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ORIGINAL PAPER

Trajectory tracking sliding mode control of underactuated AUVs

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Abstract This paper deals with the control of underactuated autonomous underwater vehicles (AUVs). AUVs are needed in many applications such as the exploration of oceans, scientific and military missions, etc. There are many challenges in the control of AUVs due to the complexity of the AUV model, the unmodelled dynamics, the uncertainties and the environmental disturbances. A trajectory tracking control scheme is proposed in this paper; this control scheme is designed using the sliding mode control technique in order to be robust against bounded disturbances. The control performance of an example AUV, using the proposed method, is evaluated through computer simulations. These simulation studies, which consider different reference trajectories, show that the proposed control scheme is robust under bounded disturbances.

Keywords Autonomous underwater vehicles · AUV · Underactuated · Trajectory tracking · Sliding mode control

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1 Introduction

Autonomous underwater vehicles (AUVs) play a major role in the exploration of oceans, and in scientific and military missions. These vehicles are required to execute different types of missions without the interaction of human operators while performing well under a variety of load conditions and with unknown sea currents.

The complexity of the AUV dynamics makes their control a challenging task. These challenges include the AUV nonlinear dynamics, the unmodelled dynamic effects, the system's uncertainties and the environmental disturbances. Furthermore, the common AUV prototypes are underactuated where the number of control inputs is less than the vehicles degrees of freedom. These challenges make the problem of controlling AUVs very attractive to researchers. Hence, these challenges along with the wide applications of autonomous underwater vehicles are the main motivation for undertaking this work.

The six degrees of freedom (DOF) model of AUVs is highly nonlinear and very complex. Different approaches were used in order to deal with the complexity and the nonlinearity of the model. The high nonlinearity of the model was handled in some works through the linearization of the model around operating points. Some researchers reduced the complexity of the AUV model by linearizing it about an operating forward speed such as the work in [1]. A common approach used in controlling AUVs is to divide the 6 DOF model of the AUV into lightly interacting models for lateral motion and vertical motion. To reduce the complexity of the overall model of the AUV, the authors of [2] divided the model into three subsystems for speed control, steering and diving. Then, they designed a controller using the sliding mode control technique.

Most available AUVs are underactuated. It is known that an underactuated system cannot be stabilized to an equilibrium point(s) by any continuous time-invariant feedback controller [3]; this implies that discontinuous control laws are needed to solve such a problem. Therefore, the sliding mode control technique is adopted in this paper for the design of controllers for AUVs. Furthermore, since AUVs often operate in harsh underwater environments, the designed control laws need to be robust against unmodelled dynamics, model uncertainties and external disturbances due to ocean currents and waves. This is another reason that motivates us to consider the sliding mode control technique which is known for its robustness that provides superior tracking performance even when bounded disturbances are acting on the AUV or with parameters or model uncertainties.

Motion control of AUVs may have different control objectives or strategies such as trajectory tracking, path following and way-point tracking. The trajectory tracking refers to the design of control laws so that the vehicle tracks a desired, time-parametrized trajectory, while the path following requires the path to be independent of time and is expressed in terms of its geometric description [4]. For the way-point tracking control problem, it is required to guide the AUV through a series of way points between the vehicle's starting position and a desired final position.

The focus of this paper is the trajectory tracking control problem of AUVs restricted to the horizontal plane (i.e., lateral motion). Many research works were conducted for controlling AUVs and marine vehicles with different control techniques such as sliding mode control [2,5–7], higher-order sliding mode [8], adaptive control [9–13], learning control [14], Neural network control [15–18], fuzzy control [19,20], Lyapanov-based techniques [21,22] and Lyapanov's direct method [23]. In [6], a trajectory tracking sliding mode controller is designed for surface vessels. Nonlinear Lyapanov-based techniques are used in [21,22] to develop trajectory tracking controllers for underactuated ships. Lyapanov's direct method is used in [23] to solve the trajectory tracking control problem of underactuated ships.

The main contribution of this paper is the proposal of a sliding mode control law for the trajectory tracking problem of AUVs in the horizontal plane. The proposed control scheme solves the trajectory tracking problem for general cases of reference trajectories rather than for some special cases as it was done in the literature such as [6,21–23]. The control design proposed by Ashrafiuon et al. [6] does not solve the trajectory tracking problem for some cases. This issue is solved in this work. In [21–23], restrictions were made on the rotational motion of the vehicle which imply that the vehicle cannot track straight lines. These problems are overcome in this paper by proposing a new design for the AUV's desired velocities in order to guarantee that the trajectory tracking problem can be solved for the general case of reference trajectories. Then, a control law is designed that forces the vehicle to track the proposed desired velocities where no restrictions are made on the yaw velocity which guarantees the tracking of straight lines.

The organization of this paper is as follows. In Sect. 2, a model of the AUV in the horizontal plane is presented. Section 3 presents the problem formulation of the trajectory tracking control of AUVs considered in this paper. The control design is then proposed in Sect. 4. In Sect. 5, the performance of the proposed control scheme is validated using computer simulations. Moreover, robustness studies are presented in Sect. 6. Finally, conclusions of this work are summarized in Sect. 7.

2 AUV modeling in the horizontal plane

In order to study the dynamics of an AUV, the analysis is done by separating the study of the geometrical aspects of the motion which is referred to as *kinematics* from the analysis of the forces causing the motion which is the *kinetics* [24]. Generally, the motion of AUVs is described in 6 degrees of freedom (DOFs) which corresponds to the set of independent displacements and rotations that describes the vehicle's position and orientation. The lateral dynamics of AUVs is the focus of this study which is referred to as the surge (longitudinal motion), the sway (lateral motion) and the yaw (rotation about the vertical axis), as depicted in Fig. 1. The remaining DOFs are the heave (vertical motion), the roll (rotational motion about the longitudinal axis) and the pitch (rotational motion about the lateral axis).

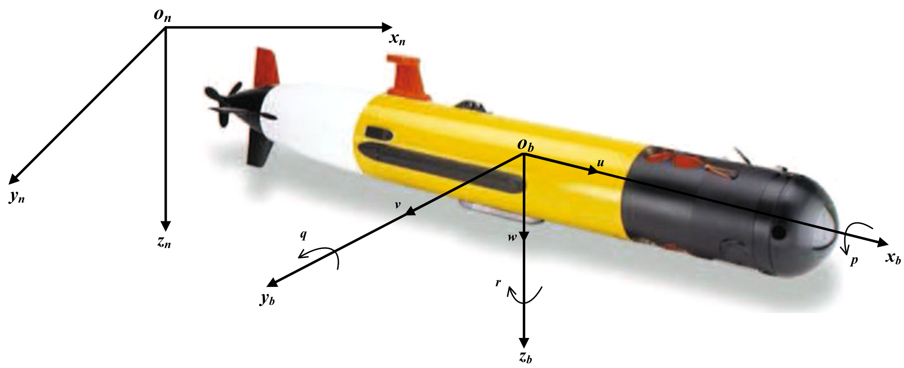

Fig. 1 The earth-fixed and body-fixed reference frames for an AUV

In addition, special reference frames need to be defined to study the motion of AUVs, and they are the *Earth-fixed* $\{n\}$ and the *body-fixed* $\{b\}$ reference frames. These frames are such that [25]:

- The **Earth-fixed** frame $\{n\} = (x_n, y_n, z_n)$ is called the North-East-Down frame (NED), and it is considered to be inertial. The vehicle's coordinates in this frame are described relative to a fixed origin o_n defined in the center of this frame.
- The **body-fixed** frame (BODY) $\{b\} = (x_b, y_b, z_b)$ is a moving frame fixed to the vehicle. Its origin o_b can be defined at the center of the vehicle. The axes of this frame are usually chosen to coincide with the principal axes of inertia.

