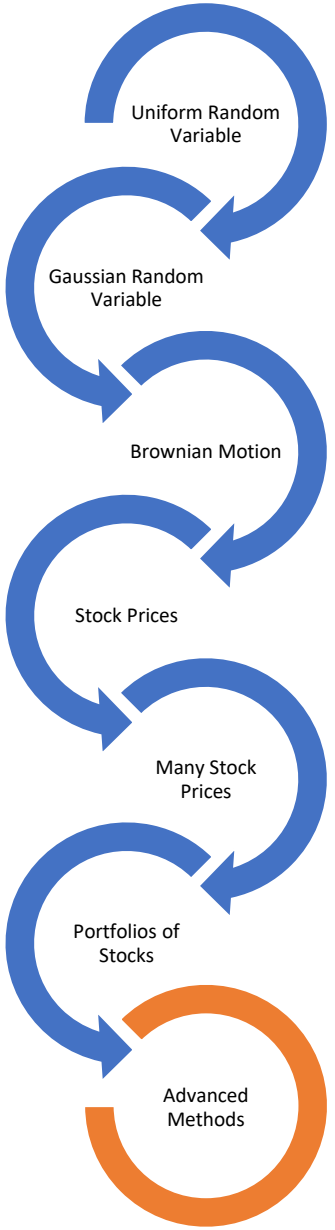
Introduction to Numerical Methods in Finance

MMF Summer Workshop No. 1927H

June 2024

Course Agenda

- Module 1: Uncertainty and Simulating the Future
  1. Random Variables
  2. Random Walks
  3. Portfolios of Random Walks
- Module 2: Risk and Reward in the Future
  1. Value at Risk
  2. Pricing Derivatives
  3. PDE's and Copulas
- Module 3: Predicting the Future
  1. Credit Risk
  2. Econometrics/Factor Analysis
  3. **Optimization**/Statistical Pattern Recognition
- What you should be able to do after this course:
  - ✓ Price a tailor-made derivative instrument
  - ✓ Calculate the risk of a portfolio
  - ✓ Calculate probability of default and credit risk
  - Create a macro-economic forecast
  - Optimize the performance of a portfolio of risks



Admin

- Assignments
  - 1 Projects – 100%
  - No final exam

- References that might be helpful
- Chatfield. The Analysis of Time Series: An Introduction, 1997.
- Martinez and Martinez. Computational Statistics Handbook with MATLAB, 2002
- Alexander. Market Models, 2001
- Neftci. Principles of Financial Engineering, 2004.
- Brandimarte. Numerical Methods in Finance and Economics, 2006.
- Lewinson. Python for Finance Cookbook, 2022.

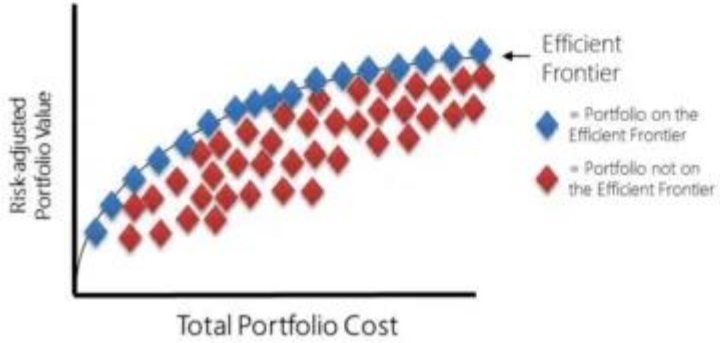
- By the end of this summer workshop you should be able to clearly articulate and calculate optimal portfolio for a wide range of assets with the potential of including
  - Climate Risk
  - Forward Looking Views of the world

- Structure of Lecture
  - 3 hours 2-3 topics
  - Describe a focus problem for the lecture using all 3 topics to solve
  - Introduction
  - Theory, methods, and tools
  - Solve the focus problem together
  - 1-2 Breaks and questions

- Send homework to taha.jaffer@gmail.com

Introduction

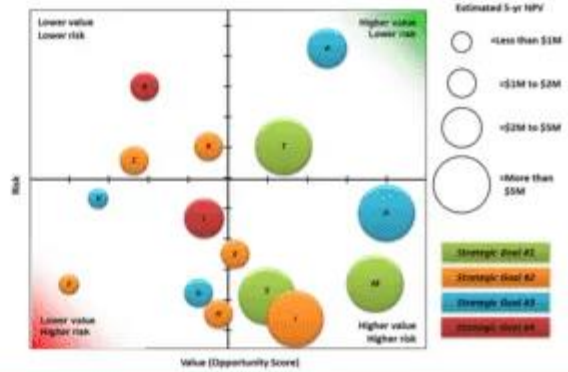
FOUR TYPES OF OPTIMIZATION



COST-VALUE OPTIMIZATION

(Efficient Frontier Analysis)

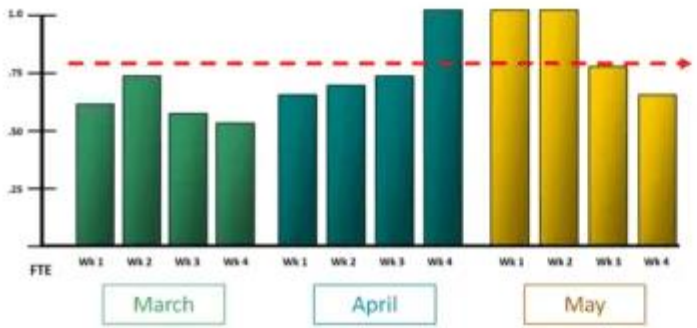
Constraint: Budget



WORK TYPE OPTIMIZATION

(Portfolio Balancing)

Constraint: Categorical Designations



RESOURCE OPTIMIZATION

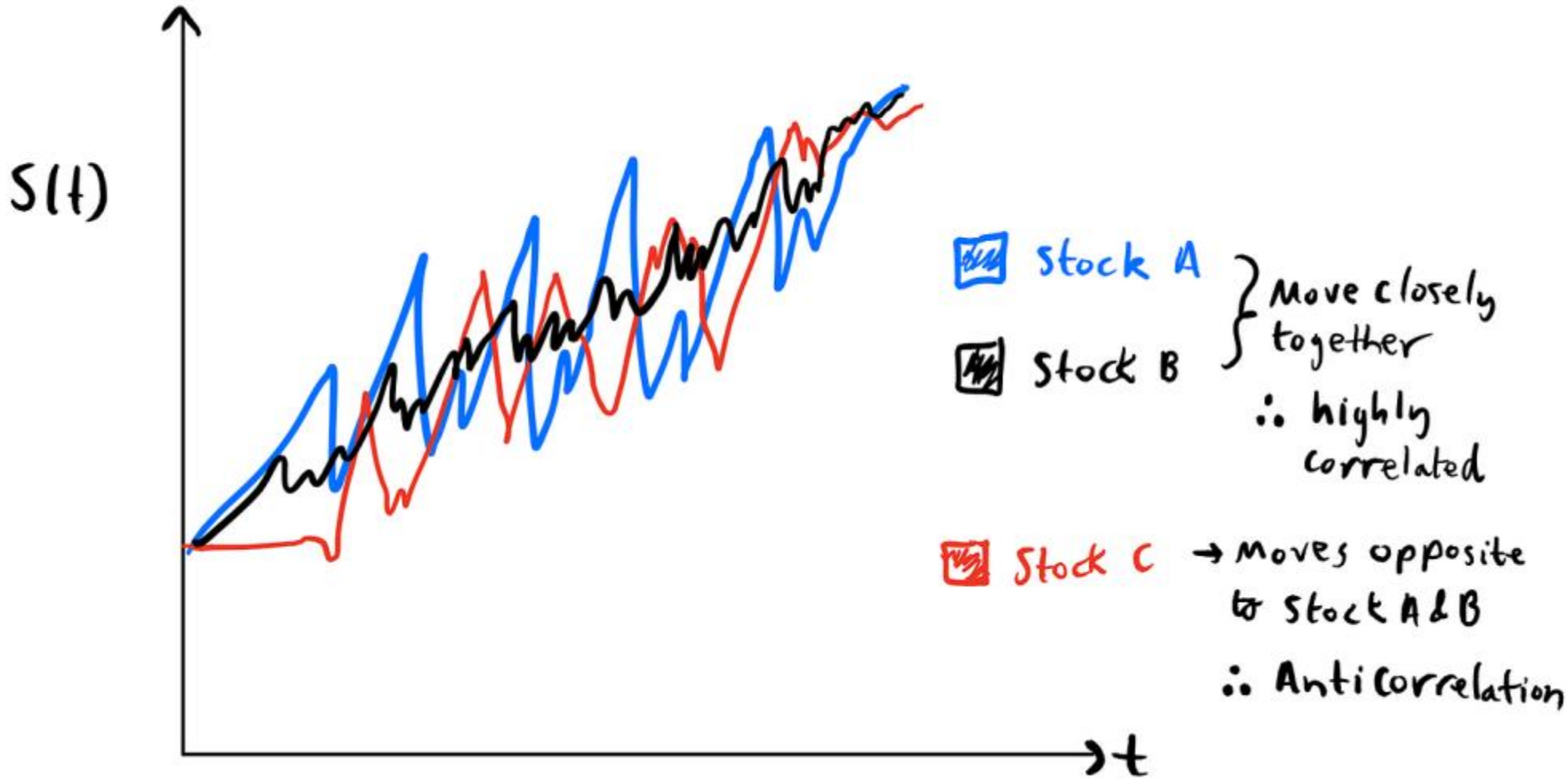
(Resource Capacity Analysis)

