

ECONOMICS 113: Mathematical Economics

Fall 2019

Basic information

Lectures	Tu/Th 11:00-12:20 HSS 2154
Instructor	Prof. Alexis Akira Toda
Office hours	Th 9:50-10:50, ECON 211
Email	atoda@ucsd.edu
Webpage	https://sites.google.com/site/aatoda111/ (Go to Teaching → Mathematical Economics)
TA	Peicong (Keri) Hu, Econ 123, p5hu@ucsd.edu

Course description

Carl Friedrich Gauss said mathematics is the queen of the sciences (and number theory is the queen of mathematics).¹ Paul Samuelson said economics is the queen of the social sciences.² Not surprisingly, modern economics is a highly mathematical subject.

Mathematical economics studies the mathematical foundations of economic theory in the approach known as the Arrow-Debreu model of general equilibrium. Partial equilibrium (things like demand and supply curves), which you have probably learned in Econ 100ABC, considers each market separately. General equilibrium (GE for short), on the other hand, considers the economy as a whole, taking into account the interaction of all markets.

Econ 113 is probably the most mathematically advanced undergraduate course offered at UCSD Economics Department, but it should have a high return. In the course, we will develop a mathematical model of classical economic thoughts like Bentham's "greatest happiness principle" and Smith's "invisible hand", and prove theorems. Then we will apply the theory to international trade, finance, social security, etc. Time permitting, I will talk about my own research.

Prerequisites

A year of calculus and a year of upper division microeconomic theory (at UCSD these courses are Math 20ABC and Economics 100ABC) are required. Students with strong mathematical background (typically including one quarter of real analysis, UCSD Math 140A or 142A) may enroll without economics prerequisites. **Although not required, because the lectures are "proof-based", prior or concurrent enrollment in Math 109 or Math 140A is highly recommended.** Students with inadequate

¹http://en.wikiquote.org/wiki/Carl_Friedrich_Gauss

²Samuelson "Economics", 10th edition, preface.

mathematical background should take other mathematics courses before enrolling in Econ 113.

Textbook

The textbook for this course is [1], written by UCSD Emeritus Professor Ross Starr, who has taught this course for many years before I took over. This book does not contain many examples but is self-contained and pedagogic in that it proceeds from the easy and special case to the difficult and general case. An added bonus is an accessible proof of the Brouwer fixed point theorem. [2, Chapters 1–7] is roughly the same level as [1], but contains lots of examples and exercises and thus may complement your study (though not required). Those who wish to pursue a Ph.D. in economics should get [3, 4], which are standard graduate-level texts. However, [3] may not be useful for learning due to its conciseness.

The course will use a lot of math, as the course title suggests. Relevant topics are basic linear algebra (but not much), calculus, convex analysis, and constrained optimization. For the last two topics, either [5] or my lecture notes for Econ 205 Mathematics for Economists (posted at my website) might be useful, but I will cover them briefly in class so you don't need to study in advance.

Preliminary course outline

1. Introduction
2. Definition of Arrow-Debreu model
3. Crash course in convex analysis and convex programming
4. Quasi-linear model: mathematical formulation of Bentham's "greatest happiness principle"
5. First and second welfare theorems: mathematical formulation of Smith's "invisible hand"
6. Existence of equilibrium (correspondences, maximum theorem, Brouwer and Kakutani fixed point theorems)
7. International trade and comparative advantage
8. Capital asset pricing theory (CAPM)

Assignments

There are no assignments for this course (because it is impossible to monitor you to do the assignments by yourselves). However, each lecture note will contain a few exercises, though typically at a higher level than the exam questions. In addition, *you can download past exams from my personal website listed above*, so make sure to check it out. Solve them by yourself: this is the best way to understand the material and get prepared for the exams. Time permitting, we will go through the solution of some of the problems during the class.