The considered model in this paper is the 3DOF model of the AUV in the horizontal plane derived in [25]. The kinematic equations of this model are such that:

$$\begin{aligned} \dot{x} &= u\cos\psi - v\sin\psi \\ \dot{y} &= u\sin\psi + v\cos\psi \\ \dot{\psi} &= r \end{aligned} \tag{1}$$

where u and v are the surge and the sway linear velocities of the vehicle, respectively, r represents the vehicle's yaw angular velocity, x and y are the coordinates of the vehicle's center of mass, and ψ describes its orientation. The linear and angular velocities (i.e., (u, v, r)) are defined in the body-fixed frame $\{b\}$, and the vehicle's position coordinates and orientation (i.e., (x, y, ψ)) are defined in the earth-fixed frame $\{n\}$.

In order to write the equations of motion, the following standard notation is used: m is vehicle's mass, I_z is the vehicle's moment of inertia about the z-axis, X_u, Y_v and N_r are the linear damping terms, and $X_{\dot{u}}$, $Y_{\dot{v}}$ and $N_{\dot{r}}$ are the hydrodynamic added mass terms in the surge, the sway and the yaw directions of motion, respectively. The 3 DOF equations of motion are obtained by neglecting the heave, the roll and the pitch motions such that [25]:

$$\begin{aligned} \dot{u} &= M_1(X_u u + a_{23} v r + \tau_u) \\ \dot{v} &= M_2(Y_v v + a_{13} u r) \\ \dot{r} &= M_3(N_r r + a_{12} u v + \tau_r) \end{aligned} \tag{2}$$

where $M_1 := 1/(m - X_{\dot{u}})$, $M_2 := 1/(m - Y_{\dot{v}})$, $M_3 := 1/(I_z - N_{\dot{r}})$, $a_{12} := Y_{\dot{v}} - X_{\dot{u}}$, $a_{13} := X_{\dot{u}} - m$ and $a_{23} := m - Y_{\dot{v}}$. The control inputs are the surge force τ_u and the yaw moment τ_r.

Notice that in the above model, there is no actuation in the sway equation of motion. Therefore, the problem being considered corresponds to an underactuated control problem with only two actuators that generate a force and a moment in the surge and yaw directions (τ_u and τ_r, respectively).

3 Problem formulation of the trajectory tracking of AUVs

3.1 Coordinate transformation

Before formulating the trajectory tracking control problem tackled in this paper, we define the following position tracking errors:

$$\begin{aligned} x_e &= x - x_d \\ y_e &= y - y_d \end{aligned} \tag{3}$$

where x_d and y_d are the coordinates of the desired, time-varying positions.

By taking the time derivatives of the position errors in (3), and using (1), the obtained position error dynamics are as follows:

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} - \begin{bmatrix} \dot{x}_d \\ \dot{y}_d \end{bmatrix} \tag{4}$$

Moreover, we define the following velocity tracking errors:

$$\begin{aligned} e_u &= u - u_d \\ e_v &= v - v_d \end{aligned} \tag{5}$$

where u_d and v_d are the desired surge and sway velocities, respectively. By differentiating (5) with respect to time and using (2), the following equations are obtained,

$$\begin{aligned} \dot{e}_u &= M_1(X_u u + a_{26} v r + \tau_u) - \dot{u}_d \\ \dot{e}_v &= M_2(Y_v v + a_{16} u r) - \dot{v}_d. \end{aligned} \tag{6}$$

3.2 Problem formulation

The goal of this work is to design robust control laws for the surge force τ_u and the yaw moment τ_r so that the position of the AUV can track a desired, time-varying trajectory. A block diagram representation of the AUV trajectory tracking control problem is depicted in Fig. 2.

Therefore, the control problem tackled in this paper can be formulated as follows:

Consider the AUV model in the horizontal plane described by (1) *and* (2). *Derive a robust control law that generates the surge force τ_u and the yaw moment τ_r in order to guarantee that the vehicle's actual position $(x(t), y(t))$ tracks a desired, time-varying trajectory $(x_d(t), y_d(t))$.*

4 Sliding mode trajectory tracking control design

This section presents the proposed control law design to solve the trajectory tracking control problem of under-actuated AUVs. In order to reduce the complexity of the control design, it is divided into two stages. The first stage handles the design of the desired linear surge and sway velocities (i.e u_d and v_d, respectively) on the kinematic level. Then, a control law is designed for the surge force and yaw moment to ensure the asymptotic convergence of the surge and sway velocities to the designed desired values using the sliding mode control technique. The designed desired velocities will guarantee the asymptotic convergence of the position tracking errors to zero while no control is available over the heading angle ψ since only two controllers are available. Therefore, a stability analysis is made to ensure that the yaw angular velocity r remains bounded under the application of the proposed control law.

4.1 Control design motivation

The control design proposed in this paper is motivated by the work done in [6] where a trajectory tracking sliding mode controller is designed for surface vessels. This controller design provided sliding surfaces that guarantee the asymptotic convergence of the surge and sway velocities to their desired values while the yaw velocity remains bounded. It uses a first-order sliding surface for the surge velocity tracking error and a second-order sliding surface for the sway velocity tracking error. This control scheme, detailed in [6], was investigated in [26], and some of its drawbacks were highlighted. Trajectory tracking controllers were considered for other applications. For example, an adaptive trajectory tracking controller was designed for a unicycle-like mobile robot in [27].

The trajectory tracking control design proposed in [6] has some drawbacks since the desired surge and sway velocities are chosen in terms of the time derivatives of the reference position as follows:

$$\begin{aligned} u_d &= \dot{x}_d \cos\psi + \dot{y}_d \sin\psi \\ v_d &= -\dot{x}_d \sin\psi + \dot{y}_d \cos\psi \end{aligned} \tag{7}$$

It is claimed in [26] that this choice of desired velocities solve the tracking control problem for special cases. This can be clear by considering a tracking control problem with the following reference trajectories:

$$x_d = a(t) + C_1, \quad y_d = b(t) + C_2, \tag{8}$$

where C_1 and C_2 are constants, and $a(t)$ and $b(t)$ are differentiable time-varying functions.

