Report 1
Electric (Langmuir) probes

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Abstract

We do a systematic analysis of the variation of plasma parameters and goodness of fit as a function of both a specified cut-off voltage and time. We found that greater values of the cut-off voltage generally lead to poorer quality fits, indicating that our model is deficient in these regions. The study of the variation of the plasma parameters as a function of time in 1 cycle of a discharge in ISTTOK revealed the formation and death of the plasma, alongside visible plasma fluctuations, as expected theoretically.

Theoretical introduction

Electric probes, also known as *Langmuir* probes, are diagnostics devices utilized in order to determine plasma parameters such as the electron temperature and density, as well as the plasma potential. These consist of one or more electrodes which are inserted into a plasma, with a constant or time-varying electric potential between the various electrodes or between them and the surrounding vessel. We can then utilize the measured currents and potentials in this system in order to determine the aforementioned physical properties of the plasma. *Langmuir* probes are the simplest form of diagnostic possible: they merely consist of a conductor immersed into the plasma. However, as we shall see, the interpretation of the collected data is fairly complicated, since the probe perturbs the plasma. In order to progress, we need to make some assumptions about the behaviour of the plasma close to the probe. We shall assume that a positive sheath of very small thickness is formed around the probe, entirely equivalent to the sheath formed near a wall in a plasma-wall interaction (due to *Debye* shielding). This, alongside *Bohm's* criterion, allows us to deduce the following I-V characteristic[1]:

$$I = I_{sat}\left(1 - e^{\frac{e\left(V_{bias}-V_f\right)}{kT_e}}\right) \tag{1}$$

Together with another set of equations:

$$I_{sat} = 0.5 \cdot eAnc_s \tag{2}$$

$$c_s = \sqrt{\frac{k(T_e+T_i)}{m_i}} \tag{3}$$

$$V_p = V_f + 3kT_e/e \tag{4}$$

Nomenclature

c_s	Ion speed of sound
e	Electron charge
I_{sat}	Ion saturation current
k	Boltzmann constant
m_i	Ion mass
n	Electron density
T_e	Electron temperature
T_i	Ion temperature
V_f	Floating potential
V_{bias}	Voltage applied to the *Langmuir* probe
V_p	Plasma potential

In our case in particular, we also assume $T_i = 0$ (see footnote 1), which means eq. 3 takes the simplified form: $c_s = \sqrt{\frac{kT_e}{m_i}}$.

Objectives

The work itself consist of interpreting the data collected with *Langmuir* probes at ISTTOK. More specifically, we shall study:

1. The effect of the cut-off voltage on the estimated plasma parameters for two/three I-V characteristic curves showing a deviation from the predicted exponential behaviour;

2. And the temporal evolution of the plasma parameters along one plasma cycle ($\approx 25ms$).

I shall later refer to these two points as 1^{st} and 2^{nd} parts of the work.

Procedure, results and discussion

I will now digress on what I wrote for my data analysis script (*Python*). For the 1^{st} part, I utilized data from shot 35605, namely, channels 'CHANNEL_030' for the collected current and 'CHANNEL_026' for the

[1] There's some other extra assumptions that we make in order to arrive at an analytic expression: $B = 0$, $Z = 1$, $T_i = 0$, no collisions in sheath, plane probe, $1D$ scenario and all particles absorbed by the probe.

applied voltage. First and foremost, I simply read the data and plotted the values in the graphs that can be seen in the figures below. From the full discharge (figure 1), I selected two consecutive I-V characteristic curves from the second cycle of the discharge, which corresponds to the following time interval: [50500.5, 53899.5] μs (figure 2).

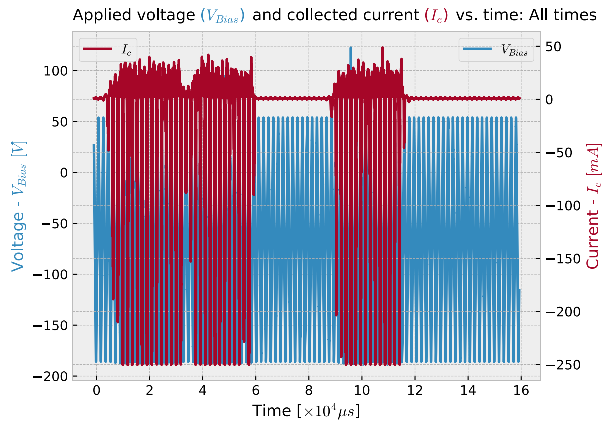

Figure 1: Values of 'CHANNEL_030' (collected current) and 'CHANNEL_026' (applied voltage) as a function of time for shot 35605.

