Trajectory tracking sliding mode control of AUV

Cheng Ouyang
Department of Electrical and Computer Engineering
University of Florida
Gainesville, United States
chengouyang@ufl.edu

Kai Wei
Department of Electrical and Computer Engineering
University of Florida
Gainesville, United States
kai.wei@ufl.edu

***Abstract*—A robust motion controller is necessary for the successful operation of AUVs. There are many challenges in the control of AUVs due to the complexity of the AUV model, unmodelled dynamics, uncertainties, and environmental disturbances. In order to solve the problem of trajectory tracking control of AUV, this paper envisions a trajectory tracking control scheme that uses an adaptive sliding mode controller to be robust to bounded perturbations. This results in limited yaw movement.**

***Index Terms*—trajectory tracking, sliding mode control, auv**

I. Introduction

Autonomous underwater vehicles (AUVs) play a major role in the exploration of oceans, and in scientific and military missions. These vehicles are required to execute different types of missions without the interaction of human operators while performing well under a variety of load conditions and with unknown sea currents.

Most available AUVs are underactuated. It is known that an underactuated system cannot be stabilized to an equilibrium point(s) by any continuous time-invariant feedback controller, this implies that discontinuous control laws are needed to solve such a problem. Therefore, the sliding mode control technique is adopted in this paper for the design of controllers for AUVs. Motion control of AUVs may have different control objectives or strategies such as trajectory tracking, path following and way-point tracking. The focus of this paper is the trajectory tracking control problem of AUVs restricted to the horizontal plane (i.e., lateral motion). By targeting the expected speed of the AUV, we overcome the problem that we can only track paths under special circumstances to ensure that the trajectory tracking problem can be solved under the general circumstances of the reference trajectory. Then, a control law is designed that forces the vehicle to track the proposed desired speed with no limit on yaw speed, thus guaranteeing tracking of a straight line.

Sliding mode control (SMC) is one of the common strategies to deal with uncertain control systems. The main feature of SMC is the robustness against parameter variations and external disturbances.

II. Model and control design

A. Auv modeling in the horizontal plane

First, we need to model the AUVs. The motion of AUVs is described in 6 degrees of freedom (DOFs) which corresponds to the set of independent displacements and rotations that describes the vehicle's position and orientation.

These six degrees of freedom are surge (longitudinal motion) x, the sway (lateral motion) y and the yaw (rotation about the vertical axis) ψ, heave (vertical motion) u, the roll (rotational motion about the longitudinal axis) v and the pitch (rotational motion about the lateral axis) r.

$$\begin{aligned}
\dot{x} &= u\cos\psi - v\sin\psi \\
\dot{y} &= u\sin\psi + v\cos\psi \\
\dot{\psi} &= r \\
\dot{u} &= M_1(X_u u + a_{23} vr + \tau_u) \\
\dot{v} &= M_2(Y_v v + a_{13} ur) \\
\dot{r} &= M_3(N_r r + a_{12} uv + \tau_r)
\end{aligned} \tag{1}$$

where $M_1 := 1/(m - X_{\dot{u}})$, $M_2 := 1/(m - Y_{\dot{v}})$, and $M_3 := 1/(I_z - N_{\dot{r}})$. $a_{12} := Y_{\dot{v}} - X_{\dot{u}}$, $a_{13} := X_{\dot{u}} - m$, and $a_{23} := m - Y_{\dot{v}}$.are also defined. The control inputs are the surge force τ_u and the yaw moment τ_r.

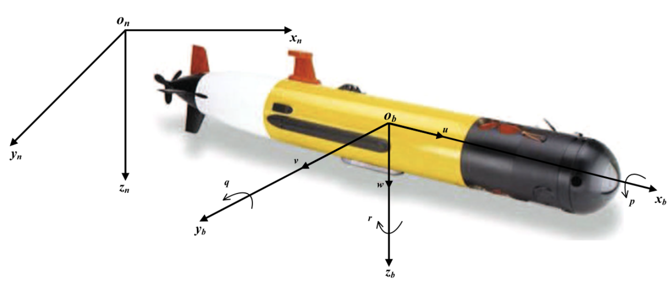

Fig. 1: The earth-fixed and body-fixed reference frames for an AUV.

B. Sliding mode trajectory tracking control design

We define the following position tracking errors:

$$\begin{aligned}
x_e &= x - x_d \\
y_e &= y - y_d
\end{aligned} \tag{2}$$

The time derivative of tracking error equations are:

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} - \begin{bmatrix} \dot{x}_d \\ \dot{y}_d \end{bmatrix} \tag{3}$$

we define the following velocity tracking errors:

$$\begin{aligned} e_u &= u - u_d \\ e_v &= v - v_d \end{aligned} \tag{4}$$

The time derivative of tracking error equations are:

$$\begin{aligned} \dot{e}_u &= M_1(X_u u + a_{23} v r + \tau_u) - \dot{u}_d \\ \dot{e}_v &= M_2(Y_v v + a_{13} u r) - \dot{v}_d \end{aligned} \tag{5}$$

We want to design robust control laws for the surge force τ_u and the yaw moment τ_r so that the position of the AUV can track a desired, time-varying trajectory, and guarantee that the vehicle's actual position $(x(t), y(t))$ tracks a desired, time-varying trajectory $(x_d(t), y_d(t))$.