Notice that the desired velocities in (7) depend on the derivatives of the reference trajectories $\dot{x}_d$ and $\dot{y}_d$ which implies that the constant parameters C_1 and C_2 in (8) do not affect the desired velocities. Therefore, the choice of the desired velocities in (7) solve the trajectory tracking problem for special cases with appropriate values of C_1 and C_2. The desired values will be chosen differently in this paper to overcome this problem. In [26], the proposed solution for this problem was to use the following desired velocities:

$$\begin{aligned} u_d &= \dot{x}_d \cos\psi + \dot{y}_d \sin\psi - k x_e \cos\psi - k y_e \sin\psi \\ v_d &= -\dot{x}_d \sin\psi + \dot{y}_d \cos\psi + k x_e \sin\psi - k y_e \cos\psi \end{aligned} \tag{9}$$

A similar approach will be followed in this paper while taking into account the work in [27] in order to design a trajectory tracking sliding mode control law for the lateral motion of AUVs.

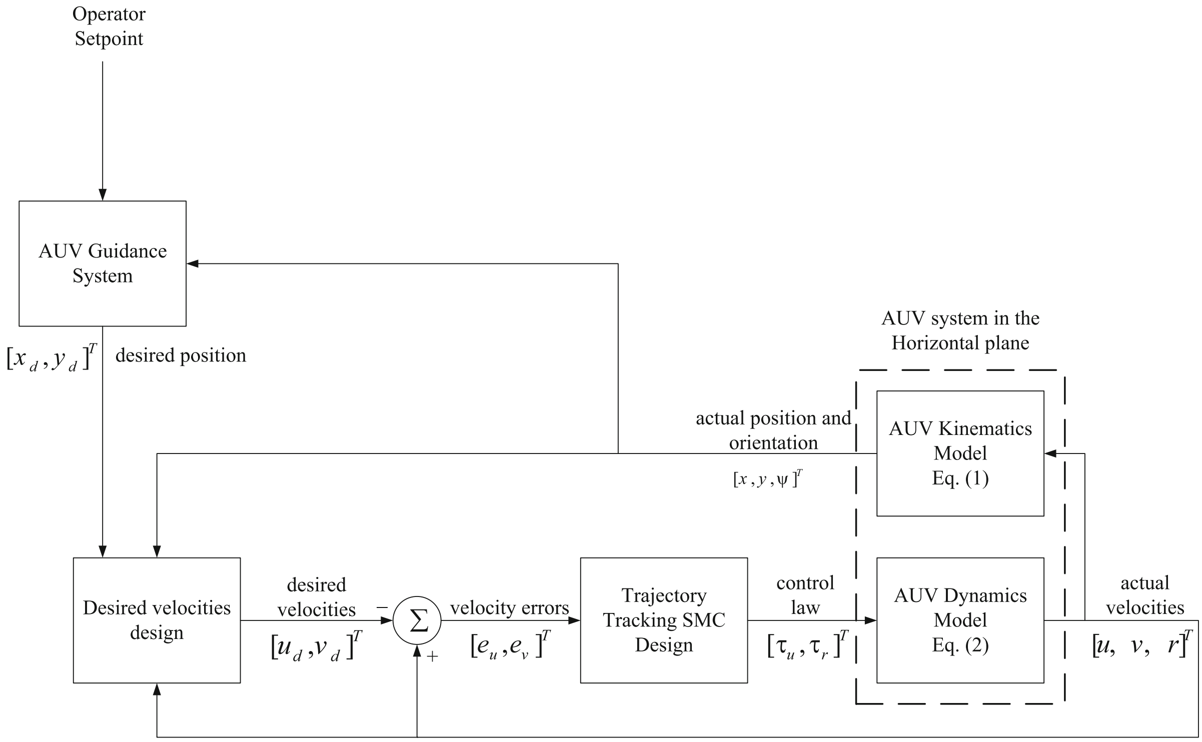

Fig. 2 A block diagram of the AUV's trajectory tracking control system

Note that the developed control law in this paper differs from the work done by Ashrafiuon et al. [6] on the kinematic level by considering a new design for the desired velocities to overcome the discussed problems. On the dynamic level, the derived control law will use similar sliding surfaces but will differ in design of the controller since the desired velocities to be tracked are completely different.

The derived control law in this section assumes that all the states are measurable and are available for feedback.

4.2 Design of the desired velocities

Proposition 1 *Let the desired surge and sway velocities be such that,*

$$\begin{bmatrix} u_d \\ v_d \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} \dot{x}_d + l_x \tanh\left(-\frac{k_x}{l_x} x_e\right) \\ \dot{y}_d + l_y \tanh\left(-\frac{k_y}{l_y} y_e\right) \end{bmatrix} \tag{10}$$

where $k_x, k_y > 0$ are controller gains and $l_x, l_y > 0$ are saturation constants.

If the velocity errors e_u and e_v in (5) converge to zero, then it is guaranteed that the position tracking errors (x_e, y_e) asymptotically converge to $(0, 0)$.

Proof Equation (1) leads to the following equation:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \tag{11}$$

By substituting (10) and (11) into (5), one can get the following:

$$\begin{bmatrix} e_u \\ e_v \end{bmatrix} = \bar{\boldsymbol{R}}_h \begin{bmatrix} \dot{x}_e - l_x \tanh\left(-\frac{k_x}{l_x} x_e\right) \\ \dot{y}_e - l_y \tanh\left(-\frac{k_y}{l_y} y_e\right) \end{bmatrix} \tag{12}$$

where

$$\bar{\boldsymbol{R}}_h = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix}.$$

where the matrix $\bar{\boldsymbol{R}}_h$ is non-singular since $|\bar{\boldsymbol{R}}_h| = 1$. Therefore, (12) implies that the asymptotic convergence of the velocity tracking errors e_u and e_v to zero leads to both $\dot{x}_e - l_x \tanh\left(-\frac{k_x}{l_x} x_e\right)$ and $\dot{y}_e - l_y \tanh\left(-\frac{k_y}{l_y} y_e\right)$ converging to zero as well. Hence, if the velocity errors converge to zero then one obtains,

$$\begin{aligned}\dot{x}_e &= l_x \tanh\left(-\frac{k_x}{l_x}x_e\right)\\ \dot{y}_e &= l_y \tanh\left(-\frac{k_y}{l_y}y_e\right)\end{aligned} \tag{13}$$

The next step is to prove that (x_e, y_e) converge to $(0, 0)$. To this end, the following Lyapunov function candidate is selected:

$$V_1 = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 \tag{14}$$

Differentiating this Lyapanov function with respect to time along the dynamics in (13) yields the following:

$$\begin{aligned}\dot{V}_1 &= x_e\dot{x}_e + y_e\dot{y}_e\\ &= -l_x x_e \tanh\left(\frac{k_x}{l_x}x_e\right) - l_y y_e \tanh\left(\frac{k_y}{l_y}y_e\right)\end{aligned} \tag{15}$$

Since $k_x, k_y > 0$ and $l_x, l_y > 0$, it is clear from (15) that $\dot{V}_1 < 0$ for $(x_e, y_e) \neq (0, 0)$. Thus, it can be concluded that both x_e and y_e asymptotically converge to zero.