Constraint: Human resource availability

SCHEDULE OPTIMIZATION

(Project Sequencing)

Constraint: Timing and Dependencies

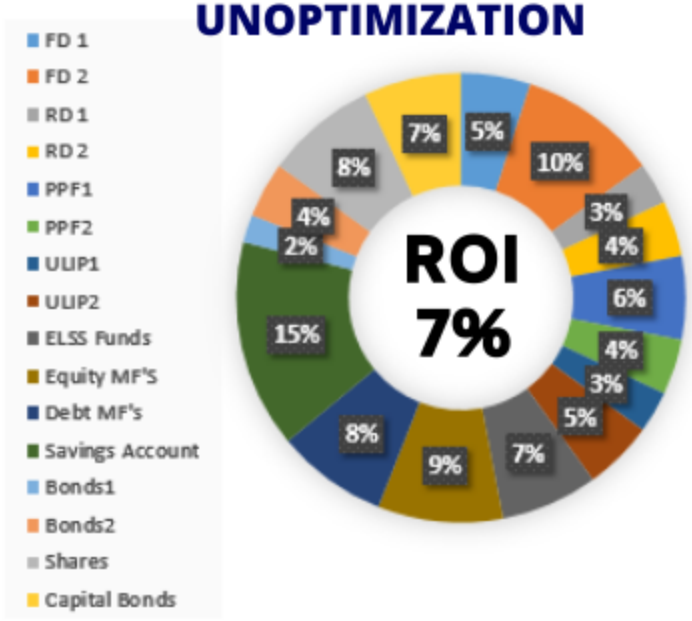


Stock A
Stock B
} Move closely together
∴ highly correlated

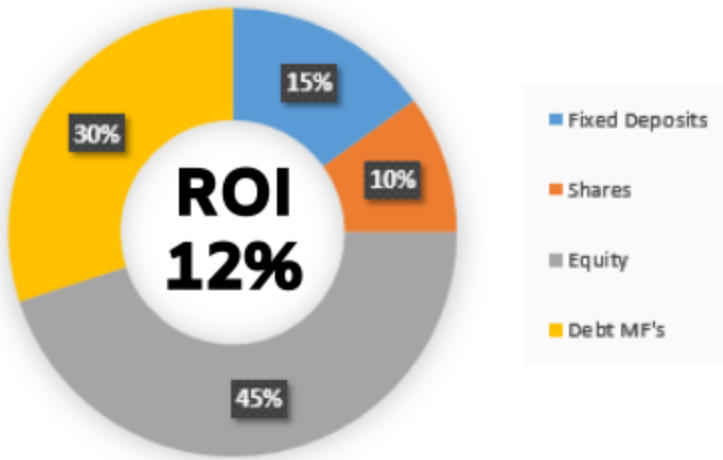
Stock C → moves opposite to Stock A & B
∴ Anti correlation

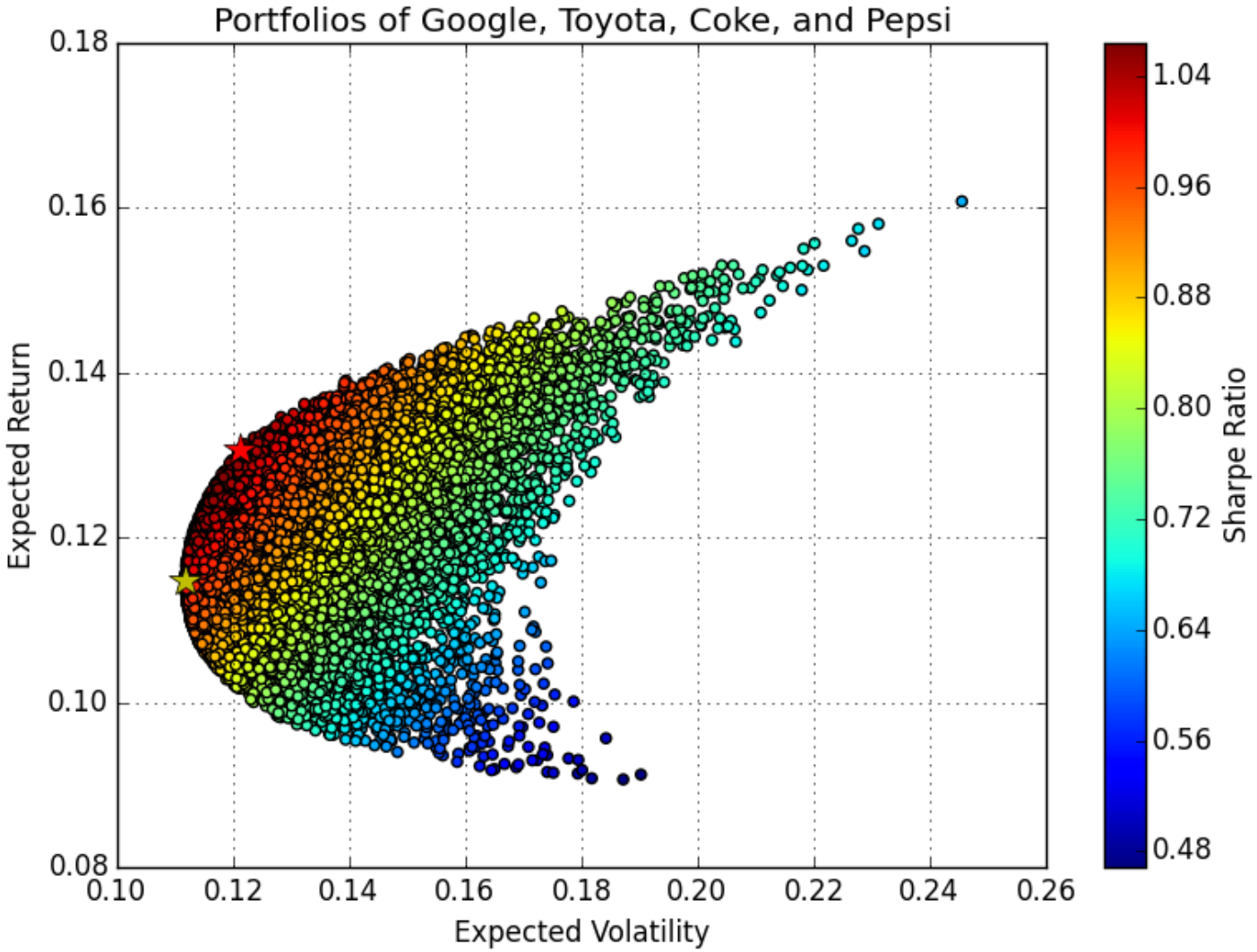
IF PORTFOLIO OPTIMIZATION PROMISES OUTSTANDING RETURNS, WOULDN'T YOU TAKE IT?