Fig.2 is the block diagram of the AUV trajectory tracking control system.

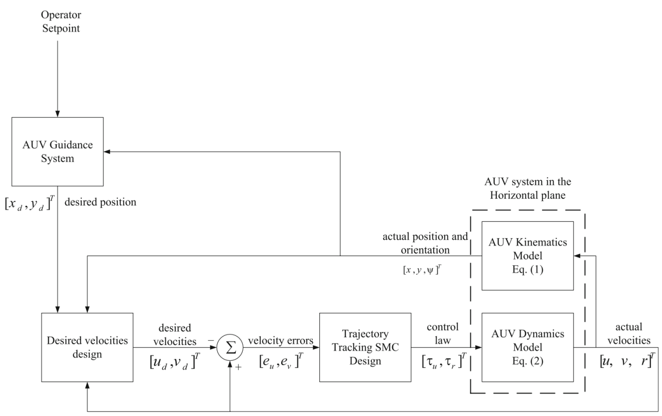

Fig. 2: A block diagram of the AUV's trajectory tracking control system.

In order to reduce the complexity of the control design, it is divided into two stages. The first stage handles the design of the desired linear surge and sway velocities (i.e., u_d and v_d, respectively) on the kinematic level. Let the desired surge and sway velocities be such that,

$$\begin{bmatrix} u_d \\ v_d \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} \dot{x}_d + l_x \tanh\left(-\frac{k_x}{l_x} x_e\right) \\ \dot{y}_d + l_y \tanh\left(-\frac{k_y}{l_y} y_e\right) \end{bmatrix} \tag{6}$$

Then, we can get the velocity errors as following:

$$\begin{bmatrix} e_u \\ e_v \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} \dot{x}_e - l_x \tanh\left(-\frac{k_x}{l_x} x_e\right) \\ \dot{y}_e - l_y \tanh\left(-\frac{k_y}{l_y} y_e\right) \end{bmatrix} \tag{7}$$

Then, a control law is designed for the surge force and yaw moment to ensure the asymptotic convergence of the surge and sway velocities to the designed desired values using the sliding mode control technique.

We will choose the sliding surfaces such that:

$$\begin{aligned} S_1 &= e_u + \lambda_1 \int_0^t e_u(\tau)\, d\tau \\ S_2 &= \dot{e}_v + \lambda_3 e_v + \lambda_2 \int_0^t e_v(\tau)\, d\tau \end{aligned} \tag{8}$$

The trajectory tracking surge and yaw control laws are chosen such that:

$$\begin{aligned} \tau_u &= \tau_{u,\text{eq}} + \tau_{u,\text{sw}} \\ \tau_r &= \tau_{r,\text{eq}} + \tau_{r,\text{sw}} \end{aligned} \tag{9}$$

Where,

$$\begin{aligned} \tau_{u,\text{eq}} &= -X_u u - a_{23} v r + \frac{1}{M_1}\dot{u}_d - \frac{1}{M_1}\lambda_1 e_u \\ \tau_{u,\text{sw}} &= \frac{1}{M_1}\left(-k_1 S_1 - W_1 \text{sign}(S_1)\right) \\ \tau_{r,\text{eq}} &= -N_r r - a_{12} u v + \frac{1}{b}\left(-M_2(Y_v \dot{v} + a_{13}\dot{u} r)\right) \\ &\quad + \frac{1}{b}\left(\Gamma - \lambda_3 \dot{e}_v - \lambda_2 e_v\right) \\ \tau_{r,\text{sw}} &= \frac{1}{b}\left(-k_2 S_2 - W_2 \text{sign}(S_2)\right) \end{aligned} \tag{10}$$

where the following definitions are made

$$\begin{aligned} b &= M_3(M_2 a_{13} u + u_d) \\ \Gamma &= -\dddot{x}_d \sin\psi + \dddot{y}_d \cos\psi - \ddot{x}_d r \cos\psi \\ &\quad - \ddot{y}_d r \sin\psi - \dot{u}_d r + \Upsilon_1 r \cos\psi + \Upsilon_2 r \sin\psi \\ &\quad + \dot{\Upsilon}_1 \sin\psi - \dot{\Upsilon}_2 \cos\psi \\ \Upsilon_1 &= k_x \dot{x}_e \,\text{sech}^2\left(-\frac{k_x}{l_x} x_e\right) \\ \Upsilon_2 &= k_y \dot{y}_e \,\text{sech}^2\left(-\frac{k_y}{l_y} y_e\right) \end{aligned} \tag{11}$$

Finally, we need to prove the stability of the tracking system through simulation.