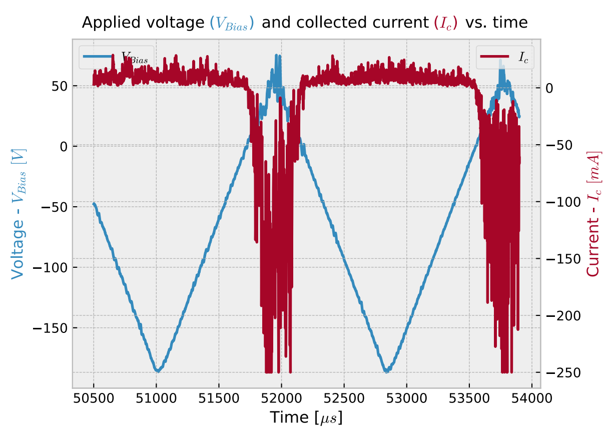

Figure 2: Detail of the graph from figure 1. Two consecutive I-V characteristic curves were selected from the second cycle of the discharge.

We now intend to plot the current (I) against the applied voltage (V_{bias}), so we can then fit[2] this graph to expression (1) and thus retrieve the desired plasma parameters as fit parameters. However, we must keep in mind that our current signal saturates at $-250\ mA$ and, as such, there will be a handful of points for high V_{bias} that are not real points, in the sense that they would have a more negative current than $-250\ mA$, if not for the signal's saturation. Therefore, these points shouldn't be considered in our fit. Not only that, but our model itself is also not perfect[3]. And its imperfections start to become more and more evident as V_{bias} increases. At some point, our model is no longer valid. This means we need to find the values of V_{bias} for which our model is indeed correct. This motivates us wanting to study the plasma parameters' variations as a function of the considered cut-off voltage. This cut-off voltage is simply the value of V_{bias} above which we will no longer be considering points for our fit. Below, in figure 3, I present the results of such a fit for $V_{cut-off} = 15$ V.

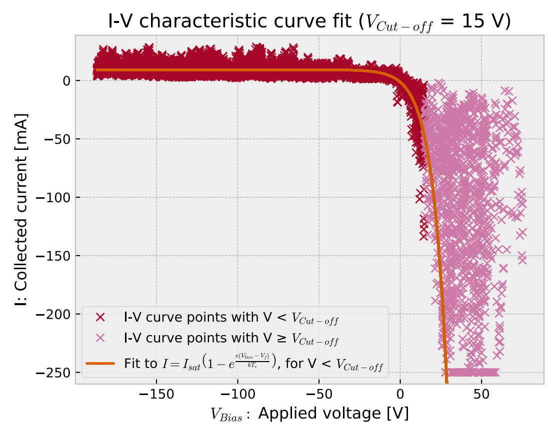

Figure 3: I-V characteristic curve fit using the 2 curves seen in figure 2 as input values, for $V_{cut-off} = 15$ V. Points with $V \geq V_{cut-off}$ are not considered for the fit. Fit parameter values and other quantities are presented in table 1.

Variable	Value
I_{sat}	9.18 ± 0.09 [mA]
V_f	-0.98 ± 0.26 [V]
T_e	8.39 ± 0.17 [eV]
c_s	$(4.01 \pm 0.04)\times 10^4$ [m/s]
n_e	$(4.77 \pm 0.07)\times 10^{20}$ [m^{-3}]
V_p	24.2 ± 0.6 [V]
SS_{Res}/N	38.24

Table 1: Values of the fit parameters (I_{sat}, V_f and T_e) retrieved directly from the fit of figure 3 and other related quantities, computed through equations (2), (3) and (4), for $V_{cut-off} = 15$ V.

Note that SS_{Res}/N represents the sum of squared

[2] I used scipy.optimize.curve_fit to fit. Note: In order to prevent overflow, I had to fit considering $e = k = 1$. After the fit, I transformed back to real physical values.