Therefore, if the vehicle's surge and sway velocities converge to the desired velocities proposed in (10), then the asymptotic convergence of the position tracking errors (x_e, y_e) to $(0, 0)$ is guaranteed. □

Remark 1 In Eq. (10), the design parameters l_x and l_y are chosen properly according to the physical limitations on the vehicle's velocities. The choice of the controller gains k_x and k_y determines how fast the trajectories converge to zero and can be tuned so that the performance of the system is robust against bounded disturbances. One of the possible approaches used to properly choose the controller gains is by minimizing a desired cost function.

4.3 Controller design

In order to design the trajectory tracking sliding mode controller, we will choose the sliding surfaces such that:

$$S_1 = e_u + \lambda_1 \int_0^t e_u(\tau)d\tau \tag{16}$$

$$S_2 = \dot{e}_v + \lambda_3 e_v + \lambda_2 \int_0^t e_v(\tau)d\tau, \tag{17}$$

where $\lambda_1, \lambda_2, \lambda_3 > 0$. Using (2) and (6), the time derivatives of these sliding surfaces are such that:

$$\dot{S}_1 = M_1(X_u u + a_{26}vr + \tau_u) - \dot{u}_d + \lambda_1 e_u \tag{18}$$

$$\begin{aligned}\dot{S}_2 &= M_2(Y_v\dot{v} + a_{16}\dot{u}r + a_{16}uM_6(N_r r + a_{12}uv + \tau_r))\\ &\quad - \ddot{v}_d + \lambda_3\dot{e}_v + \lambda_2 e_v\end{aligned} \tag{19}$$

In order to ensure a finite-time convergence of these sliding surfaces to zero, the following dynamics will be imposed on the sliding surfaces S_1 and S_2,

$$\dot{S}_1 = -k_1 S_1 - W_1 sign(S_1) \tag{20}$$

$$\dot{S}_2 = -k_2 S_2 - W_2 sign(S_2) \tag{21}$$

where $k_1, k_2 \geq 0$, $W_1, W_2 > 0$.

Therefore, the trajectory tracking surge and yaw control laws are chosen such that:

$$\tau_u = \tau_{u,eq} + \tau_{u,sw} \tag{22}$$

$$\tau_r = \tau_{r,eq} + \tau_{r,sw}, \tag{23}$$

where,

$$\begin{aligned}\tau_{u,eq} &= -X_u u - a_{26}vr + \frac{1}{M_1}\dot{u}_d - \frac{1}{M_1}\lambda_1 e_u\\ \tau_{u,sw} &= \frac{1}{M_1}\Big(-k_1 S_1 - W_1 sign(S_1)\Big)\\ \tau_{r,eq} &= -N_r r - a_{12}uv + \frac{1}{b}\Big(-M_2(Y_v\dot{v} + a_{16}\dot{u}r)\Big)\\ &\quad + \frac{1}{b}\Big(\Gamma - \lambda_3\dot{e}_v - \lambda_2 e_v\Big)\\ \tau_{r,sw} &= \frac{1}{b}\Big(-k_2 S_2 - W_2 sign(S_2)\Big),\end{aligned}$$

where the following definitions are made,

$$\begin{aligned}b &= M_6(M_2 a_{16}u + u_d)\\ \Gamma &= -\dddot{x}_d \sin\psi + \dddot{y}_d \cos\psi - \ddot{x}_d r \cos\psi\\ &\quad - \ddot{y}_d r \sin\psi - \dot{u}_d r + \Upsilon_1 r \cos\psi + \Upsilon_2 r \sin\psi\\ &\quad + \dot{\Upsilon}_1 \sin\psi - \dot{\Upsilon}_2 \cos\psi\\ \Upsilon_1 &= k_x\dot{x}_e sech^2\left(-\frac{k_x}{l_x}x_e\right)\\ \Upsilon_2 &= k_y\dot{y}_e sech^2\left(-\frac{k_y}{l_y}y_e\right).\end{aligned} \tag{24}$$

Theorem 1 *Consider the AUV system in the horizontal plane described by* (1) *and* (2). *Let the velocity tracking errors be defined as in* (5) *with the desired velocities* *be chosen as in* (10)*. When the control laws for the surge force* τ_u *and the yaw moment* τ_r *proposed in* (22) *and* (23) *are applied to the AUV, then the asymptotic convergence of the velocity tracking errors* (e_u, e_v) *to* $(0, 0)$ *is guaranteed. Furthermore, the AUV's position errors* (x_e, y_e) *asymptotically converge to* $(0, 0)$ *while the yaw motion remains bounded.*

Proof Taking the first and second time derivatives of (10), one obtains the following:

$$\begin{bmatrix} \dot{u}_d \\ \dot{v}_d \end{bmatrix} = r \begin{bmatrix} -\sin\psi & \cos\psi \\ -\cos\psi & -\sin\psi \end{bmatrix} \begin{bmatrix} \dot{x}_d + l_x \tanh\left(-\frac{k_x}{l_x} x_e\right) \\ \dot{y}_d + l_y \tanh\left(-\frac{k_y}{l_y} y_e\right) \end{bmatrix} + \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} \ddot{x}_d - k_x \dot{x}_e sech^2\left(-\frac{k_x}{l_x} x_e\right) \\ \ddot{y}_d - k_y \dot{y}_e sech^2\left(-\frac{k_y}{l_y} y_e\right) \end{bmatrix} \tag{25}$$

and

$$\ddot{v}_d = \Gamma - u_d \dot{r} \tag{26}$$

where Γ is defined in (24).
Consider the following Lyapanov function candidate:

$$V_2 = \frac{1}{2} S_1^2 + \frac{1}{2} S_2^2$$

By differentiating V_2 with respect to time and using (18) and (19), one can get,

$$\begin{aligned} \dot{V}_2 = {} & S_1[M_1(X_u u + a_{26} v r + \tau_u) - \dot{u}_d + \lambda_1 e_u] \\ & + S_2[M_2(Y_v \dot{v} + a_{16} \dot{u} r + a_{16} u M_6(N_r r + a_{12} u v + \tau_r))] \\ & + S_2[-\ddot{v}_d + \lambda_3 \dot{e}_v + \lambda_2 e_v] \end{aligned} \tag{27}$$

By using (22), (23) and (26), the time derivative of V_2 can be written as follows:

$$\begin{aligned} \dot{V}_2 = {} & S_1[M_1(X_u u + a_{26} v r + \tau_u) - \dot{u}_d + \lambda_1 e_u] \\ & + S_2[M_2(Y_v \dot{v} + a_{16} \dot{u} r + a_{16} u M_6 \dot{r}) - \ddot{v}_d + \lambda_3 \dot{e}_v + \lambda_2 e_v] \\ = {} & -k_1 S_1^2 - W_1 S_1 sign(S_1) + S_2[M_2(Y_v \dot{v} + a_{16} \dot{u} r)] \\ & + S_2[-\Gamma + b(N_r r + a_{12} u v + \tau_r) + \lambda_3 \dot{e}_v + \lambda_2 e_v] \\ = {} & -k_1 S_1^2 - W_1 |S_1| - k_2 S_2^2 - W_2 |S_2| \end{aligned} \tag{28}$$