III. Lyapunov-based Stability Analysis

Consider the following Lyapunov function candidate:

$$V_2 = \frac{1}{2} S_1^2 + \frac{1}{2} S_2^2$$

The time derivative of V_2 can be written as follows:

$$\begin{aligned} \dot{V}_2 &= S_1[M_1(X_u u + a_{23} v r + \tau_u) - \dot{u}_d + \lambda_1 e_u] \\ &\quad + S_2[M_2(Y_v \dot{v} + a_{13} u \dot{r} + a_{13} u M_3 \dot{r}) - \ddot{v}_d + \lambda_3 \dot{e}_v + \lambda_2 e_v] \\ &= -k_1 S_1^2 - W_1 S_1 \text{sign}(S_1) + S_2[M_2(Y_v \dot{v} + a_{13} u \dot{r})] \\ &\quad + S_2[-\Gamma + b(N_r r + a_{12} u v + \tau_r) + \lambda_3 \dot{e}_v + \lambda_2 e_v] \\ &= -k_1 S_1^2 - W_1 |S_1| - k_2 S_2^2 - W_2 |S_2| \end{aligned} \tag{12}$$

Since $k_1, k_2 \geq 0$ and $W_1, W_2 \geq 0$, it is obviously that $\dot{V}_2 < 0$ for $(S_1, S_2) \neq 0$. This implies that the sliding surface can reach zero in finite time. Furthermore, on the sliding surface, we can obtain the following dynamics:

$$\dot{e}_u = -\lambda_1 e_u \tag{13}$$

$$\ddot{e}_v = -\lambda_3 \dot{e}_v - \lambda_2 e_v \tag{14}$$

Since $\lambda_1, \lambda 2$ and λ_3 are positive scalars, the dynamics ensure the asymptotic convergence of (e_u, e_v) to (0,0).

According to Eq. (7), if the velocity errors converge to zero then we can get:

$$\begin{aligned} \dot{x}_e &= l_x \tanh\left(\frac{-k_x}{l_x} x_e\right) \\ \dot{y}_e &= l_y \tanh\left(\frac{-k_y}{l_y} y_e\right) \end{aligned} \tag{15}$$

The next step is to prove that (x_e, y_e) converge to $(0, 0)$. We can choose the following Lyapunov function candidate:

$$V_1 = \frac{1}{2} x_e^2 + \frac{1}{2} y_e^2$$

Differentiating this Lyapunov function we can get:

$$\begin{aligned} \dot{V}_1 &= x_e \dot{x}_e + y_e \dot{y}_e \\ &= -l_x x_e \tanh\left(\frac{k_x}{l_x} x_e\right) - l_y y_e \tanh\left(\frac{k_y}{l_y} y_e\right) \end{aligned} \tag{16}$$

Since $k_x, k_y > 0$ and $l_x, l_y > 0$, it is clearly that $\dot{V}_1 < 0$ when $(x_e, y_e) \neq (0, 0)$. Thus, it can be concluded that both x_e and y_e asymptotically converge to zero.

TABLE I: The REMUS AUV model parameters

Parameter	Value	Units
m	30.48	kg
I_z	3.45	kg m^2
X_u	-8.8065	kg/s
Y_v	-65.5457	kg/s
N_r	-6.7352	kg/s
$X_{\dot{u}}$	-0.93	kg
$Y_{\dot{v}}$	-35.5	kg
$N_{\dot{r}}$	-35.5	kg m^2

IV. Simulation Results

The simulation are done using the MATLAB software. The parameters of the model are based on the REMUS autonomous underwater vehicle, as listed in Table 1. The initial values are assumed to be zero, indicating that the vehicle is at rest: $x(0) = y(0) = \psi(0) = u(0) = v(0) = r(0) = 0$.

To avoid the chattering problem, we use hyperbolic tangent function to approximate signum function such that $sign(a) \approx \tanh(\gamma a)$ where γ is a positive scalar which can be chosen to get a very good approximation.

We simulate for three different cases to check system's stability and performance, These cases are as follows:

- Case I: Linear motion ($C_1 = 0$ and $C_2 = 0$)

$$x_d(t) = t, \quad y_d(t) = t$$

- Case II: Linear motion ($C_1 \neq 0$ and $C_2 \neq 0$)

$$x_d(t) = t + 5, \quad y_d(t) = 2$$

- Case III: Circular motion ($C_1 = 0$ and $C_2 = 0$)

$$x_d(t) = \cos(t), \quad y_d(t) = \sin(t)$$

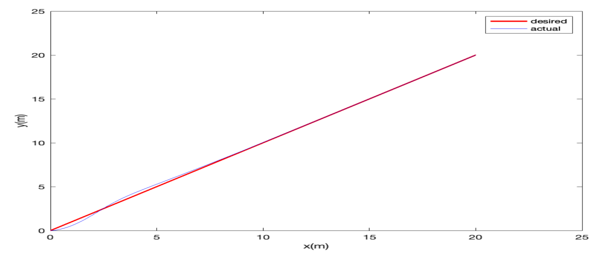

Fig. 3: The actual and desired paths of the AUV using the proposed trajectory tracking controller for case I

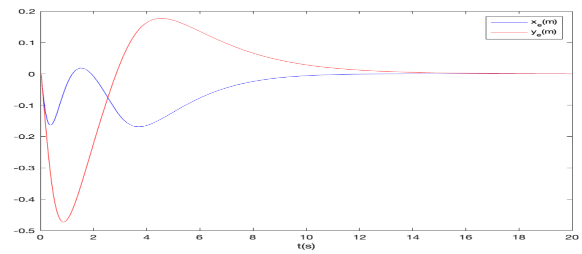

Fig. 4: The position tracking errors of the AUV versus time using the proposed trajectory tracking controller for case I

In these simulations, the design parameters are taken to be $l_x = l_y = 2, k_x = k_y = 0.5, \lambda_1 = 2, \lambda_2 = 7, \lambda_3 = 8, k_1 = 0, k_2 = 0, W_1 = 0.5$ and $W_2 = 3$. The simulation results for case I are presented in Figs. 3, 4, 5, 6 and 7, the results for both case II and III are shown in appendix.