[3] Mostly due to the presence of magnetic fields in the plasma, which were unaccounted for in our model. See footnote 1 where we mention that we assume $B = 0$.

residues normalized to the number of points considered in the fit. This will work as a goodness of fit parameter. These were the results obtained for $V_{cut-off}$ = 15 V. Afterwards, all I did was repeat this analysis for a sweep in $V_{cut-off}$, considering 1000 points going from 0 to 70 V. There is no point in considering cut-off voltages much below 0 V, since, then, we would be 'losing'[4] the exponential behaviour of our I-V characteristic curve, which would lead to terrible fits overall. Following this procedure, these were the obtained results:

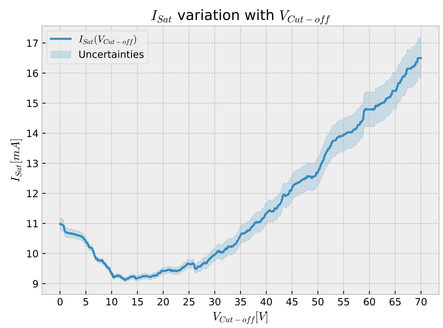

Figure 4: I_{sat}'s variation with the cut-off voltage.

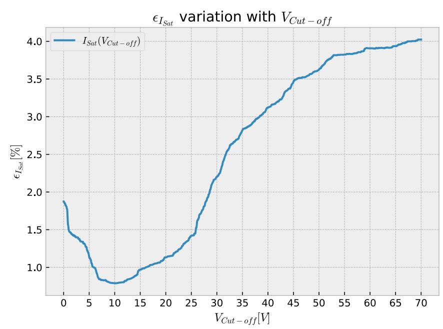

Figure 5: I_{sat}'s relative uncertainty's variation with the cut-off voltage.

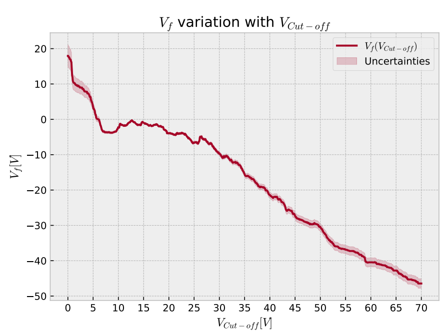

Figure 6: V_f's variation with the cut-off voltage.

[4]As in not considering.

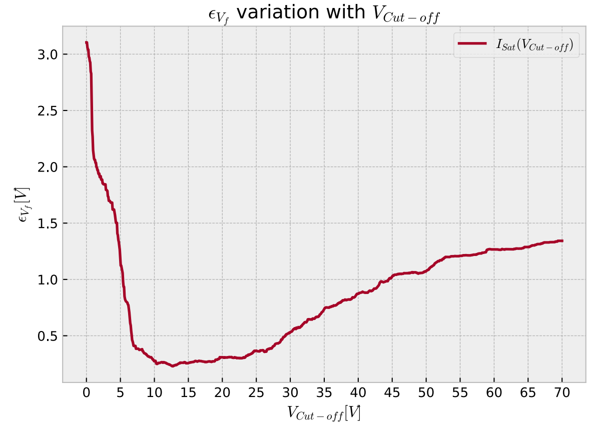

Figure 7: V_f's **absolute** uncertainty's variation with the cut-off voltage.

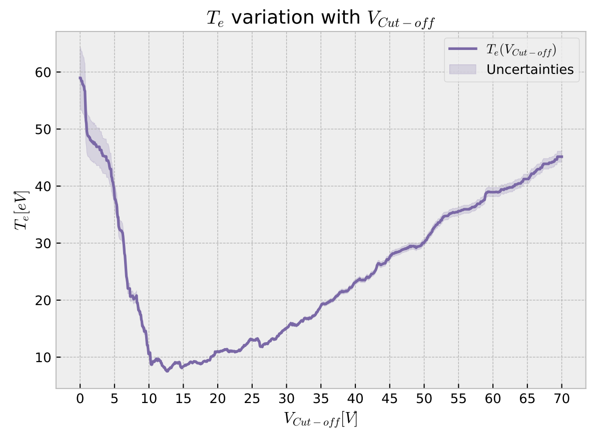

Figure 8: T_e's variation with the cut-off voltage.