It is obvious from (28) that $\dot{V}_2 < 0$ for $(S_1, S_2) \neq (0, 0)$ since $k_1, k_2 \geq 0$ and $W_1, W_2 > 0$. This implies that the sliding surfaces in (16) and (17) reach zero in finite time. Furthermore, on the sliding surfaces (i.e., $S_1 = S_2 = 0$), the following dynamics are obtained,

$$\dot{e}_u = -\lambda_1 e_u \tag{29}$$

$$\ddot{e}_v = -\lambda_3 \dot{e}_v - \lambda_2 e_v. \tag{30}$$

Since λ_1, λ_2 and λ_3 are chosen to be positive scalars, the dynamics in (29), (30) ensure the asymptotic convergence of (e_u, e_v) to $(0, 0)$. That is, the surge and sway velocities converge to the desired ones given in (10). Hence, it can be concluded that the asymptotic convergence of the position errors (x_e, y_e) to $(0, 0)$ is guaranteed based on Proposition 1.

Also, the sway dynamics will be such that:

$$\dot{v}_d = M_2(Y_v v_d + a_{16} u_d r) \tag{31}$$

Substituting for $\dot{v}_d$ from (25) in (31) yields,

$$\begin{aligned} & -\left(\ddot{x}_d - k_x \dot{x}_e sech^2(-\tfrac{k_x}{l_x} x_e)\right) \sin\psi \\ & + \left(\ddot{y}_d - k_y \dot{y}_e sech^2(-\tfrac{k_y}{l_y} y_e)\right) \cos\psi = M_2 Y_v v_d \\ & \qquad + (1 + M_2 a_{16}) u_d r \end{aligned} \tag{32}$$

Since $u_d > 0$ (forward motion), and from (10), (13), it is obvious that u_d, v_d, $\dot{x}_e$ and $\dot{y}_e$ are bounded. Therefore, it is guaranteed that the yaw velocity r will remain bounded.

Moreover, choose the following Lyapunov function candidate:

$$V_3 = \frac{1}{2} r^2 \tag{33}$$

By differentiating V_3 with respect to time along the dynamics in (2), the following is obtained:

$$\dot{V}_3 = r\dot{r} = M_6 r(N_r r + a_{12} u v + \tau_r) \tag{34}$$

Knowing that $N_r < 0$ (a damping term) and $M_6 > 0$, the condition $\dot{V}_3 < 0$ is satisfied when:

$$a_{12} u v + \tau_r < -N_r r \quad if \;\; r > 0$$

$$a_{12} u v + \tau_r > -N_r r \quad if \;\; r < 0$$

which implies the following,

$$|N_r r| > |a_{12} u v + \tau_r|. \tag{35}$$

Therefore, $\dot{V}_3 < 0$ if the inequality (35) is satisfied. Moreover, $\dot{V}_3 < 0$ implies that V_3 is a decreasing function which means that $|r|$ is decreasing as well from (33).

This means that the proposed controller guarantees the boundedness of the yaw velocity in general, and it ensures the asymptotic convergence of r to zero in some cases where the condition in (35) is satisfied. It can be shown from the yaw controller in (23) that this condition is met when the vehicle is required to track a straight line.

So, the control laws proposed in (22) and (23) guarantee the asymptotic convergence of (x_e, y_e) to $(0, 0)$ while the yaw motion remains bounded. □

Table 1 The REMUS AUV model parameters

Parameter	Value	Units
m	30.48	kg
I_z	3.45	$kg\, m^2$
X_u	−8.8065	kg/s
Y_v	−65.5457	kg/s
N_r	−6.7352	kg/s
$X_{\dot{u}}$	−0.93	kg
$Y_{\dot{v}}$	−35.5	kg
$N_{\dot{r}}$	−35.5	$kg\, m^2$

5 Simulation results

The trajectory tracking sliding mode control law proposed in the previous section is applied to the 3 DOF lateral motion model of the AUV, which is described by the kinematic and dynamic equations of motion in (1) and (2), respectively. The proposed controllers are simulated for different cases in order to check the system's stability and performance. The simulations are done using the MATLAB software. In the simulations, the parameters of the model are those of the REMUS autonomous underwater vehicle, and they are listed in Table 1. In all simulations, the initial values are taken to be zero (the vehicle is at rest) such that $x(0) = y(0) = \psi(0) = u(0) = v(0) = r(0) = 0$.

In addition, the discontinuous signum function used in the control laws is approximated by the hyperbolic tangent function which is continuous such that $sign(a) \approx \tanh(\gamma a)$ where γ is a positive scalar which can be chosen to get a very good approximation. This approximation is used in order to avoid the chattering problem.

The control law proposed in (22) and (23) is used in order to force the vehicle to track the desired velocities designed in (10). Three cases for the reference trajectories in the form of (8) are considered. These cases are as follows:

– Case I: linear motion ($C_1 = 0$ and $C_2 = 0$)

$$x_d(t) = t, \quad y_d(t) = t$$

– Case II: linear motion ($C_1 \neq 0$ and $C_2 \neq 0$)

$$x_d(t) = t + 5, \quad y_d(t) = 2$$

– Case III: Circular motion ($C_1 = 0$ and $C_2 = 0$)

$$x_d(t) = cos(t), \quad y_d(t) = sin(t)$$

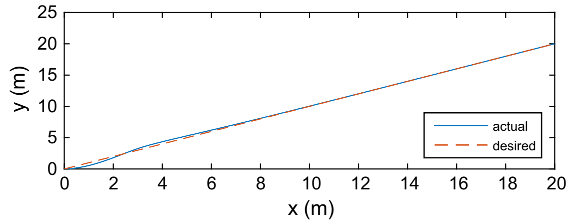

Fig. 3 The actual and desired paths of the AUV using the proposed trajectory tracking controller for case I

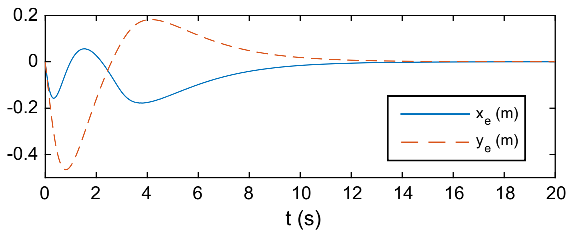

Fig. 4 The position tracking errors of the AUV versus time using the proposed trajectory tracking controller for case I

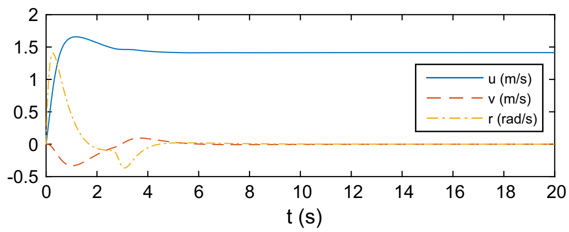

Fig. 5 The velocities of the AUV versus time using the proposed trajectory tracking controller for case I

In these simulations, the design parameters are taken to be $l_x = l_y = 2$, $k_x = k_y = 0.5$, $\lambda_1 = 2$, $\lambda_2 = 7$, $\lambda_3 = 8$, $k_1 = 0$, $k_2 = 0$, $W_1 = 0.5$ and $W_2 = 3$. The simulation results for case I are presented in Figs. 3, 4, 5, 6 and 7, the results for case II are shown in Figs. 8, 9, 10, 11 and 12, and the results obtained for case III are given in Figs. 13, 14, 15, 16 and 17.