Take case I as an example, In Figure 3, the red line is the actual actual and desired path of the vehicle. We can see that the AUV successfully tracked the desired trajectory.

The asymptotic convergence of the vehicle's position tracking errors x_e and y_e to zero is clear as shown in Figure 4.

Figures 5 shows that the yaw motion is bounded in each case. This figure also show the surge, sway and yaw velocities versus time.

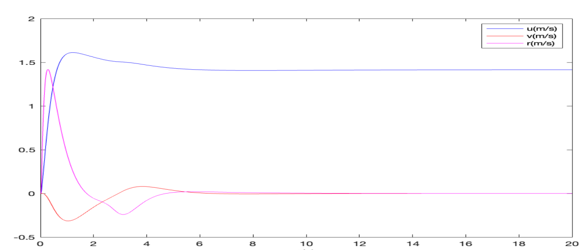

Fig. 5: The velocities of the AUV versus time using the proposed trajectory tracking controller for case I

Also, in figure 6 , we can see that it takes about 6 seconds for the velocities tracking errors converging to zero.

Moreover, the simulated surge force and yaw moment (controller output) versus time are presented in Figure 7.

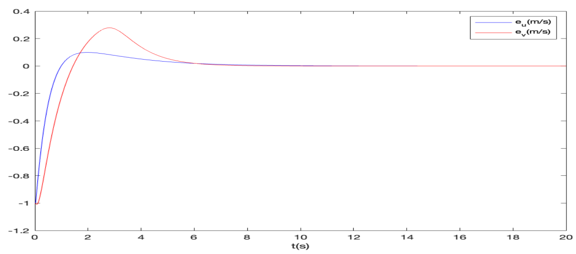

Fig. 6: The velocity tracking errors of the AUV versus time using the proposed trajectory tracking controller for case I

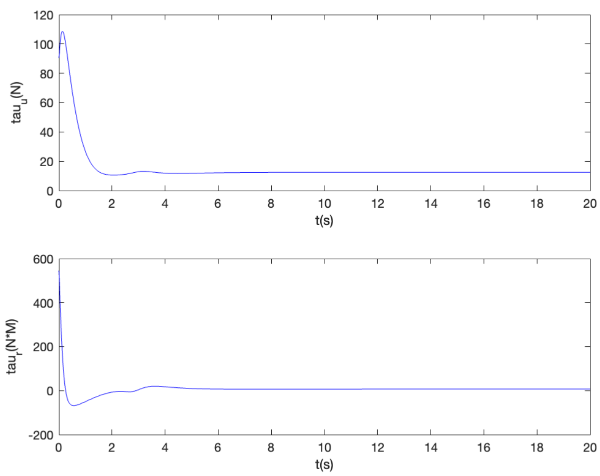

Fig. 7: The surge and yaw control laws of the AUV versus time using the proposed trajectory tracking controller for case I

V. Discussion

A. Change controller gain

Controller gain affects the dynamic characteristics of the system in sliding mode. Gain selection often needs to be tailored to specific system requirements and performance metrics. When the gain is larger, the system will respond faster, but may introduce more oscillations and noise; when the gain is smaller, the system response will be smoother, but it may result in a slower response.

Take case 3 as example, we changed controller gains W_1 and W_2 from 0.5 and 3 to 0.05 and 0.3. Figure.8 are the position tracking errors of the AUV. We can see that position tracking errors can be bounded but can not be canceled.

We changed controller gains k_1, k_2, W_1 and W_2 from 0, 0, 0.5 and 3 to 10, 10, 50 and 300. Figure.9 are the velocity tracking errors of the AUV. We can see that high controller gains brings high-frequency oscillation to the system.

B. Change switching surface parameters

Changing the sliding mode surface parameters will have an impact on the performance and robustness of the control system, and the specific impact varies depending on many factors.

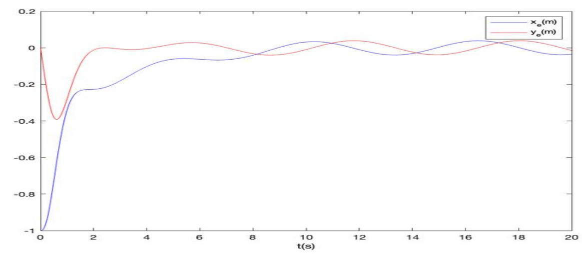

Fig. 8: The position tracking errors of the AUV when k decrease

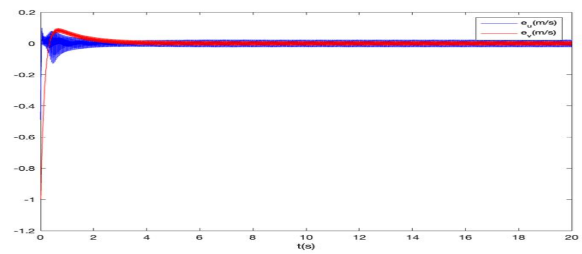

Fig. 9: The velocity tracking errors of the AUV when k and w increase

The design of sliding mode control usually involves the slope of the sliding mode surface, which will directly affect the response speed of the system. A larger slope may result in faster system response, but may also introduce more high-frequency oscillations. A smaller slope will cause the system to respond more slowly.