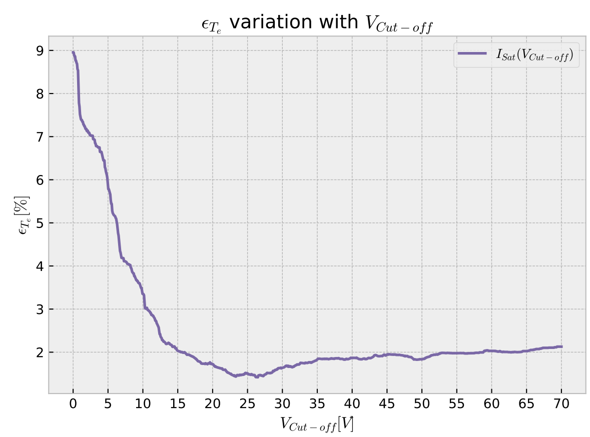

Figure 9: T_e's relative uncertainty's variation with the cut-off voltage.

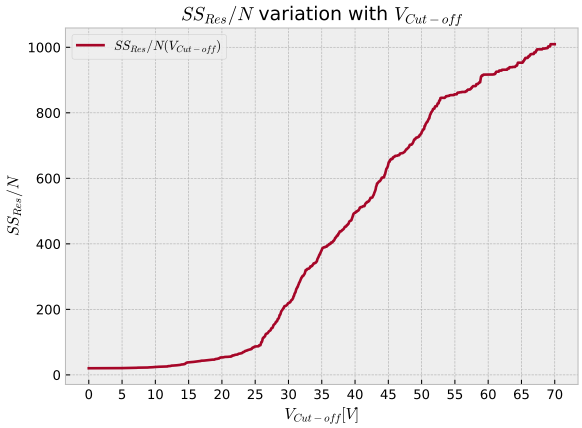

Figure 10: SS_{Res}/N's variation with the cut-off voltage.

Let us now explore all this information. Starting with our goodness of fit parameter, we can see that our fits are fairly reasonable in quality for lower values of $V_{cut-off}$, until about 25 V. From this point onwards, SS_{Res}/N starts increasing more steeply, indicating lower quality fits. This is probably due to what I mentioned earlier on in the discussion: The higher we go in $V_{cut-off}$ the more inadequate our model is and the more 'non-real' points we consider, leading to overall worse fit results. I'd also like to point out that SS_{Res}/N takes, overall, reasonably large values, which indicates that our fit has not fully captured the data (or, maybe, that the error variance has been underestimated[5]). This result isn't all too surprising considering the amount of simplifications we had to make in our model in order to arrive at a concrete analytic expression for the I-V characteristic. With this in mind, we can now look at the rest of the graphs. We can see that the ion saturation current appears to have a slow increasing tendency, with its uncertainty also increasing with $V_{cut-off}$, as predicted by our analysis of graph 10. Note the minimum around $10-15$ V (for I_{sat}), which we could attempt to relate to the most probable value of I_{sat}. In order for this to be true, there would need to be no significant plasma fluctuations in the time interval associated with the fit. This might not always be true! In my case, I considered two curves, which might slightly mitigate these fluctuations. Ideally, though, more should be considered in order for this argument to be more robust. Nevertheless, this would yield $I_{sat} \approx 9.25\ mA$. I did not put much effort in the rigorous determination of this value, provided the unreliability of the argument presented above. Somewhat identical considerations could be made about T_e's and V_f's variation with $V_{cut-off}$. I would just like to point out that, for small values of $V_{cut-off}$, $\epsilon_{T_e} > \epsilon_{I_{sat}}$, as one would expect from a fit of the kind of figure 3. Looking at all the graphs as a whole, our model seems to be reasonably valid for $V_{cut-off} < 20$.

Moving onto the 2^{nd} part, here I utilized data from shot 35604, namely, channels 'CHANNEL_030' for the collected current and 'CHANNEL_026' for the applied voltage. In order to study the plasma parameters' variations in time, I selected only the 1^{st} cycle of the discharge for a total time period of roughly 30 μs: [3500.5, 33500.0] μs (see figure 11). Afterwards, I broke down this time interval into smaller intervals (16 in this case, since we seemed to have about 16 I-V characteristic curves in figure 11), in each of which I would fit the data to expression (1), just like before. This way, I was able to obtain the plasma parameters' variation with time. The results are presented below[6].