The actual and desired paths of the vehicle are presented in Figs. 3, 8 and 13 for cases I, II and III, respectively. The asymptotic convergence of the vehicle's position tracking errors x_e and y_e to zero for the three cases is clear as shown in Figs. 4, 9 and 14. Figures 5, 10 and 15 show that the yaw motion is bounded in each case. These figures also show the surge, sway and yaw velocities versus time for each case. Also, the velocities tracking errors versus time are shown in Figs. 6, 11 and 16. Moreover, the simulated surge force and yaw

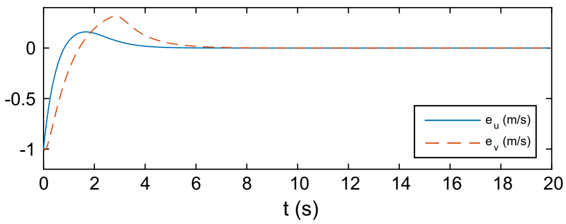

Fig. 6 The velocity tracking errors of the AUV versus time using the proposed trajectory tracking controller for case I

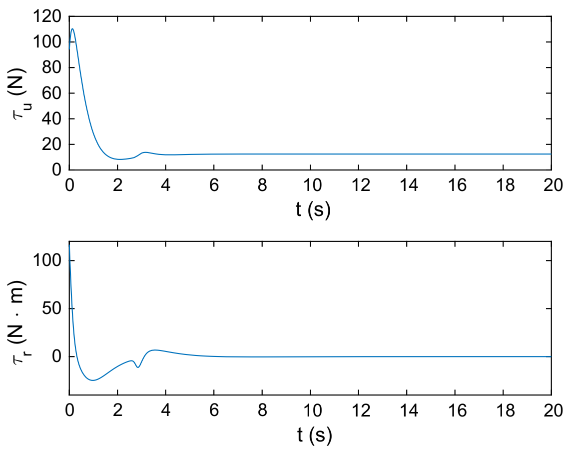

Fig. 7 The surge and yaw control laws of the AUV versus time using the proposed trajectory tracking controller for case I

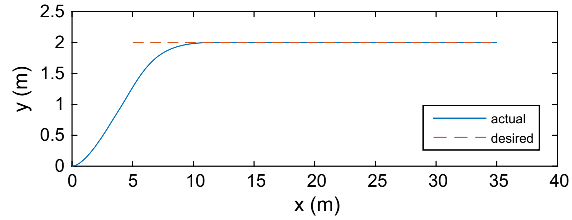

Fig. 8 The actual and desired paths of the AUV using the proposed trajectory tracking controller for case II

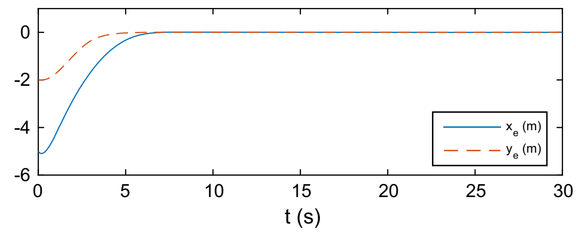

Fig. 9 The position tracking errors of the AUV versus time using the proposed trajectory tracking controller for case II

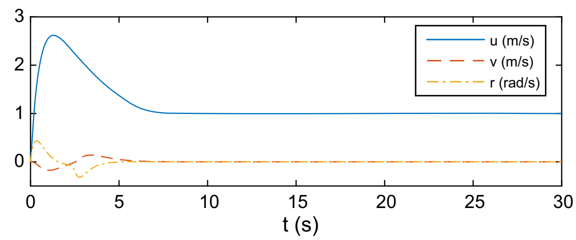

Fig. 10 The velocities of the AUV versus time using the proposed trajectory tracking controller for case II

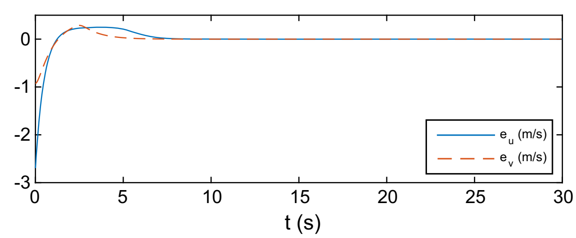

Fig. 11 The velocity tracking errors of the AUV versus time using the proposed trajectory tracking controller for case II

moment versus time are presented in Figs. 7, 12 and 17 for each of the three cases considered.

The applied controllers forced the sliding surfaces S_1 and S_2 to reach zero in about 3 s for case I which causes the switching in the controllers as can be seen in Fig. 7. After that, the velocity tracking errors take about 1 s for e_u and 1.5 s for e_v to settle as can be seen from Fig. 6. This means that after about 4.5 s, the desired surge and sway velocities are tracked; the position tracking errors asymptotically converge to zero with a settling time of about 12 s as can be seen from Fig. 4. The same analysis holds for cases II and III but with different settling times. These results show that the proposed control scheme works well for the trajectory tracking of AUVs.

6 Robustness studies

This section provides simulation studies to test the robustness of the proposed trajectory tracking control scheme. The disturbances are included in the AUV model represented by Eqs. (1), (2). The new AUV kinematic and dynamic equations are,

$$\begin{aligned}
\dot{x} &= u \cos\psi - v \sin\psi \\
\dot{y} &= u \sin\psi + v \cos\psi \\
\dot{\psi} &= r \\
\dot{u} &= M_1(X_u u + a_{26} v r + \tau_u) + d_1(t)
\end{aligned} \qquad (36)$$

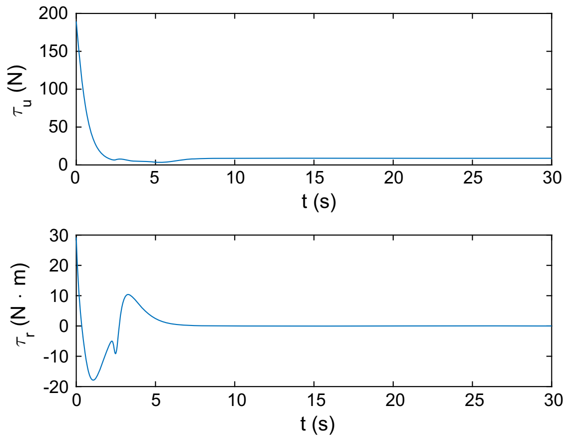

Fig. 12 The surge and yaw control laws of the AUV versus time using the proposed trajectory tracking controller for case II