We changed sliding surface parameters λ_1, λ_2, and λ_3 from 2, 7, and 8 to 0.2, 0.7, and 0.8 respectively.

Figure.10 are the position tracking errors. We can see that the system responded slower and it took about 18 seconds for the position tracking errors to converge to 0.

C. Robustness studies

In this section, We do simulation to test the robustness of the proposed trajectory tracking control scheme. The disturbances

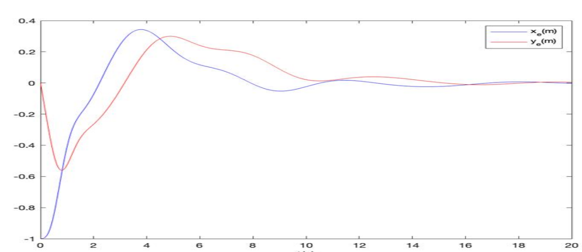

Fig. 10: The position tracking errors of the AUV when λ decrease

are included in the AUV model. The new AUV kinematic and dynamic equations are,

$$
\begin{aligned}
\dot{x} &= u\cos\psi - v\sin\psi \\
\dot{y} &= u\sin\psi + v\cos\psi \\
\dot{\psi} &= r \\
\dot{u} &= M_1(X_u u + a_{23}vr + \tau_u) + d_1(t) \\
\dot{v} &= M_2(Y_v v + a_{13}ur) \\
\dot{r} &= M_3(N_r r + a_{12}uv + \tau_r) + d_2(t)
\end{aligned}
\tag{17}
$$

The terms d_1 and d_2 represent the external disturbances (e.g. back flow) and model uncertainties. The bounds on the disturbances are assumed known (or estimated) in order to properly select the controller gains and consequently suppress the disturbance effects.

The following three disturbance scenarios are considered in the simulations,

- Constant Disturbances,

$$d_1(t) = 0.2, \quad d_2(t) = 0.15$$

- Sinusoidal Disturbances,

$$d_1(t) = 0.15\cos(t), \quad d_2(t) = 0.2\sin(t)$$

- Disturbances for a period of time,

$$
\begin{aligned}
d_1(t) &= 0.5[u_s(t-5)u_s(t-6)], \\
d_2(t) &= 2[u_s(t-5)u_s(t-6)]
\end{aligned}
$$

where $u_s(t)$ is the unit step function defined as,

$$u_s(t) = \begin{cases} 1, & \text{if } t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

The simulation results for the case II with constant disturbance are presented in Figs. 11, 12, 13, 14. The actual and desired paths of the AUV are shown in Figs. 11. Figures 12 show the position errors of the AUV versus time. Figures 13 show the velocity errors versus time. The surge and yaw controllers for the case of disturbance is depicted in Figs. 14. These figures clearly indicate that the proposed control scheme is robust to bounded uncertainties. The simulation results of sinusoidal disturbance and disturbances for a period time are in appendix.

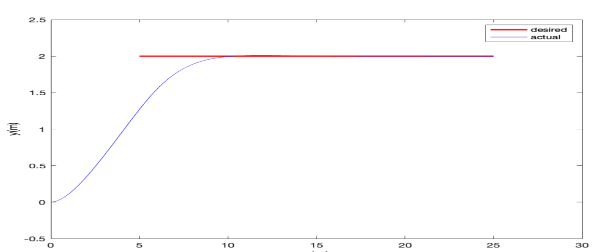

Fig. 11: The actual and desired paths of the AUV using the proposed trajectory tracking controller with constant disturbances

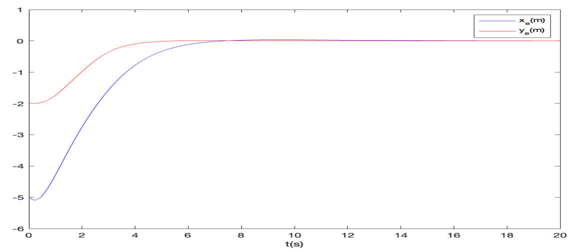

Fig. 12: The position errors of the AUV versus time using the proposed trajectory tracking controller with constant disturbances

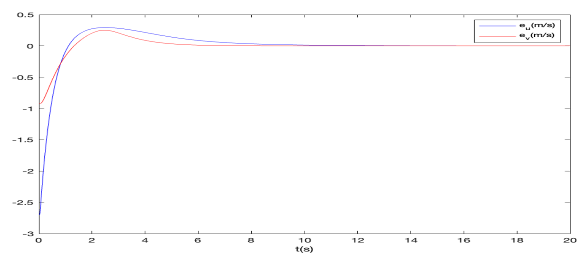

Fig. 13: The velocity errors of the AUV versus time using the proposed trajectory tracking controller with constant disturbances

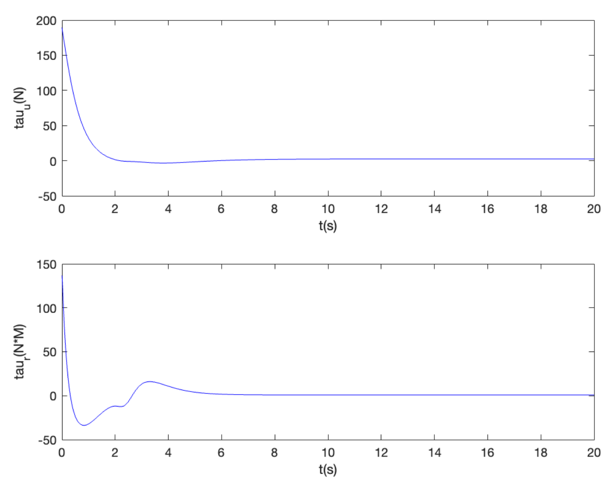

Fig. 14: The surge and yaw control laws of the AUV versus time using the proposed trajectory tracking controller with constant disturbances

When we increase the disturbances d_1 and d_2 to 2 and 1.5 respectively, the system will become unstable. It can be seen from picture 15 that the position error of the AUV tends to diverge.