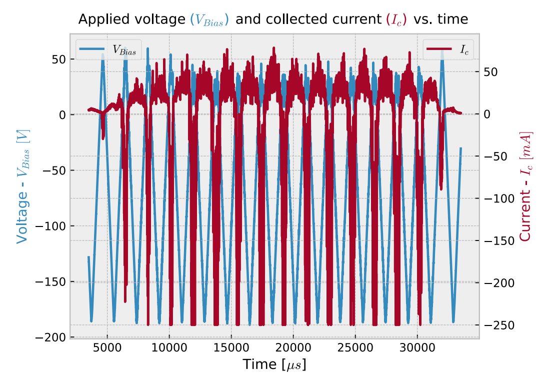

Figure 11: Values of 'CHANNEL_030' (collected current) and 'CHANNEL_026' (applied voltage) as a function of time for shot 35604.

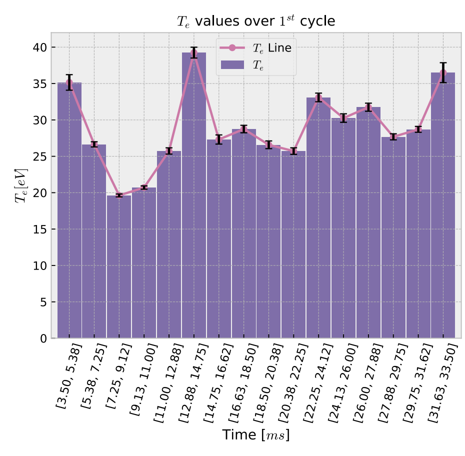

Figure 12: T_e's variation with time over the 1^{st} cycle of figure 11.

[5]In fact, if we look at figures 5 and 9, the values of the relative uncertainties are indeed fairly small. They might have been underestimated.

[6]Note that in this case, we need not consider a cut-off voltage.

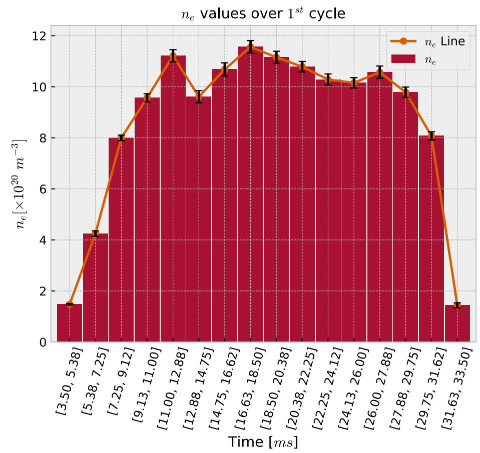

Figure 13: n_e's variation with time over the 1^{st} cycle of figure 11.

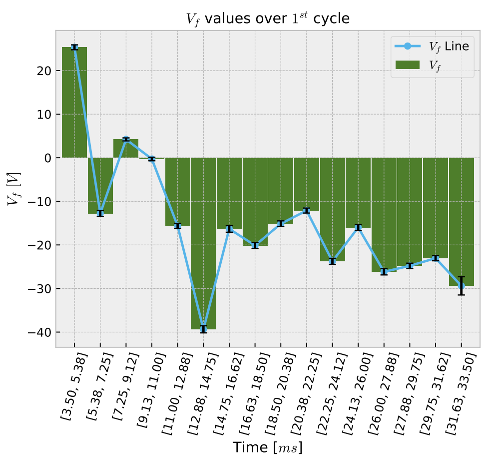

Figure 14: V_f's variation with time over the 1^{st} cycle of figure 11.

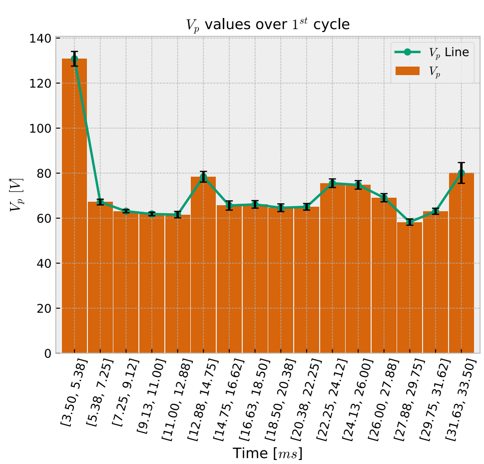

Figure 15: V_p's variation with time over the 1^{st} cycle of figure 11.

