STA 2503 Project - 2 **Dynamic Hedging**

In this project you will investigate Delta-Gamma hedging within the Black-Scholes model.

You have believe that an asset price process $S = (S_t)_{t \geq 0}$ follows the Black-Scholes model. The asset's current price is \$10, you have just sold 10,000 units of an at-the-money $\frac{1}{4}$ year (=63 days) call on this asset, and you wish to hedge it. Call this option g.

You may trade in an at-the-money call with maturity 0.3 year (call this option h), the stock, and the bank account. As well, you will account for transaction costs by assuming you are charged \$0.005 per share on equity transactions and \$0.005 per option on option transactions. You may only trade integer value of stocks and options.

The remaining model parameters are

$$\mu = 10\%, \qquad \sigma = 25\%, \qquad \text{and} \qquad r = 5\%$$

and you hedge daily.

1. Compare the profit and loss distribution assuming you Delta hedge and when you Delta-Gamma hedge using 5,000 simulated paths. How do they vary as μ varies?

2. Plot the position you hold in the asset and the hedging option (when Delta-Gamma hedging) for two sample paths – one that ends in the money and one that ends out-of-the money. Set the random number seed so that the asset sample paths when Delta and Delta-Gamma hedging are the same – so you can compare them.

3. Suppose that the real-world $\mathbb{P}$ volatility is $\sigma \in \{20\%, 22\%, \dots, 30\%\}$, but you still sold the option using $\sigma = 25\%$, and hedge assuming that volatility is 25%. Compare again, the Delta and Delta-Gamma hedging cases.

Comment on any observations.