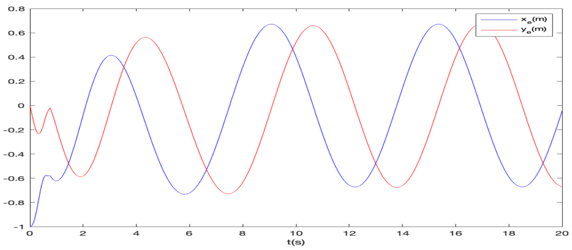

Fig. 15: The position tracking error of AUV when d increase

VI. Conclusion

A trajectory tracking sliding mode control scheme is proposed for the control of the lateral motion of AUVs. This control mechanism aims to precisely align the AUV's position with a predefined, dynamically evolving trajectory. To ascertain the efficacy of the proposed control design, it was implemented on an AUV model followed by a series of simulations under various trajectory scenarios. The outcomes of these simulations reveal that the control strategy consistently achieved accurate trajectory tracking across all tested scenarios. Furthermore, the results demonstrate a significant robustness of the control system against a range of bounded external disturbances.

References

[1] T. Elmokadem, M. Zribi, and K. Youcef-Toumi, "Trajectory tracking sliding mode control of underactuated AUVs," Nonlinear Dynamics, vol. 84, pp. 1079–1091, 2016, Springer.

APPENDIX

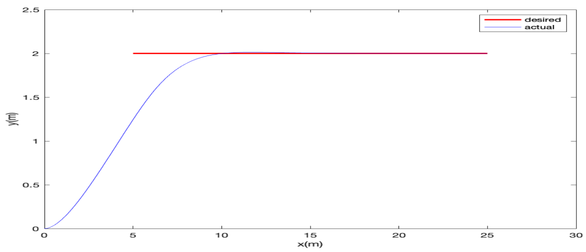

Fig. 16: The actual and desired paths of the AUV using the proposed trajectory tracking controller for case II

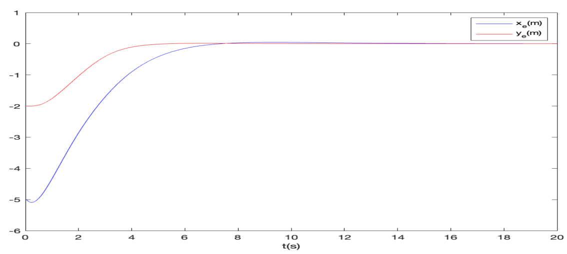

Fig. 17: The position tracking errors of the AUV versus time using the proposed trajectory tracking controller for case II

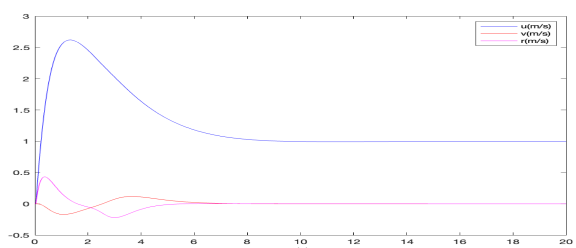

Fig. 18: The velocities of the AUV versus time using the proposed trajectory tracking controller for case II

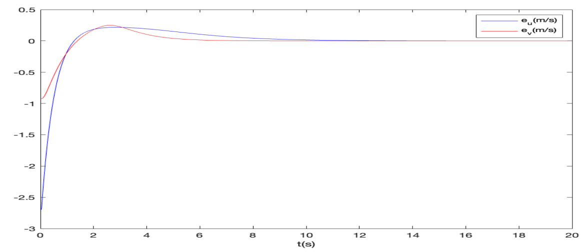

Fig. 19: The velocity tracking errors of the AUV versus time using the proposed trajectory tracking controller for case II

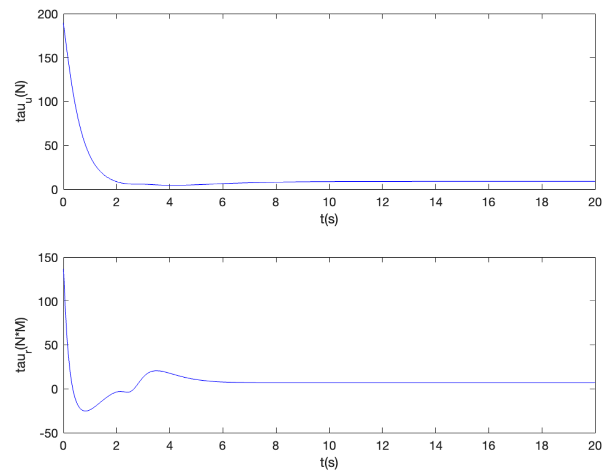

Fig. 20: The surge and yaw control laws of the AUV versus time using the proposed trajectory tracking controller for case II

