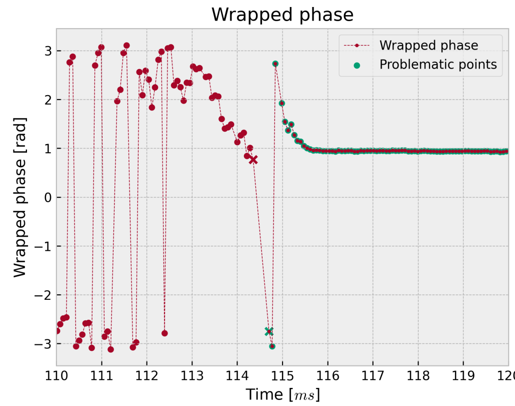

Figure 6: Unwrapped phase as a function of time (detail from $t \in [110, 120]$ ms).

These were obtained utilising **`unwrap`**'s default parameters (the ones we just discussed). In figure 4, I also plot the plasma current's absolute value, divided by 500. This should give us a guideline of how we expect the unwrapped phase to behave. From this figure, it is evident that a 2π fringe jump has occurred: We start with an unwrapped phase of around 0 [rad] and we end at ≈ 7, instead of at around 0 again, like we expected. And judging from the plasma current's profile, we suspect that the fringe jump occurred close to the end of the graph (at ≈ 115 ms), since that's where the two plotted profiles appear to diverge. Upon closer manual inspection, I was able to identify exactly where the jump occurred. Below, I present the graphs that allowed me to arrive at such conclusions. (Note that this fringe jump appears to be a consequence of the unwrapping procedure, rather than ambiguity when the phase variation is larger than 2π).

The relatively big jump ($\Delta y^* = 2.7614$) in figure 5 immediately caught my attention. I figured this was probably where the jump had happened. To test this hypothesis, I looked at the wrapped phase graph (figure 6). In green I mark what I believe are the problematic points, that is, points affected by the fringe jump. These will later have to be shifted downwards by a factor of 2π. Anyways, the jump appears to have occurred when going from the last red point to the first green point (marked with '`x`'). These two points are relatively far apart: they exhibit $\Delta y = -3.5218 \Rightarrow |\Delta y| > \pi$. As such, the **`unwrap`** function will be 'correcting' such big gap thinking it was due to the wrapping introduced by the **`atan2`** function, when that is not the case! It does this by summing 2π to all values after the '`x`' marked red point in figure 6. In fact, if we sum 2π to $\Delta y = -3.5218$, we get $\Delta y^* = 2.7614$, that we see clearly in figure 5, thus proving the initial hypothesis. This is what caused the fringe jump. After amending this, the following graph was obtained:

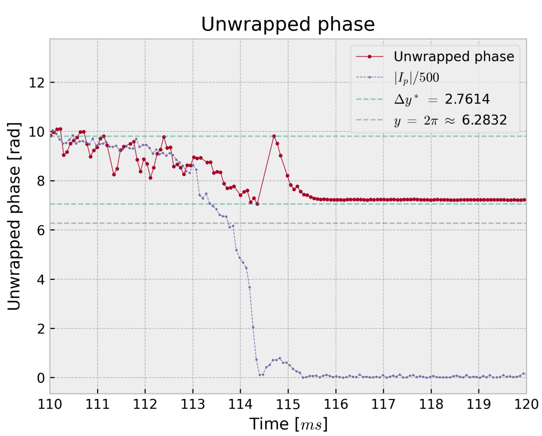

Figure 5: Unwrapped phase as a function of time (detail from $t \in [110, 120]$ ms).

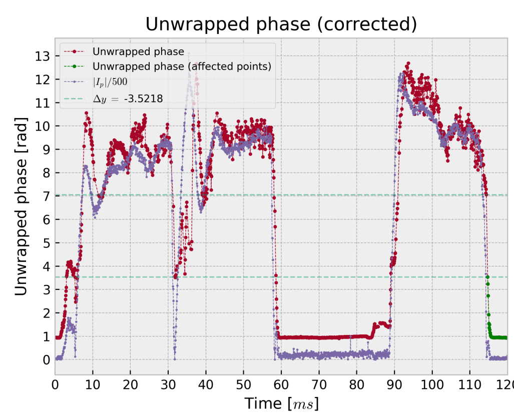

Figure 7: Corrected unwrapped phase as a function of time (detail from $t \in [110, 120]$ ms).

I would like to point out that there seems to be some systematic error in the unwrapped phase, since we do not start (nor end) at exactly 0 [rad]. This might be due to the method I utilised to remove the offsets in the I/Q plot: it might not have fully mitigated the electronics' effect. Or, alternatively, there might simply be some 'leftover plasma' in the chamber that is causing this. Anyways, we can now, finally, make use of expression (7) to compute the electron line-averaged density. These were the obtained results:

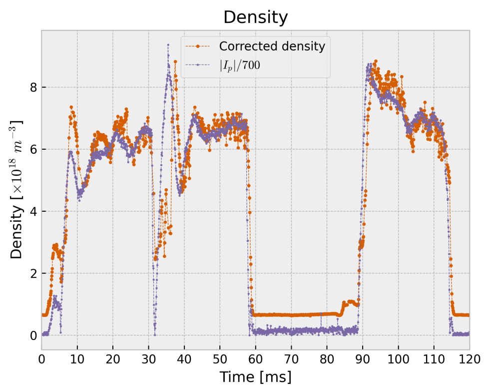

Figure 8: Corrected density as a function of time.