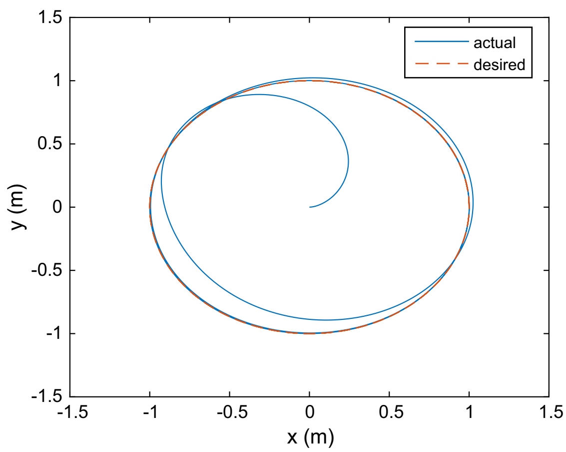

Fig. 13 The actual and desired paths of the AUV using the proposed trajectory tracking controller for case III

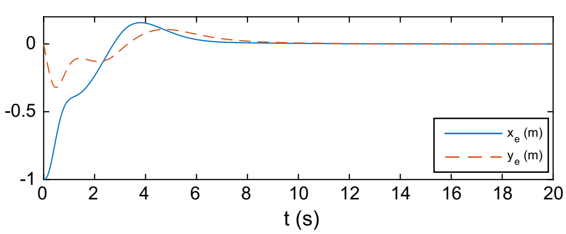

Fig. 14 The position tracking errors of the AUV versus time using the proposed trajectory tracking controller for case III

$$\dot{v} = M_2(Y_v v + a_{16}ur)$$
$$\dot{r} = M_6(N_r r + a_{12}uv + \tau_r) + d_2(t)$$

where the model parameters are as defined for (1)–(2), and the terms d_1 and d_2 represent the external distur-

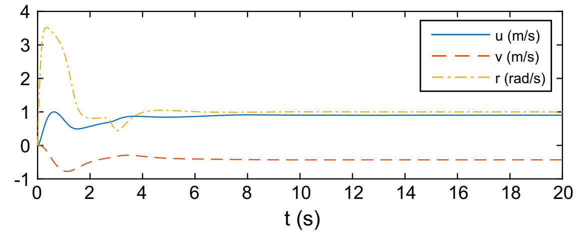

Fig. 15 The velocities of the AUV versus time using the proposed trajectory tracking controller for case III

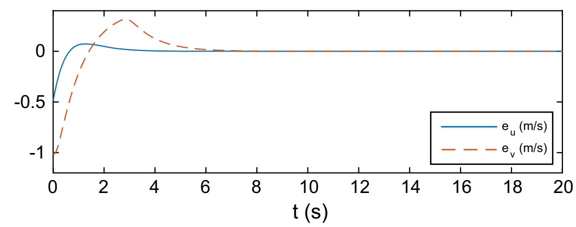

Fig. 16 The velocity tracking errors of the AUV versus time using the proposed trajectory tracking controller for case III

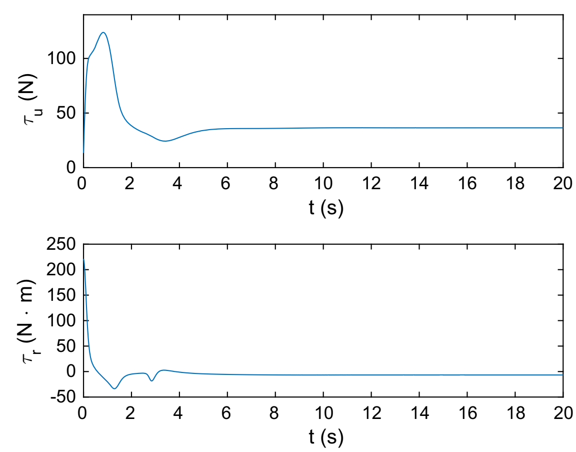

Fig. 17 The surge and yaw control laws of the AUV versus time using the proposed trajectory tracking controller for case III

bances and model uncertainties. The bounds on the disturbances are assumed known (or estimated) in order to properly select the controller gains and consequently suppress the disturbance effects.

The following three disturbance scenarios are considered in the simulations,

1. Constant Disturbances,

$$d_1(t) = 0.2, \; d_2(t) = 0.15 \tag{37}$$

2. Sinusoidal Disturbances,

$$d_1(t) = 0.15\cos(t), \; d_2(t) = 0.2\sin(t) \tag{38}$$

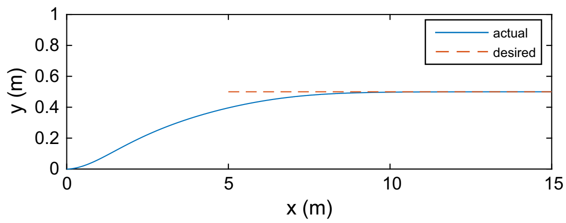

Fig. 18 The actual and desired paths of the AUV using the proposed trajectory tracking controller with constant disturbances

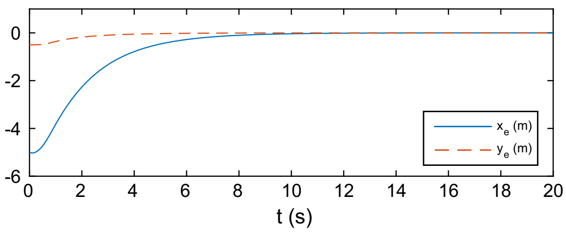

Fig. 19 The position errors of the AUV versus time using the proposed trajectory tracking controller with constant disturbances

3. Disturbances for a period of time,

$$d_1(t) = 0.5[u_s(t-5) - u_s(t-6)],$$
$$d_2(t) = 2[u_s(t-5) - u_s(t-6)] \tag{39}$$

where $u_s(t)$ is the unit step function defined as,

$$u_s(t) = \begin{cases} 1, \ t \geq 0 \\ 0, \ otherwise \end{cases}$$

The simulation results for the three cases given by (37), (38) and (39) are presented, respectively, in Figs. 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28 and 29. The actual and desired paths of the AUV are shown in Figs. 18, 22 and 26. Figures 19, 23 and 27 show the position errors of the AUV versus time. Figures 20, 24 and 28 show the velocity errors versus time. The surge and yaw controllers for the 3 cases of disturbances are depicted in Figs. 21, 25 and 29. These figures clearly indicate that the proposed control scheme is robust to bounded uncertainties.

Notice that for the third scenario, the disturbances act on the system at the time period between 5 and 6 s which affects the velocities as can be seen in Fig. 28. Figure 29 shows that the proposed controllers managed to reject the effect of the disturbances at this time period.