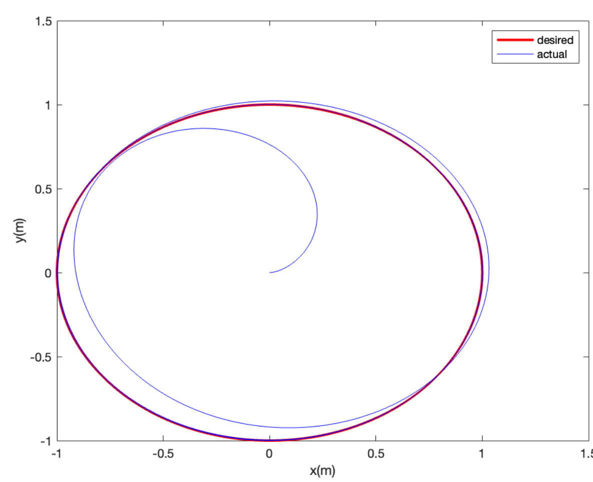

Fig. 21: The actual and desired paths of the AUV using the proposed trajectory tracking controller for case III

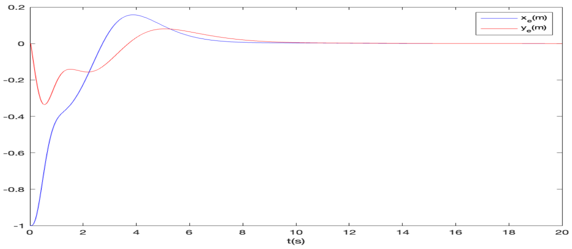

Fig. 22: The position tracking errors of the AUV versus time using the proposed trajectory tracking controller for case III

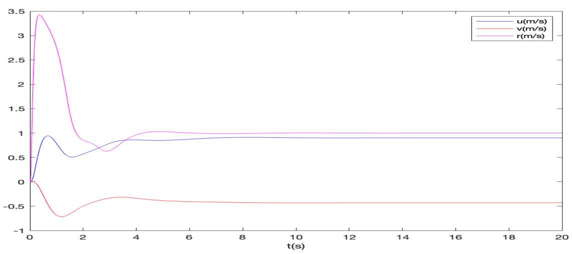

Fig. 23: The velocities of the AUV versus time using the proposed trajectory tracking controller for case III

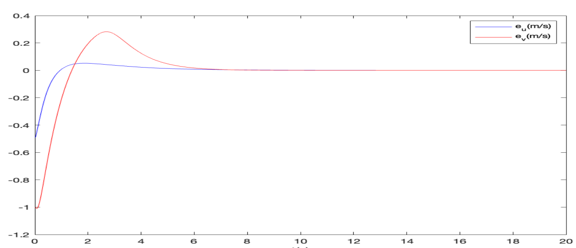

Fig. 24: The velocity tracking errors of the AUV versus time using the proposed trajectory tracking controller for case III

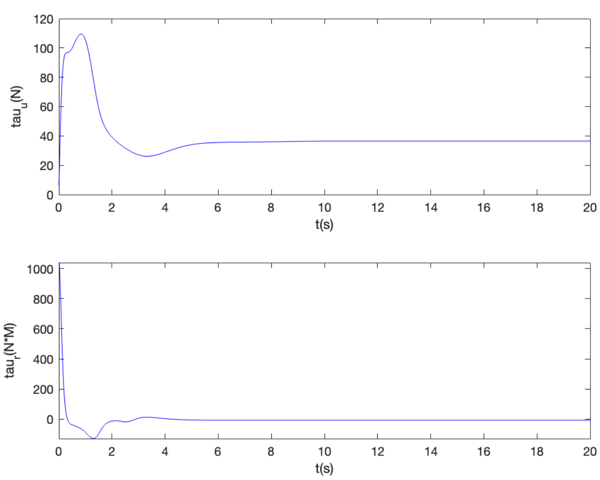

Fig. 25: The surge and yaw control laws of the AUV versus time using the proposed trajectory tracking controller for case III

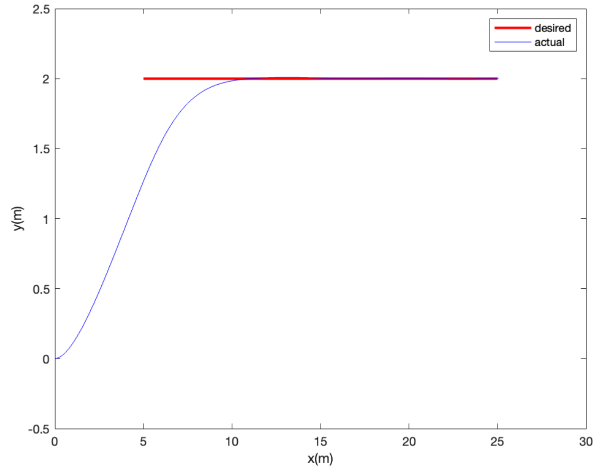

Fig. 26: The actual and desired paths of the AUV using the proposed trajectory tracking controller with sinusoidal disturbances

