The central limit theorem:

- I will consider 3 (continuous) distribution.
  - uniform;
  - gaussian;
  - lognormal (sampling).

- $x = \sum_{i=1}^{N} x_i$ uncorrelated.
- $\langle x \rangle = N\mu = N\langle x_i \rangle$.
- $\sigma_x = \sqrt{N}\,\sigma_{x_i}$

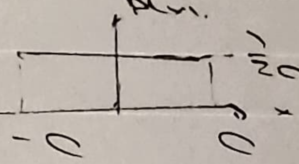
- the x_i: are independently identically distributed (i.i.d.) random
- sample: ..

Theoretical considerations:

(central limit th.)

uniform distribution:

$p(x) = \frac{1}{2a}\,\Theta(a-|x|)$



$\mu = 0$

$\sigma = \frac{a^2}{3}$

$\mu = \langle x \rangle = \int_{-a}^{a} dx\, x\, p(x)$

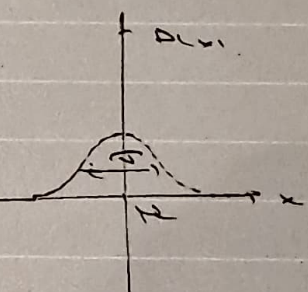
$= \int_{-a}^{a} dx\, \frac{x}{2a} = \frac{1}{2a}\left(\frac{x^2}{2}\right)\Big|_{-a}^{a} = 0.$

$\langle x^2 \rangle = \frac{1}{2a}\int_{-a}^{a} dx\, x^2 = \frac{1}{2a}\left(\frac{x^3}{3}\right)\Big|_{-a}^{a} = \cancel{\frac{1}{2a}\cdot\frac{2a^3}{3}} = \frac{a^2}{3}.$

$\sigma = \langle x^2 \rangle - \langle x \rangle^2 = \frac{a^2}{3} - 0 = \frac{a^2}{3}.$

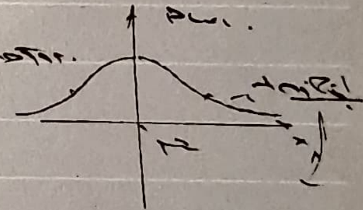
gaussian distribution:

$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$, mean: μ, variance: σ^2



Lorentzian (cauchy) distribution:

$p(x) = \frac{1}{\pi}\,\frac{\gamma}{(x-\mu)^2+\gamma^2}$, ~~mean: μ~~ μ is simply a location parameter. variance $\sigma^2 \to \infty$.



$X = \sum_i^N x_i$:

x_i	μ_X	σ^2_X	problems!
uniform	$N\cdot\mu = N\cdot 0 = 0$	$\frac{Na^2}{3}$	
gaussian	$N\mu$	$N\sigma^2$	
cauchy ?	not defined	$\to\infty$ (not defined).	

- should also focus on
- probably more "fat" tailed than the lorentzian. (binomial, e.g)
  even though it is discrete.
  - it should work to demonstrate (normality, the CLT).

Also, I use the poisson distribution:

$P(X=k) = \frac{\lambda^k}{k!}\, e^{-\lambda}$

$\mu = \sigma^2 = \lambda.$ (poisson dist. parameter)

- Intuition:

$X = \sum_{i=1}^{N} x_i$ satisfies:

$\mu_X = N\mu = N\lambda$

$\sigma^2_X = N\sigma^2 = N\lambda.$

- In addition to this explanation, I include the obtained plots (and essentially the code?) in the submission.pdf.

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