而: $\sum_{i=1}^{n} f_i(x_i)$ 有: $-\sum_{i=1}^{n} [\log(\alpha_i + x_i))$ : $f_i(x_i) = -\log(\alpha_i + x_i)$

而 $f_i^*(y) = \sup_x (x^T y - f(x)) = \sup_x (x^T y + \log(\alpha + x))$

而 $\frac{\partial(x^T y + \log(\alpha + x))}{\partial x} = y + \frac{1}{\alpha + x}$ (if $x$ 为 $R$)

则有: $y = -\frac{1}{\alpha + x} \Rightarrow x + \alpha = -\frac{1}{y}$

∴有 $x^T y + \log(\alpha + x) = (-\frac{1}{y} - \alpha) y + \log(-\frac{1}{y})$

$= -1 - \alpha y + \log(-\frac{1}{y}) = -1 - \alpha y - \log(-y)$

∴ $f_i^*(y) = -1 - \alpha y - \log(-y)$

∴有: $f^*(x) = \sum_{i=1}^{n} (-1 - \alpha_i x_i - \log(-x_i))$

而: $-(A^T \lambda + C^T v) \Rightarrow A = -I$. $b = 0$ $C = 1^T$ $d = 1$

∴有. $-(-I \cdot \lambda + 1 \cdot v) = (\lambda - 1 \cdot v) = \begin{pmatrix} \lambda_i - v \\ \vdots \\ \lambda_n - v \end{pmatrix}$

则有: 对偶函数

$g(\lambda, v) = -(\lambda^T b + v^T d) - f_0^*(\lambda - 1 \cdot v)$

$= -(\lambda^T \cdot 0 + v \cdot 1) - \sum_{i=1}^{n} (-1 - \alpha_i(\lambda_i - v) - \log(v - \lambda_i))$

$= -1 + \sum_{i=1}^{n} (1 + \alpha_i(\lambda_i - v) + \log(v - \lambda_i))$

$= n - 1 + \sum_{i=1}^{n} \alpha_i (\lambda_i - v) + \sum_{i=1}^{n} \log(v - \lambda_i)$