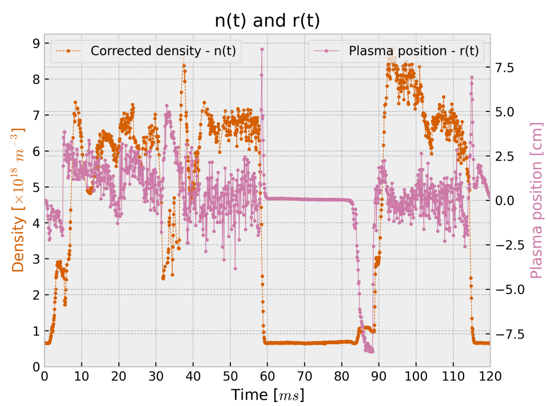

Figure 9: Corrected density and plasma position as a function of time.

After the correction, we note that the density profile now loosely follows the plasma current's[1], as expected. Also note that all the measured densities are below the cut-off density of approximately 1.2×10^{20} m^{-3}, as they should be. In figure 9, I plot both the electron LAD and the plasma position, in an attempt to establish some sort of correlation between the two. Intuitively, if the plasma is not centered (i.e. $r \neq 0$), we expect the LAD to be smaller, given that we are overshooting the cord length[2], $(z_2 - z_1)$. Not only that, but we also expect the electron density to be lower nevertheless, since we are farther away from the center of the plasma, where it should [expectedly] be maximum, further contributing to a lesser density. Through the graph's inspection, not much meaningful information can be extracted, unfortunately. We note a bump in the plasma position around 35 ms, which is accompanied by a dip in the LAD, as we predicted. Other than this, though, the only other things that stand out are the peaks at roughly 60 ms and 115 ms and the valley at around 90 ms, probably associated with the 'death' and creation of the plasma, respectively. The only reasonable conclusion we can take from this is that the density values in these regions are surely affected by a high uncertainty. Anyhow, due to the low frequency of observed correlations between position and density, we cannot affirm with 100% certainty **how** these two quantities are correlated, even though they should [intuitively] be negatively correlated[3]. To further investigate this, we could look into other shots and do a similar treatment. This way, we could accumulate more experimental observations, thus leading to an overall more significant analysis. I did not do this, however, since it is rather tedious and time consuming to manually find and correct all the fringe jumps for each individual shot. Note that I purposefully chose a shot that only had one fringe jump to save me some extra time. Besides, the method I utilised for removing the offsets does not consider points that are too far away from the fitted circle. One would expect this to cause additional fringe jumps, since there might be even greater gaps in the phase, due to us neglecting said points. Surprisingly, though, that wasn't the case. I compared the obtained results with the case in which I used all the points. After having corrected for the <u>two</u> occurring fringe jumps, I arrived at the following density profile (figure 10).

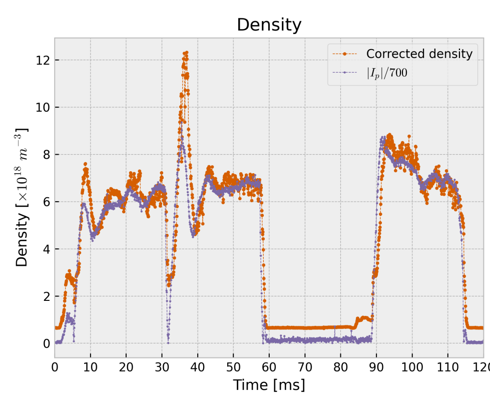

Figure 10: Corrected density as a function of time (utilising **all** the points).

[1] This time, I plot $|I_p|/700$, for better visualization.
[2] Note that the interferometer's beam runs through the center of the plasma chamber.
[3] As in: plasma strays away from the center $\implies$ LAD decreases.

As can be seen, qualitatively, the density profile itself is rather similar. However, note that the peak at roughly 35 ms is now at $12 \times 10^{18}\ m^{-3}$, when before it was at $8 \times 10^{18}\ m^{-3}$. I cannot quite pinpoint the exact reason as to why this happens, although it is clear that it is due to me excluding a big portion of the data in the first analysis I made. Perhaps I ended up removing so many points that I completely overlook an entire revolution in the I/Q plot, leading to overall smaller [unwrapped] phases and, thus, lower densities. Keep in mind that, in the initial analysis, I excluded about 2.11% of the data points, which corresponds to 346 out of 16384 points. In hindsight, this was not the best decision, as it seems that some of the behaviour of the plasma was lost in the process.

Conclusions

With this work, we were able to extract information about the electron line-averaged density by means of exploring the data collected at ISTTOK, utilising its interferometer. Through the signal's I/Q plots, we computed the phase difference between the two signals ('vacuum' and 'plasma' paths) and related this to the electron LAD. There were many technical difficulties along the way, mainly due to the unwrapping procedure. Fringe jumps were manually found and corrected. The obtained results appear to make sense. However, no meaningful correlation could be established between the plasma position and the LAD. Nevertheless, I consider this treatment a success, since we were able to effectively determine the plasma's LAD.

References

[1] "Scikit-guess's `nsphere_fit` function documentation." https://scikit-guess.readthedocs.io/en/latest/generated/skg.nsphere_fit.html, Accessed: 2022-10-23.

[2] "Numpy's `atan2` function documentation." https://numpy.org/doc/stable/reference/generated/numpy.arctan2.html, Accessed: 2022-10-23.

[3] "Numpy's `unwrap` function documentation." https://numpy.org/doc/stable/reference/generated/numpy.unwrap.html, Accessed: 2022-10-23.

[4] I. H. Hutchinson, *Principles of Plasma Diagnostics*. Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo: Cambridge University Press, 2005.