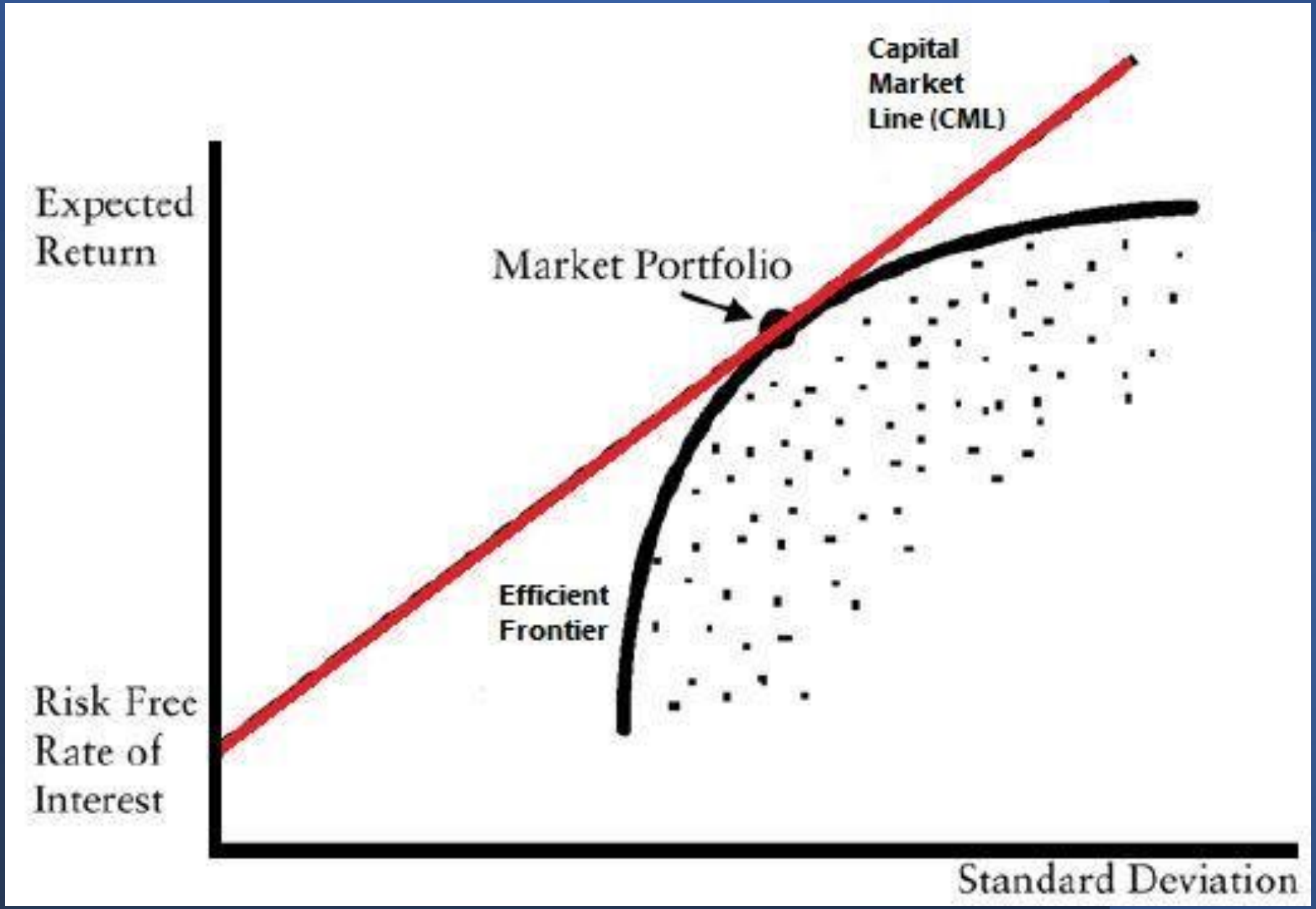
UNOPTIMIZATION



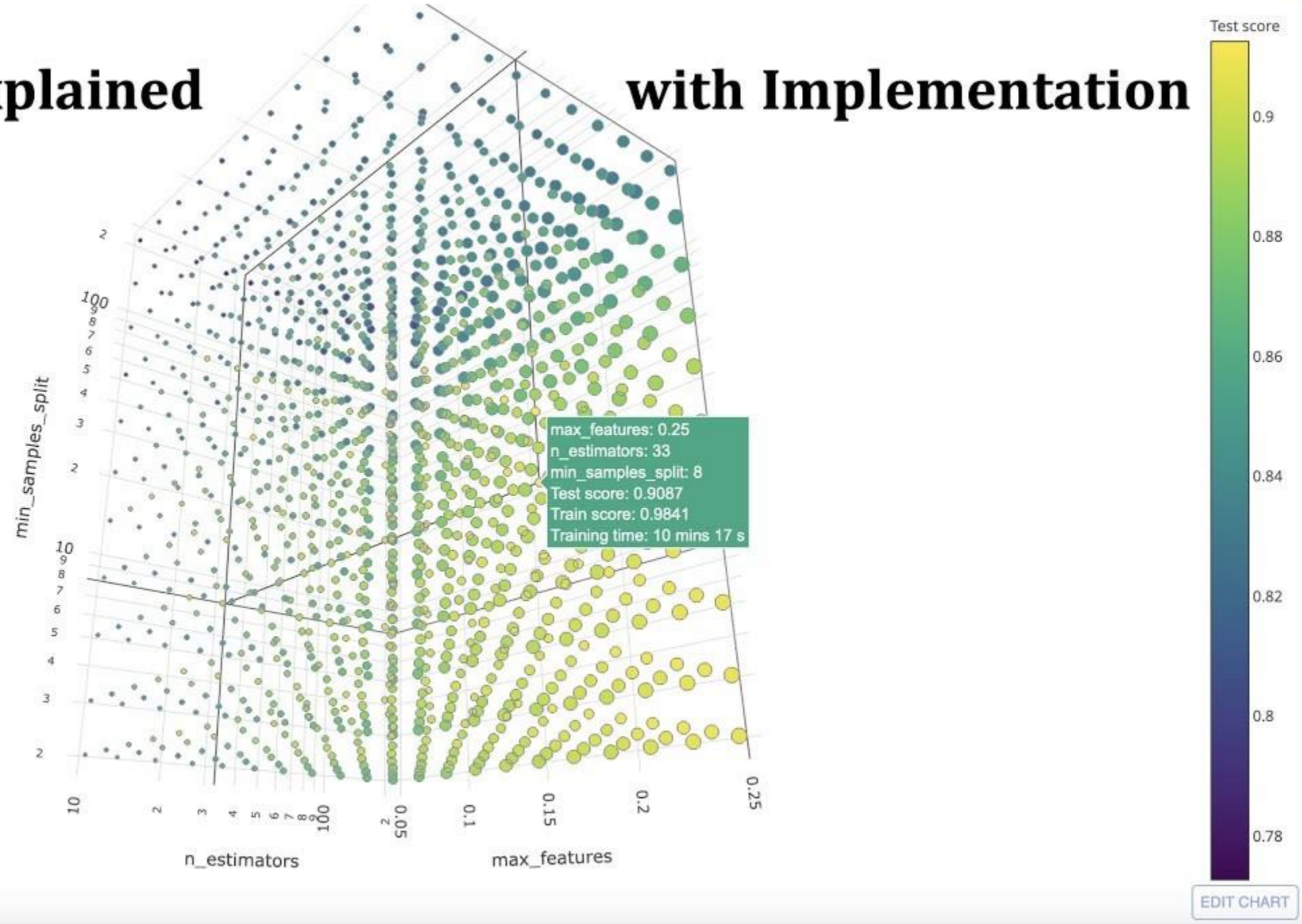
AFTER OPTIMIZATION



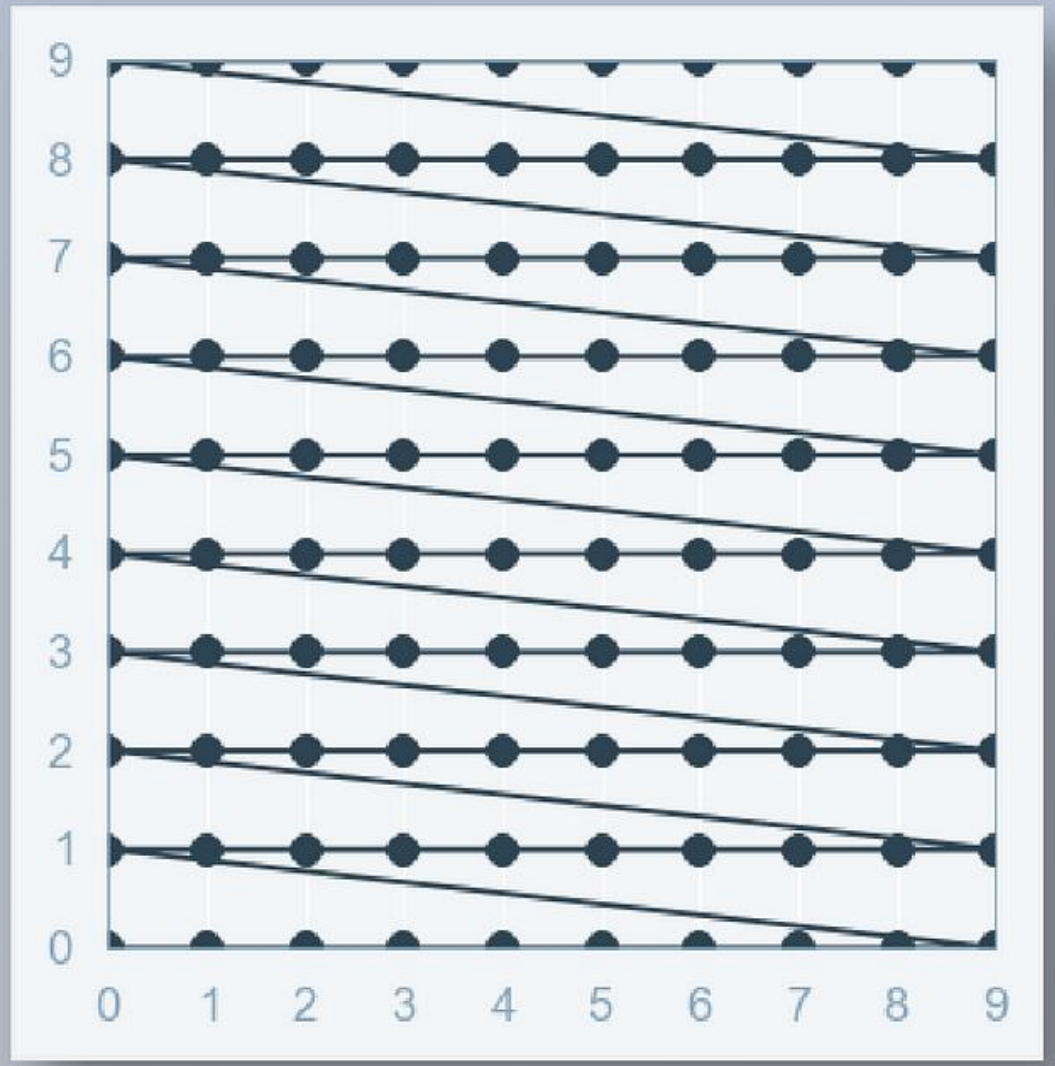

Portfolios of Google, Toyota, Coke, and Pepsi
Expected Return
Expected Volatility
Sharpe Ratio
0.18
0.16
0.14
0.12
0.10
0.08
0.10
0.12
0.14
0.16
0.18
0.20
0.22
0.24
0.26
1.04
0.96
0.88
0.80
0.72
0.64
0.56
0.48