Exams

There will be a quiz, a midterm, and a final. Please mark your calendar:

Quiz Thursday October 17, in class

Midterm Thursday November 14, in class

Final Wednesday December 11, 11:30–14:30, location TBA

The exam dates are non-negotiable. If you miss the quiz or the midterm for a documented, university approved reason (i.e., illness, family emergency, official university trip, etc.) the weight for that exam will be placed on the final. If you miss an exam for another reason (i.e., oversleep, vacation, etc.) you will receive a zero for that exam. No one will be allowed to start an exam after the first person leaves it. You are only permitted to use pens, pencils, and a straight edge (*no* calculator).

Grades

Each of the quiz, midterm, and final will be graded on some scale (say 0–100). Your course grade will be determined by the formula

$$G = 0.1 \max\{Q, M\} + 0.2M + 0.2 \max\{M, F\} + 0.5F,$$

where G is the course grade and Q, M, F are the scores on the quiz, midterm, and final. The course grade G will be converted to letter grades at my discretion (i.e., “curved”) at the end of the quarter. (So please don’t ask me questions like “What is the letter grade corresponding to x points in midterm?”)

As you can see from this formula, you are not required to take the quiz, since in theory you can still obtain a perfect course grade if $Q = 0$ and $M = F = 100$. The idea is that I would like to give you some feedback before the deadline to drop class without “W” grade. (For your information, each year the score of the first exam is a strong predictor of the course grade, and about 30% of students drop out after the first exam.)

Regrade requests must be made through a written statement *before* the start of class one week after the exam was first passed back. Extensions will only be permitted if you have a documented, university approved reason for missing the entire week after the exam was first passed back. If you request a regrade I reserve the right to regrade your entire exam and your score could go up, down, or stay the same. My regrade decision is final.

Questions

The best opportunity to ask questions is *during* the class, for two reasons. First, you can resolve your question immediately (assuming—well—I know the answer). Second, your classmates are likely to have similar questions, so they can benefit from questions being resolved and I benefit by saving time. So, don’t be shy, please ask questions. If you have a question outside of class that cannot be resolved by a Google search or discussing with your friends, you can show up during my office hour listed above (no appointment necessary).

How to do well in this course

Get your favorite math text (linear algebra and calculus) or my lecture notes of Econ 205 at hand so that you can refer if necessary. Experience tells that (this is *true*) students who regularly attend classes outperform those who don't, so come to class. Ask questions during the class whenever you don't understand. Read the lecture notes. Solve exercises and past exams (posted at my web page) without looking at the solutions. If you do (very) well in this course, you have a good chance to be admitted to good Master (Ph.D.) programs (and therefore get a lucrative job in the future).

Letter of recommendation

I wish to support students who are ambitious, and therefore if asked I usually write a letter of recommendation. Since Econ 113 is a mini-version of the first year Ph.D. microeconomics, doing well in this course signals that you are prepared for post graduate studies. Typically if you rank within the top half of the class, you might consider applying to good Master programs. Ph.D. is significantly more competitive and challenging (you should consult me if interested).

Academic integrity

I take academic dishonesty seriously. Any student found guilty of academic dishonesty will earn a failing grade for the course. In addition to this sanction, the Council of Deans of Student Affairs will also impose a disciplinary penalty.

UCSD policy:

<http://senate.ucsd.edu/manual/appendices/appendix2.pdf>

Facts about academic integrity:

<http://students.ucsd.edu/academics/academic-integrity/facts.html>

Consequences of cheating:

<http://students.ucsd.edu/academics/academic-integrity/consequences.html>

References

- [1] Ross M. Starr. *General Equilibrium Theory: An Introduction*. Cambridge University Press, second edition, 2011.
- [2] Truman F. Bewley. *General Equilibrium, Overlapping Generations Models, and Optimal Growth Theory*. Harvard University Press, Cambridge, MA, 2007.
- [3] Gerard Debreu. *Theory of Value*. Cowles Foundation Monograph 17. Yale University Press, New Haven, 1959.
- [4] Andreu Mas-Colell, Michael D. Whinston, and Jerry R. Green. *Microeconomic Theory*. Oxford University Press, Cambridge, Massachusetts, 1995.
- [5] Rangarajan K. Sundaram. *A First Course in Optimization Theory*. Cambridge University Press, NY, 1996.