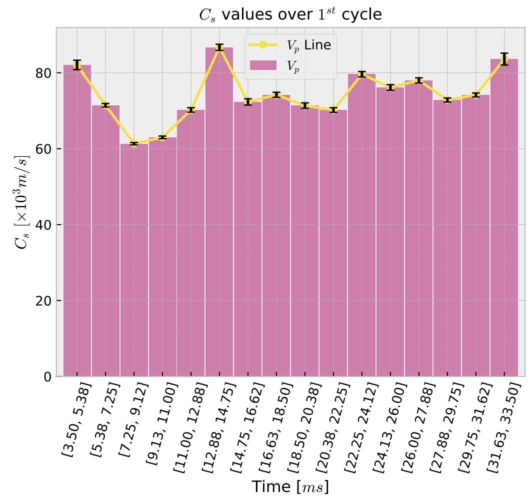

Figure 16: C_s's variation with time over the 1^{st} cycle of figure 11.

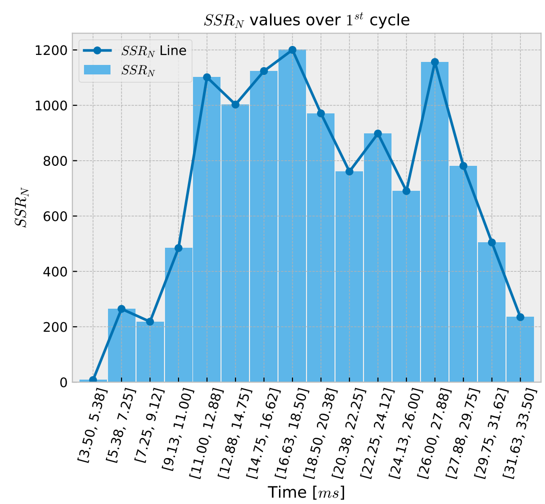

Figure 17: SS_{Res}/N's variation with time over the 1^{st} cycle of figure 11.

Note that it really isn't feasible to break the time intervals much more, since that would lead to smaller intervals with even less points, which means generally worse quality fits. From figure 13, we can see that the electron density seems to increase in the beginning, before sort of stabilizing around (10-11) $\times 10^{20}$ m^{-3}, with some fluctuations around these values being clearly visible. These are the predicted plasma fluctuations. We know these are in fact fluctuations due to the fairly low uncertainty of our results. The initially increasing tendency of the graph might be related to the creation of the plasma itself, whereas the late decreasing tendency is probably due to the plasma dieing out. Compare this with figure 12, for the electron temperature. Now, we cannot see the same increasing and decreasing tendencies as before, but what we can see are much more evident plasma fluctuations. Figures 15 and 16 show overall more stable V_p and c_s values, respectively, than what we've seen so far. V_f, however, in figure 14, shows very pronounced fluctuations. Note, also, that it is frequent for the last time interval to be associated with a greater uncertainty (black bars). This might be due to the fact that this last time interval contains many points where the plasma is already mostly 'dead', thus not following our model as well.

Conclusions

With this work, we were able to test the validity of our model for the I-V characteristic curve of a plasma - *Langmuir* probe system. We concluded that our model was reasonably valid for cut-off voltages smaller than around 20 V. For $V_{cut-off} > 20$ V, uncertainties start to climb, together with SS_{Res}/N, which implies that our model fails to faithfully capture the data in this region. When plotting the plasma parameters as a function of time, we could identify the formation and 'death' of the plasma, as well as very noticeable plasma fluctuations, like one would expect from an experience of this kind.

References

[1] "[PowerPoint from theoretical classes] Diagnostics: Electric probes," https://fenix.tecnico.ulisboa.pt/downloadFile/845043405605125/Lecture%202022.pdf, accessed: 2022-10-10.

[2] "Goodness of fit," https://en.wikipedia.org/wiki/Goodness_of_fit, accessed: 2022-10-12.

[3] "Scipy documentation," https://docs.scipy.org/doc/scipy/, accessed: 2022-10-10.