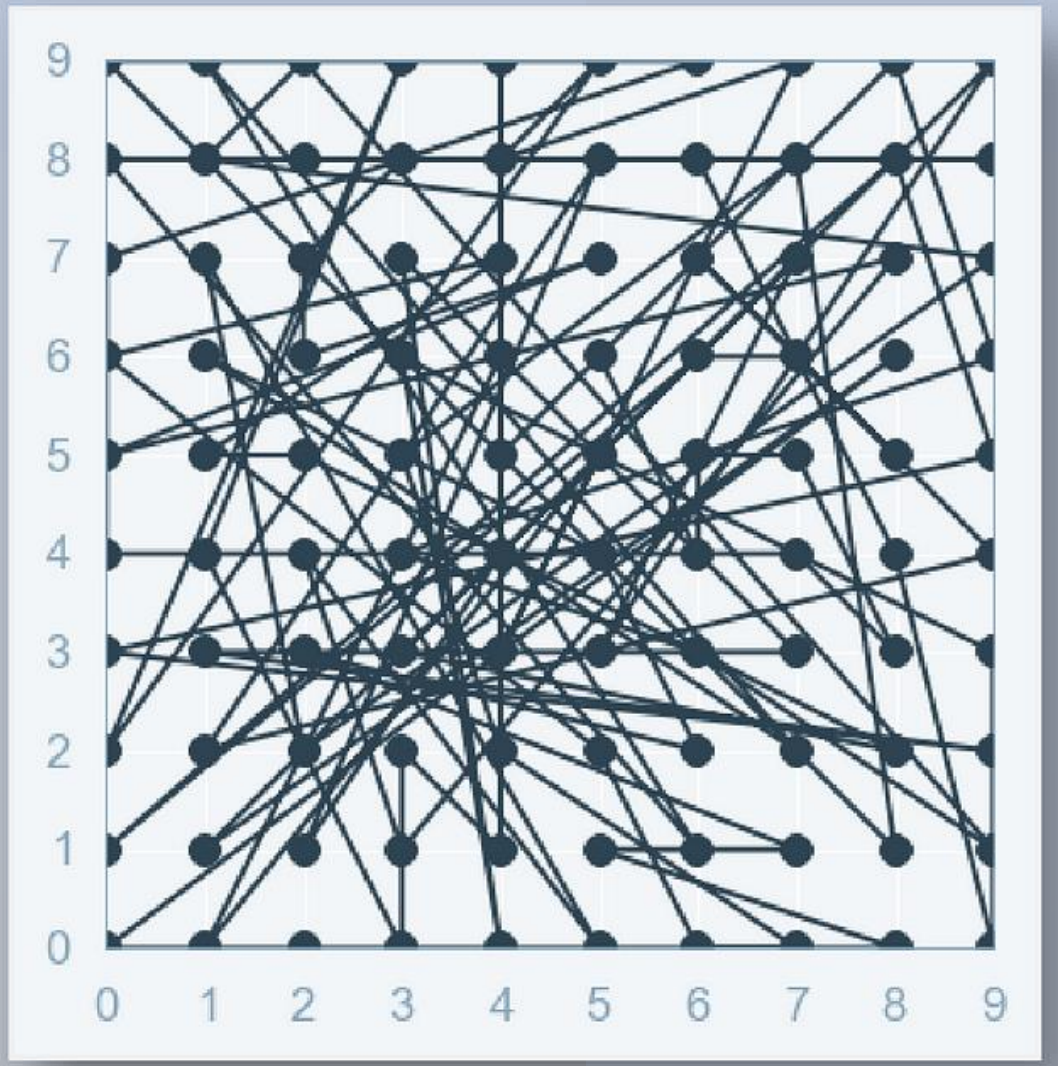

Capital
Market
Line (CML)
Expected
Return
Market Portfolio
Efficient
Frontier
Risk Free
Rate of
Interest
Standard Deviation

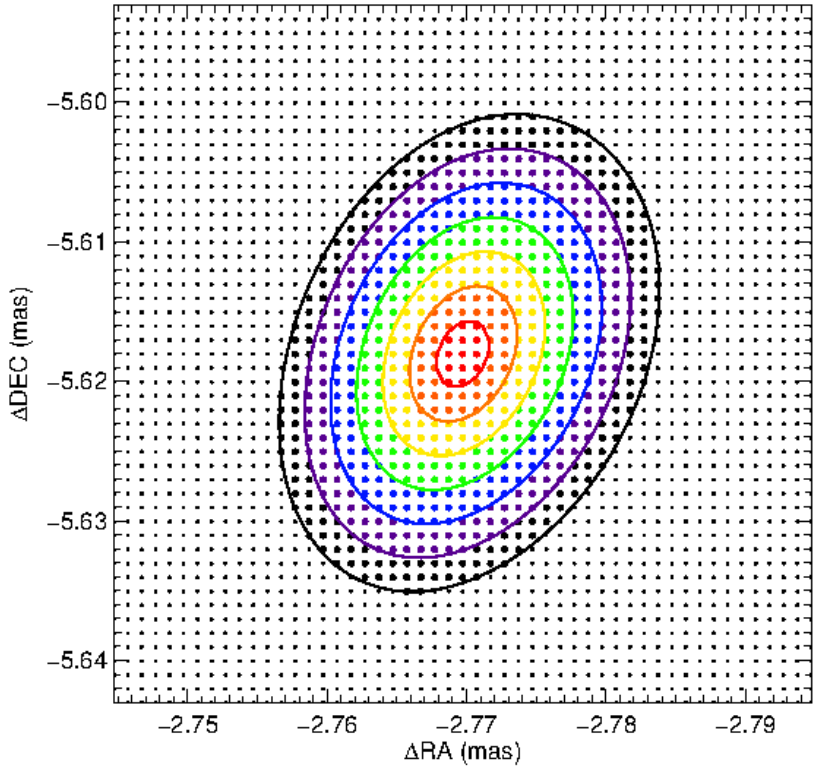
Grid Search Explained with Implementation



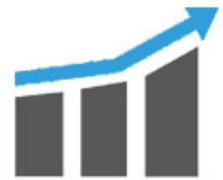
The Computing Process of Uninformed Searches

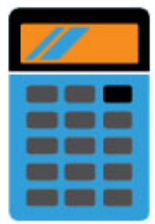





−5.60
−5.61
−5.62
−5.63
−5.64
ΔDEC (mas)
−2.75
−2.76
−2.77
−2.78
−2.79
ΔRA (mas)

Portfolio Return Formula





$$R_p = \sum(W_i * R_i)$$

The formula for portfolio volatility is:

$$\sigma_{Portfolio} = \sqrt{w_T \cdot \Sigma \cdot w}$$

- $\sigma_{Portfolio}$: Portfolio volatility
- Σ: Covariance matrix of returns
- w: Portfolio weights (w_T is transposed portfolio weights)
- $\cdot$ The dot-multiplication operator

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}$$

Workshop Project

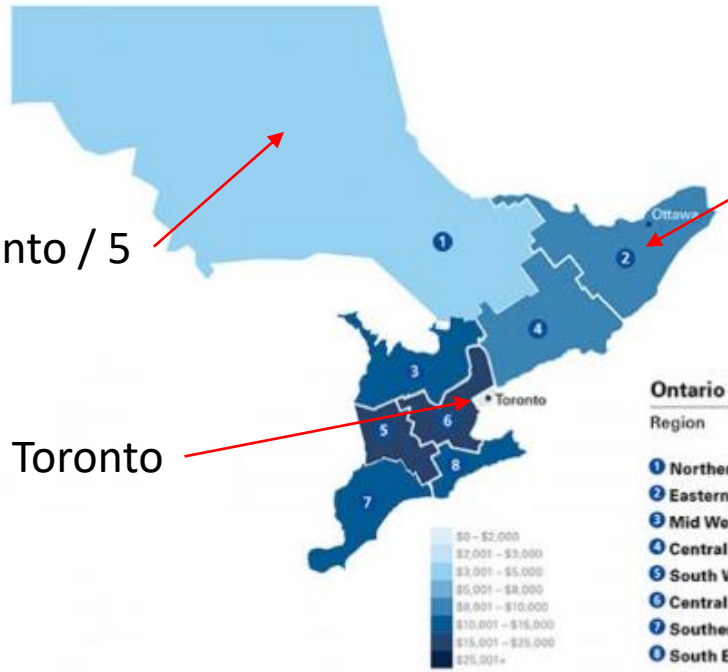
Optimal Tree Planting for Carbon Credits

- You have been retained by a sovereign wealth fund to that wants to enter the carbon credit market.
- They want to invest USD$1B in Ontario farmland and replace the crops with trees and earn carbon credits which they can exchange for USD.
- They want to buy parcels of land in Ontario only. Can you do better by investing across Canada? Does it make sense to plant trees in Nunavut?
- The key assumptions are in the following slides. You will need to fill in the blanks.
- Project Questions:
  - What is the return on an Ontario-only book of land?
  - What is the return on an equally weighted Canada-wide book of land?
  - What is the optimal holding? What is the return and Sharpe ratio? What are the three largest holding and their weights? Does this make sense?

● Tree Locations

Farmland values

STRONG INCREASES ACROSS CANADA



Toronto / 5

Toronto / 2

Toronto

Ontario farmland regions

Region	% change	Value $/acre*	Value range**
1 Northern	3.2%	$3,700	$2,000 – $5,000
2 Eastern	6.4%	$8,700	$3,200 – $14,200
3 Mid Western	8.9%	$11,200	$7,300 – $20,300
4 Central East	8.9%	$8,200	$3,300 – $18,000
5 South West	1.7%	$19,100	$12,000 – $25,800
6 Central West	8.2%	$19,600	$11,200 – $32,900
7 Southern	0.0%	$13,600	$8,200 – $20,000
8 South East	5.5%	$12,200	$7,900 – $20,700

