ECONOMICS 205: Mathematics for Economists

Summer 2020

Basic information

Lectures	M-F August 24-September 4, 16:00-18:00 Pacific Standard Time, via Zoom
Instructor	Prof. Alexis Akira Toda
Email	atoda@ucsd.edu
Webpage	`https://alexisakira.github.io/` (Go to `Teaching` → `Mathematics for Economists`)
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Course description

Modern economics requires substantial knowledge of mathematical techniques. Most of the incoming Ph.D. students should already be familiar with linear algebra and real analysis. The course Econ 205 Mathematics for Economists (commonly known as "math camp") has two purposes. First, it quickly brushes up (for those who are familiar) or introduces (for those who are not familiar) basic mathematical concepts. Second, and more importantly, it studies in detail some mathematical topics that are important for economic analysis yet generally not covered deeply in mathematics courses or textbooks at the undergraduate level. These topics include: nonnegative matrices, convex analysis, constrained optimization, and dynamic programming.

Textbook

Lectures will be based on my lecture notes posted at my website. In addition, the following textbooks may be useful, though not required.

Berge (1959) Concise textbook on topological spaces, convex analysis, and some fixed point theory. Must-have for theorists.

Luenberger (1969) Beautiful textbook on introduction to functional analysis and optimization in infinite-dimensional spaces.

Kolmogorov and Fomin (1970) Introduction to topological spaces and functional analysis.

Rudin (1976) Standard textbook in real analysis.

Simon and Blume (1994) Standard textbook on mathematics for economists.

Sundaram (1996) Introduction to optimization.

Folland (1999) Beautiful textbook on measure theory with introduction to topological spaces and functional analysis with some applications.

Horn and Johnson (2013) A standard reference book for matrix analysis (analytical aspects of linear algebra). Must-have for theorists and econometricians. The 1985 first edition is more readable (the second edition is too encyclopedic).

The lectures assume that you have basic knowledge of linear algebra and real analysis. Students with insufficient mathematical background should consult standard textbooks such as Simon and Blume (1994).

Course outline

The course closely follows the lecture notes. The goal of my lecture note is to get to mathematical results that are useful in economics as fast as possible, though rigorously. To keep the lecture note to a manageable length, I had to omit many topics. For example, neither Riemann nor Lebesgue integration appear. This is not because these topics are not important (they are) but they are treated elsewhere (Rudin, 1976; Folland, 1999), and it seems that remembering basic rules of integration seems to be enough to solve economic problems. On the other hand, I devote quite a few pages to convex analysis and constrained optimization. These topics are crucial in economics but usually not covered in detail by textbooks.

The first half (chapters 1–7) is basic, while the second half (chapters 8–15) is more advanced. There is some overlap between the first and second halves. For example, chapter 6 is an abridged version of chapters 10 and 11. The purpose here is to get to the most important results as quickly as possible for the benefit of the users of theory. However, since we are doing theory, all theorems need to be eventually proved. Since some proofs are quite technical (such as the Karush-Kuhn-Tucker theorem), I have decided to split the material into several chapters at the cost of some repetition and redundancies.

1. Quick review of linear algebra (chapter 1)
2. Quick review of topology in Euclidean spaces (chapter 2)
3. Quick review of calculus (chapters 3, 4)
4. Unconstrained optimization (chapter 5)
5. Introduction to constrained optimization (chapter 6)
6. Contraction Mapping Theorem and applications (chapter 7)
7. Convex sets and functions (chapters 8, 9)
8. Convex and nonlinear optimization (chapters 10, 11)
9. Maximum and Envelope Theorem (chapter 12)
10. Dynamic programming (chapter 14)
11. Introduction to numerical analysis (chapter 15)

By the end of the course, you should be familiar with the following concepts and results:

- Convex sets, Separating Hyperplane Theorem,

- Convex and quasi-convex functions and their properties,
- Karush-Kuhn-Tucker theorem, constraint qualification,
- Contraction Mapping Theorem,
- Implicit Function Theorem,
- Dynamic programming, value function iteration.

You will encounter items in the above list over and over again during the first year of Ph.D. and beyond. It is absolutely necessary to master these items for success. My lecture note covers all of them with complete proofs.

Although not in the list, linear algebra and real analysis are also crucial, though I do not have time to cover in detail and most students should be already familiar anyway. If not, please study as soon as possible. Unfortunately, since I studied these topics as an undergrad in Japan, I do not know which English textbook to recommend. Ask graduate students or do a Google search for recommendations.

If you plan to be a theorist or an econometrician, I also recommend you to study measure theory, probability theory, and functional analysis. Maybe there are Ph.D. level courses at the mathematics department but I don't know.

Exam

The final is on Monday September 7. It will be an open-book, take home exam with a 24 hour deadline.

Questions

The best opportunity to ask questions is *during* the class, for two reasons. First, you can resolve your question immediately (assuming—well—I know the answer). Second, your classmates are likely to have similar questions, so they can benefit from questions being resolved and I benefit by saving time. So, don't be shy, please ask questions. If you have a question outside of class that cannot be resolved by a Google search or discussing with your classmates, you can show up during my office hour listed above (no appointment necessary).

References

Claude Berge. *Espaces Topologiques: Fonctions Multivoques.* Dunod, Paris, 1959. English translation: Translated by E. M. Patterson. *Topological Spaces*, New York: MacMillan, 1963. Reprinted: Mineola, NY: Dover, 1997.

Gerald B. Folland. *Real Analysis: Modern Techniques and Their Applications.* John Wiley & Sons, Hoboken, NJ, second edition, 1999.

Roger A. Horn and Charles R. Johnson. *Matrix Analysis.* Cambridge University Press, New York, second edition, 2013.

A. N. Kolmogorov and S. V. Fomin. *Introductory Real Analysis.* Prentice-Hall, 1970. Translated and edited by R. A. Silverman.

David G. Luenberger. *Optimization by Vector Space Methods.* John Wiley & Sons, New York, 1969.

Walter Rudin. *Principles of Mathematical Analysis*. McGraw-Hill, third edition, 1976.

Carl P. Simon and Lawrence E. Blume. *Mathematics for Economists*. W. W. Norton & Company, New York, 1994.

Rangarajan K. Sundaram. *A First Course in Optimization Theory*. Cambridge University Press, NY, 1996.