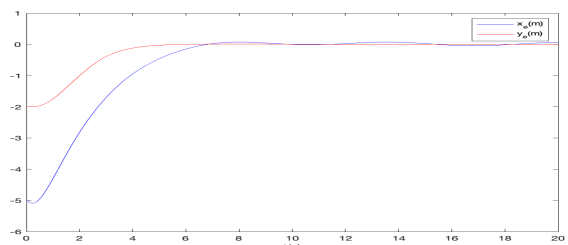

Fig. 27: The position errors of the AUV versus time using the proposed trajectory tracking controller with sinusoidal disturbances

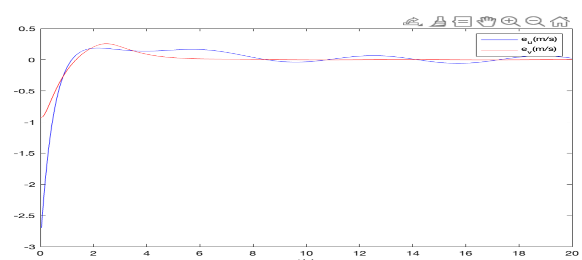

Fig. 28: The velocity errors of the AUV versus time using the proposed trajectory tracking controller with sinusoidal disturbances

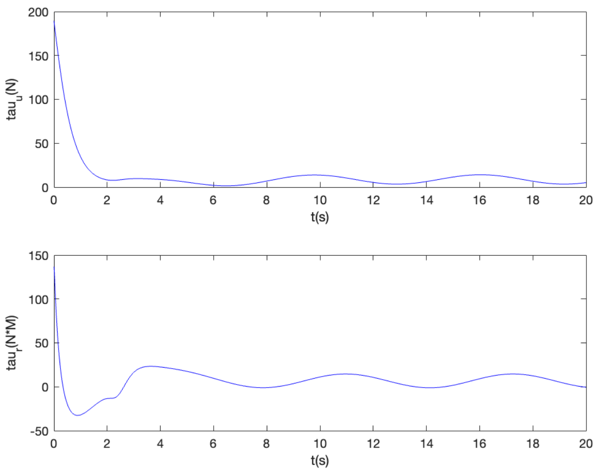

Fig. 29: The surge and yaw control laws of the AUV versus time using the proposed trajectory tracking controller with sinusoidal disturbances

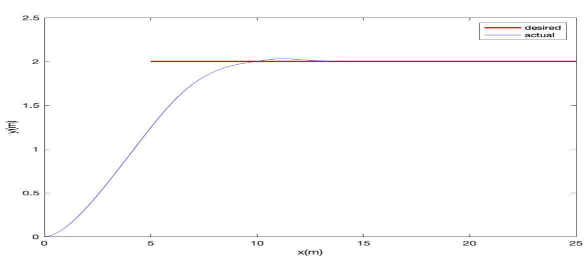

Fig. 30: The actual and desired paths of the AUV using the proposed trajectory tracking controller with disturbances for a period of time

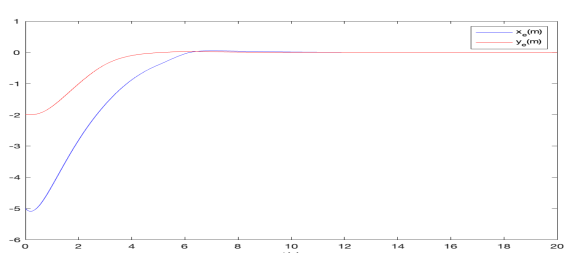

Fig. 31: The position errors of the AUV versus time using the proposed trajectory tracking controller with disturbances for a period of time

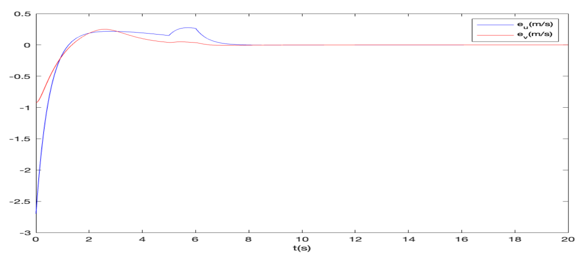

Fig. 32: The velocity errors of the AUV versus time using the proposed trajectory tracking controller with disturbances for a period of time

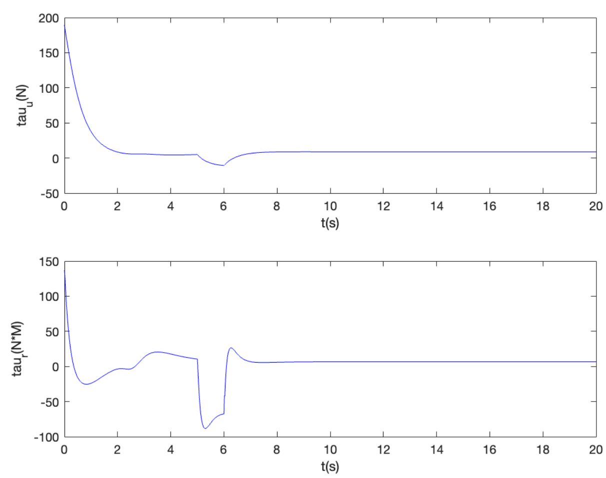

Fig. 33: The surge and yaw control laws of the AUV versus time using the proposed trajectory tracking controller with disturbances for a period of time