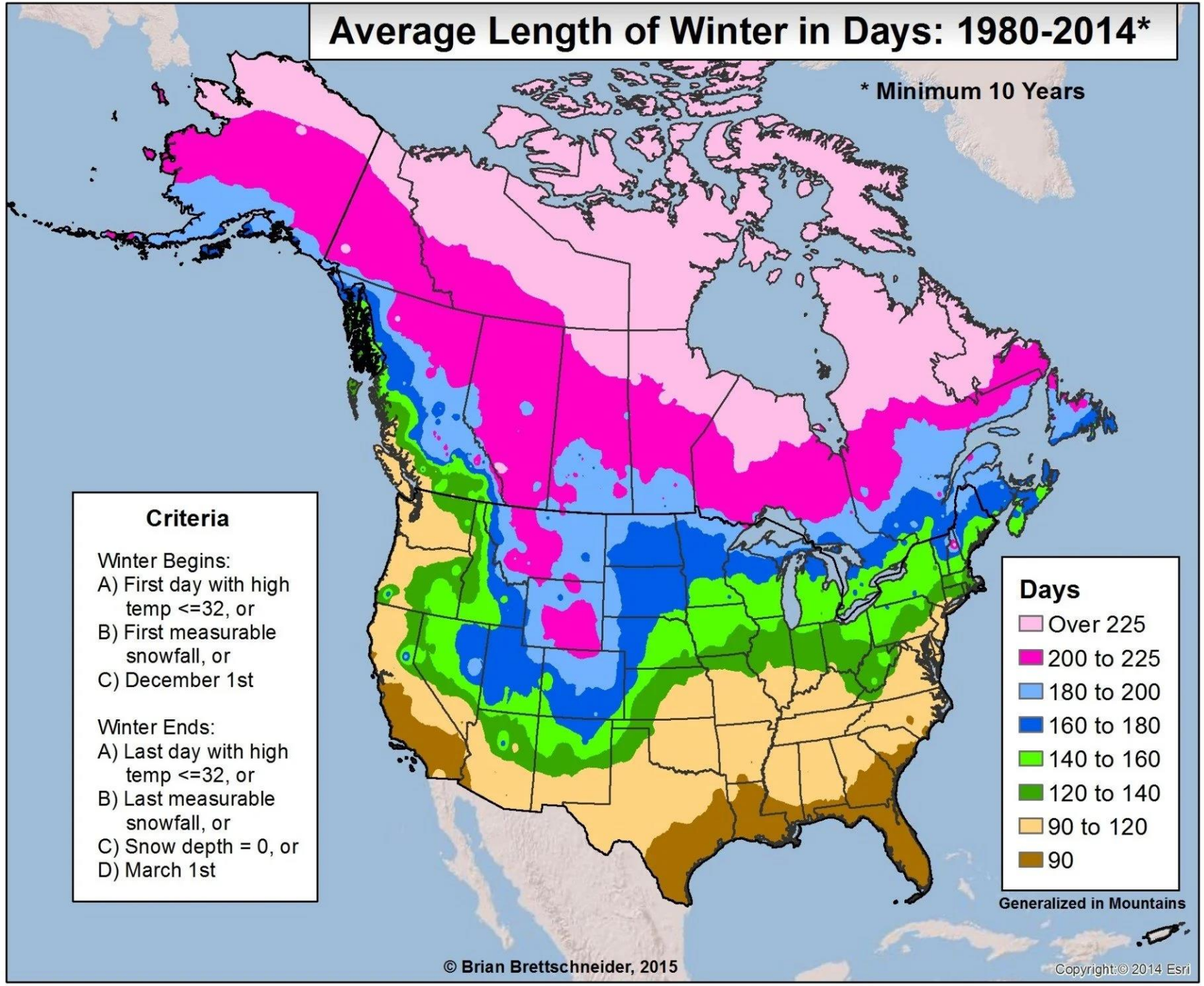
*FCC reference value $/acre.

**The value range represents 90% of the sales in each area and excludes the top and bottom 5%.

ONTARIO FARMLAND REGIONS. TABLE AND MAP COURTESY OF FARM CREDIT CANADA. FARMLAND ON THE OUTSKIRTS OF URBAN CENTRES OR WITHIN CLOSE COMMUTING DISTANCE TO LARGER URBAN AREAS ALSO DREW MORE DEMAND.

Geography	Farm land and buildings	2019	2020	2021	2022	2023
Canada (map)		Dollars				
	Value per acre	3,359	3,570	3,947	4,527	4,951
Newfoundland and Labrador (map)	Value per acre	5,847	6,623	7,497	8,133	8,642
Prince Edward Island (map)	Value per acre	4,063	4,421	4,664	5,898	6,452
Nova Scotia (map)	Value per acre	2,713	2,909	3,313	3,659	3,913
New Brunswick (map)	Value per acre	2,885	3,353	3,432	3,746	4,340
Quebec (map)	Value per acre	6,656	7,066	8,072	9,089	9,964
Ontario (map)	Value per acre	11,786	12,341	14,435	17,963	19,685
Manitoba (map)	Value per acre	2,382	2,532	2,761	3,121	3,439
Saskatchewan (map)	Value per acre	1,585	1,710	1,798	2,050	2,384
Alberta (map)	Value per acre	2,897	3,091	3,279	3,495	3,728
British Columbia (map)	Value per acre	6,855	7,449	8,812	9,987	10,056

source


Average Length of Winter in Days: 1980-2014*
* Minimum 10 Years
Criteria
Winter Begins:
A) First day with high temp <=32, or
B) First measurable snowfall, or
C) December 1st
Winter Ends:
A) Last day with high temp <=32, or
B) Last measurable snowfall, or
C) Snow depth = 0, or
D) March 1st
Days
Over 225
200 to 225
180 to 200
160 to 180
140 to 160
120 to 140
90 to 120
90
Generalized in Mountains
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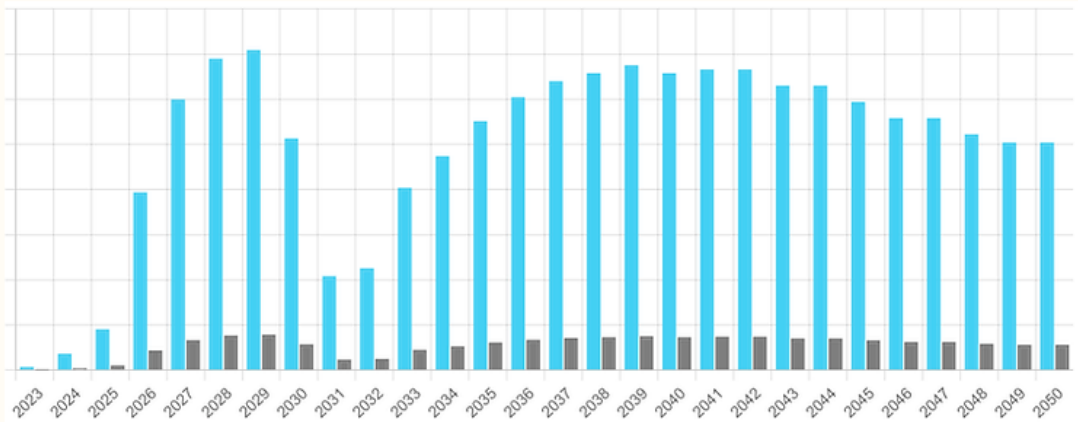
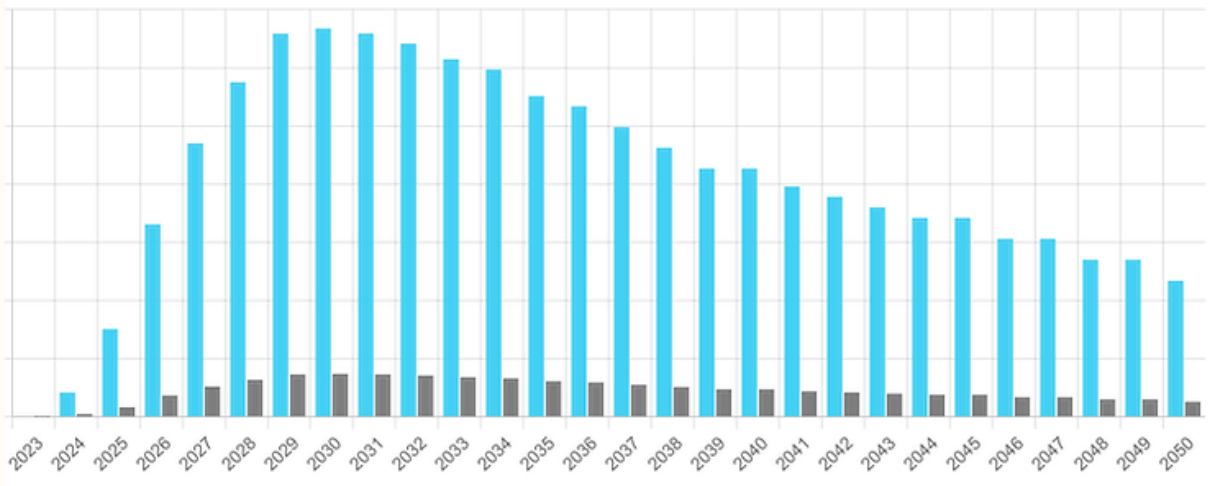
Pinus radiata