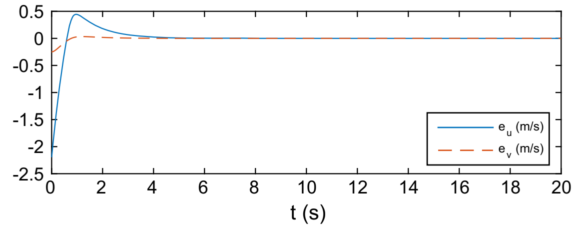

Fig. 20 The velocity errors of the AUV versus time using the proposed trajectory tracking controller with constant disturbances

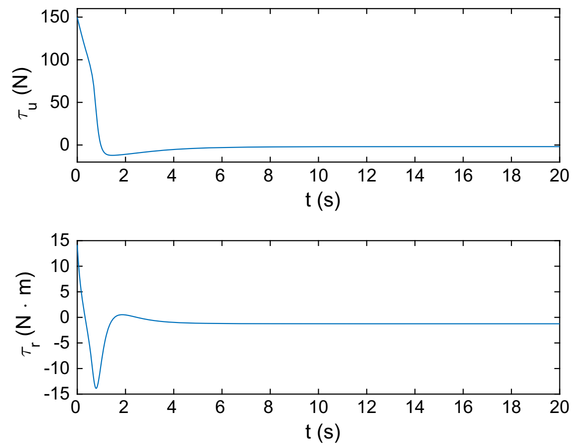

Fig. 21 The surge and yaw control laws of the AUV versus time using the proposed trajectory tracking controller with constant disturbances

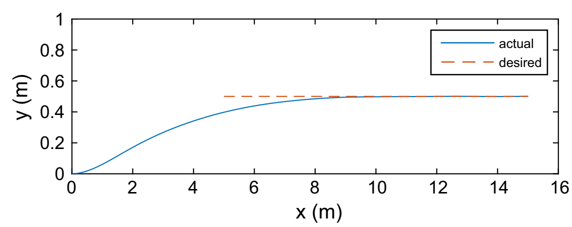

Fig. 22 The actual and desired paths of the AUV using the proposed trajectory tracking controller with sinusoidal disturbances

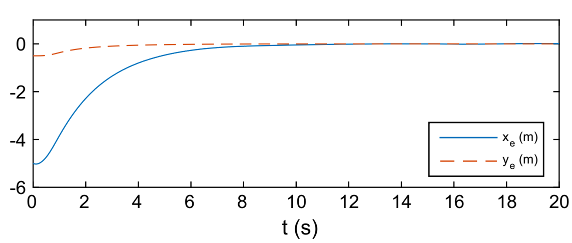

Fig. 23 The position errors of the AUV versus time using the proposed trajectory tracking controller with sinusoidal disturbances

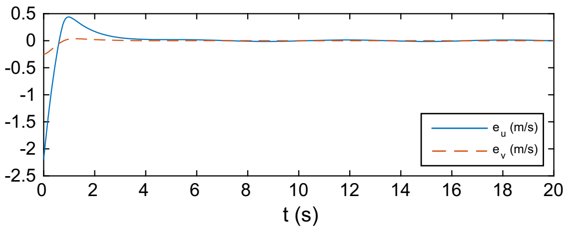

Fig. 24 The velocity errors of the AUV versus time using the proposed trajectory tracking controller with sinusoidal disturbances

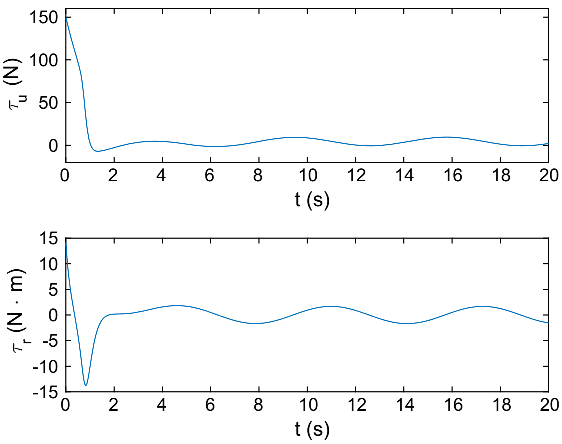

Fig. 25 The surge and yaw control laws of the AUV versus time using the proposed trajectory tracking controller with sinusoidal disturbances

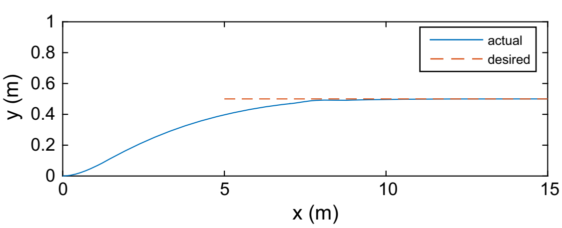

Fig. 26 The actual and desired paths of the AUV using the proposed trajectory tracking controller with disturbances for a period of time

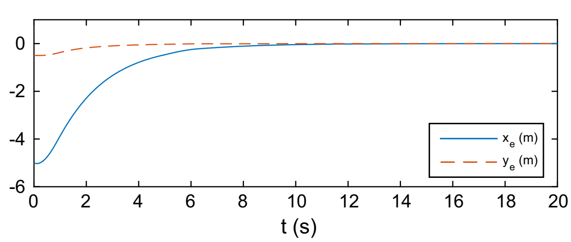

Fig. 27 The position errors of the AUV versus time using the proposed trajectory tracking controller with disturbances for a period of time

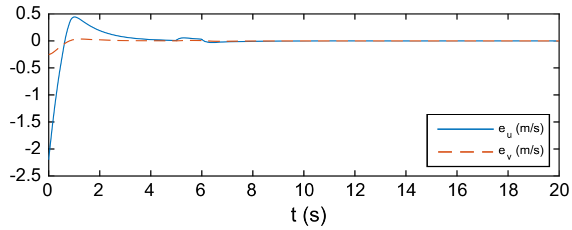

Fig. 28 The velocity errors of the AUV versus time using the proposed trajectory tracking controller with disturbances for a period of time

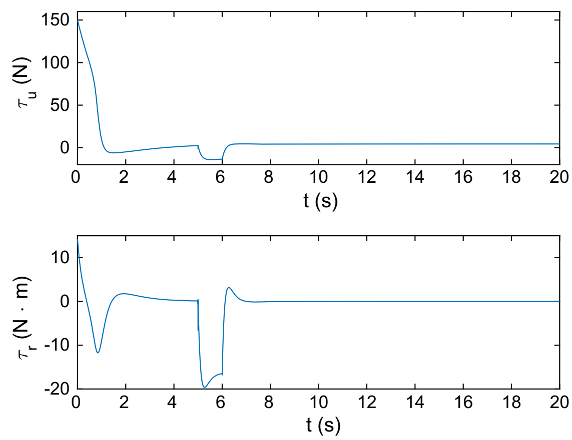

Fig. 29 The surge and yaw control laws of the AUV versus time using the proposed trajectory tracking controller with disturbances for a period of time

7 Conclusion

A trajectory tracking sliding mode control scheme is proposed for the control of the lateral motion of AUVs. The objective of this controller is to force the position of the AUV to track a desired, time-varying trajectory. The control design is validated by applying it to an AUV and simulating its performance for different cases of reference trajectories. The simulation results indicate that the proposed control scheme works well for the three different cases. Moreover, the simulation studies indicate that the proposed control scheme is robust to bounded disturbances.

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