- Pines generate high short-term carbon yield and have a reputation for being relatively cheap and easy to establish.
- Planted in 2022, 10 ha of pinus radiata could earn an average of around 221 credits per year through to 2050. If sold at a price of $65 per credit, this would equate to around $14,365 per year, on average.
- The amount of carbon credits earned in a given year will vary as the trees grow. To illustrate this, take a look at the shape of this carbon sequestration chart for pines, borrowed from one of our land assessments. Notice the initial growth spurt, then a dip around the 10 year mark? This coincides with thinning operations that have been modelled into the growth rates for this species.



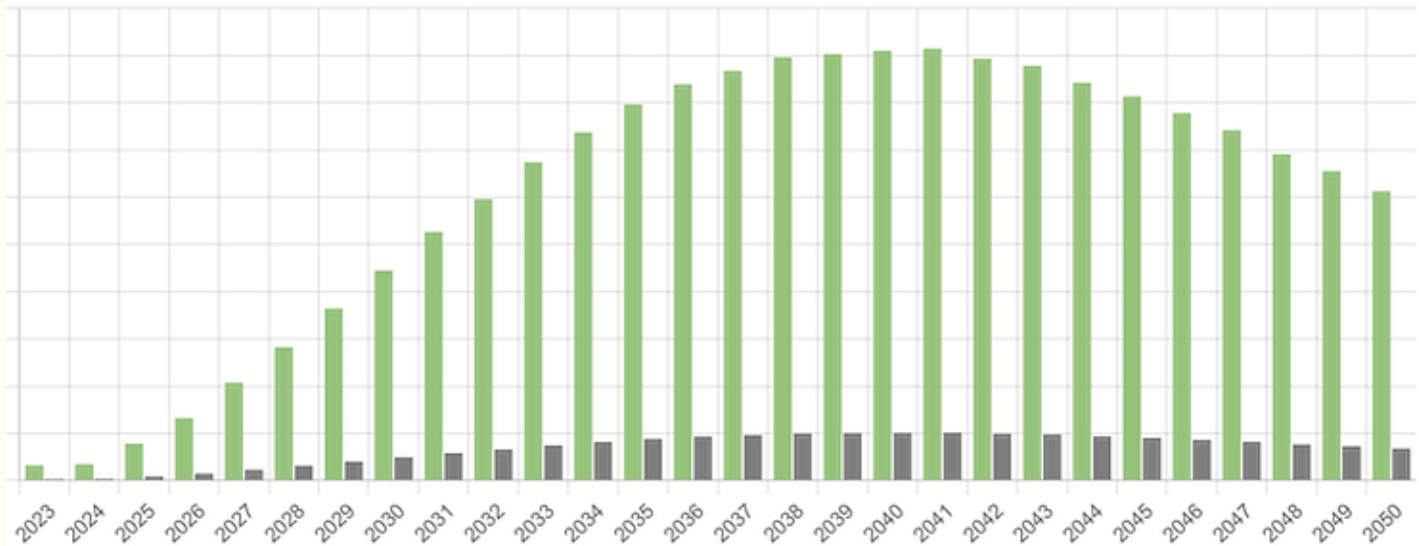
Exotic hardwood

- Exotic hardwoods also tend to grow rapidly. This category of trees includes eucalypts and tasmanian blackwood.
- Planted in 2022, your 10 ha plot of exotic hardwoods could earn an average of around 204 credits per year through to 2050. If sold at a price of $65 per credit, this would equate to around $13,260 per year, on average.
- As you'll see from the shape of this example chart for exotic hardwood, carbon sequestration increases sharply initially, then falls steadily over time:



Native forest

- Indigenous forest takes longer to ramp up, when it comes to carbon sequestration. Despite the rich variety of native species, the ETS currently lumps all indigenous species together - from kanuka to kauri - which provides no incentive to plant particular types of native trees. Note however, that there are plans to redesign the lookup tables and this opens up the exciting possibility that faster growing native species will be recognised for greater rates of carbon sequestration.
- Native planting can be expensive (depending on the species), yet tree planting may not be necessary for ETS registration if your land is already regenerating.
- As things stand, if 10 ha of native forest was planted or started regenerating in 2022, it would earn around 76 credits a year on average, between now and 2050. If sold at a price of $65 per credit, this would equate to around $4,940 per year, on average. The following chart shows the slow, sustained build in carbon credits which is typical of native forest:



CARBON CREDITS .com

Live Carbon Prices

	Last	Change	YTD
Compliance Markets			
European Union	€70.87	+0.65 %	−12.13 %
UK	£49.41	+7.41 %	+8.59 %
California	$28.66	-	0.00 %
Australia (AUD)	$33.60	+1.82 %	+0.30 %
New Zealand (NZD)	$51.25	+0.99 %	−25.89 %
South Korea	$6.36	+0.92 %	−2.88 %
China	$13.30	+3.95 %	+21.51 %
Voluntary Markets			
Aviation Industry Offset	$0.34	−5.56 %	−42.37 %
Nature Based Offset	$1.01	−2.88 %	+10.99 %
Tech Based Offset	$0.65	−2.99 %	+10.17 %

Carbon Prices by CarbonCredits.com

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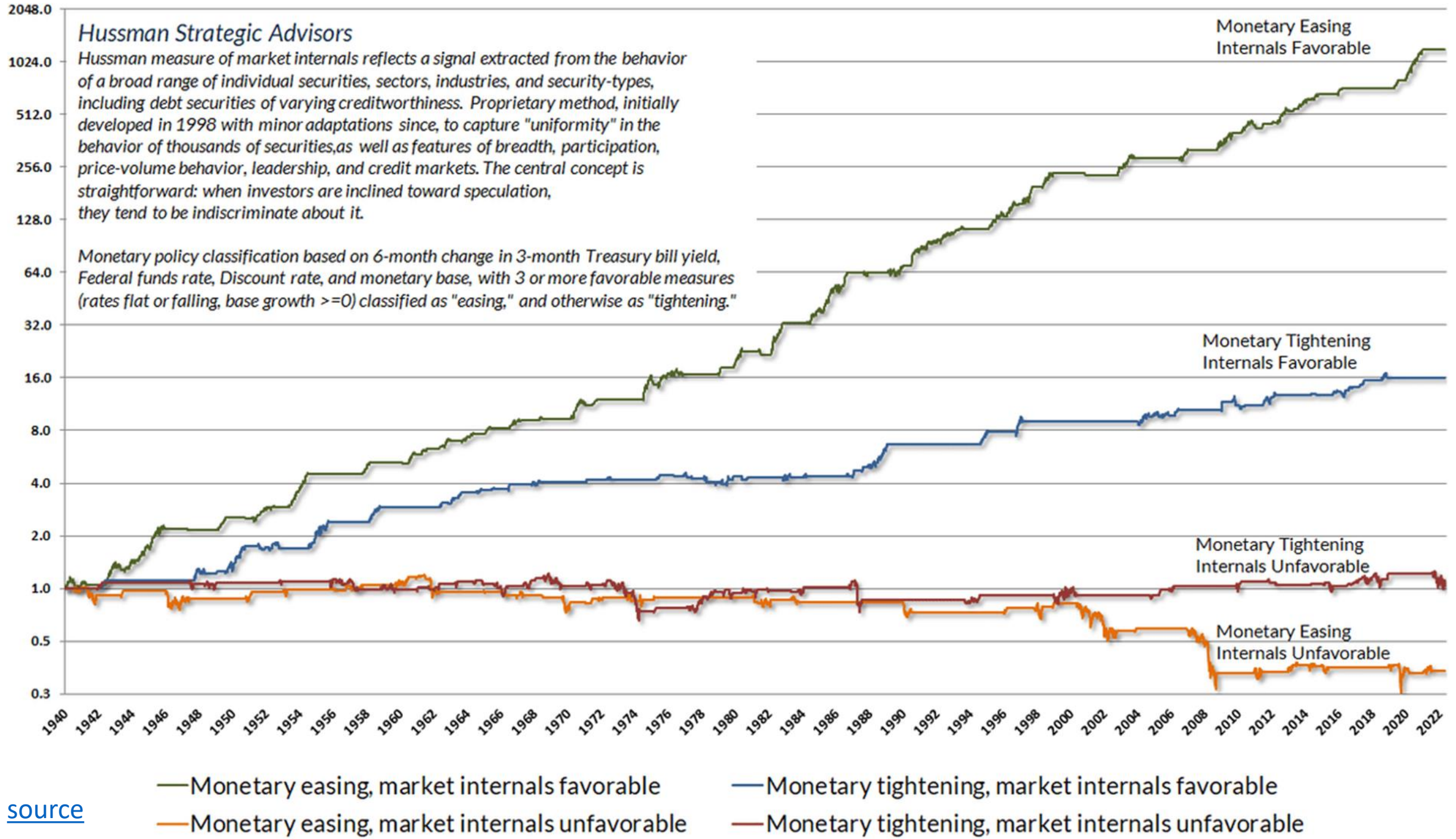
Appendix


Hussman Strategic Advisors
Hussman measure of market internals reflects a signal extracted from the behavior of a broad range of individual securities, sectors, industries, and security-types, including debt securities of varying creditworthiness. Proprietary method, initially developed in 1998 with minor adaptations since, to capture "uniformity" in the behavior of thousands of securities,as well as features of breadth, participation, price-volume behavior, leadership, and credit markets. The central concept is straightforward: when investors are inclined toward speculation, they tend to be indiscriminate about it.
Monetary policy classification based on 6-month change in 3-month Treasury bill yield, Federal funds rate, Discount rate, and monetary base, with 3 or more favorable measures (rates flat or falling, base growth >=0) classified as "easing," and otherwise as "tightening."
Monetary Easing Internals Favorable
Monetary Tightening Internals Favorable
Monetary Tightening Internals Unfavorable
Monetary Easing Internals Unfavorable
2048.0
1024.0
512.0
256.0
128.0
64.0
32.0
16.0
8.0
4.0
2.0
1.0
0.5
0.3
1940 1942 1944 1946 1948 1950 1952 1954 1956 1958 1960 1962 1964 1966 1968 1970 1972 1974 1976 1978 1980 1982 1984 1986 1988 1990 1992 1994 1996 1998 2000 2002 2004 2006 2008 2010 2012 2014 2016 2018 2020 2022
Monetary easing, market internals favorable
Monetary tightening, market internals favorable
Monetary easing, market internals unfavorable
Monetary tightening, market internals unfavorable

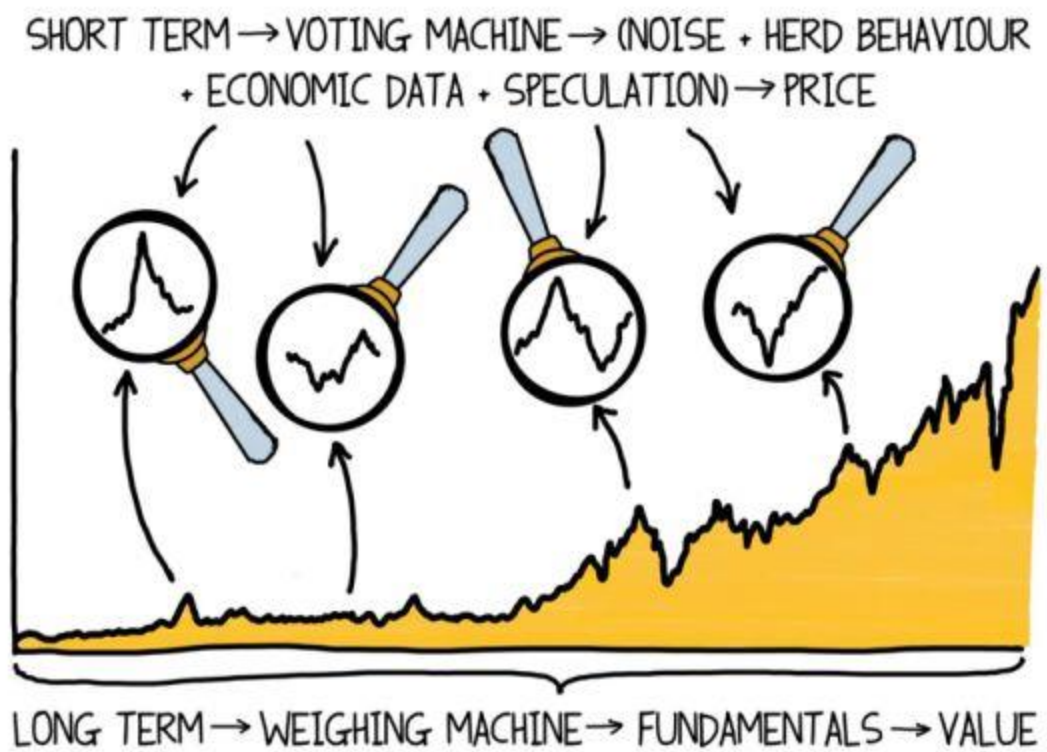
source

STOCK MARKET - VOTING OR WEIGHING MACHINE?

Vishal Khandelwal, safalniveshak.com

"Following Ben Graham's teachings, Charlie and I let our marketable equities tell us by their operating results - not by their daily, or even yearly, price quotations - whether our investments are successful. The market may ignore business success for a while, but eventually will confirm it. As Ben said: 'In the short run, the market is a voting machine but in the long run it is a weighing machine.'"

- Warren Buffett, 1987 Letter



BECOME ANTIFRAGILE

FRAGILE (Breaks under stress)

ROBUST (stays same under stress)

ANTIFRAGILE (Benefits from stress)

Life is uncertain, and often random. Things we think should happen, often don't. And things we think should not happen, often do.

However, the good thing about the randomness of Life is that it provides us with the ability to become better at dealing with, well, randomness. In Nassim Taleb lingo, randomness provides us with the opportunity to become "antifragile" – things that get better when exposed to shocks, stressors, disorders, risks, uncertainty, and randomness. Now, how do you become antifragile when it comes to investing your money? Avoid leverage, diversify adequately, practice simple and time-tested rules, avoid businesses and practices that may cause permanent capital loss, and keep a long-term perspective.

VISHAL KHANDELWAL