Q1： P&L Attribution

Using the right order, the following figure is the PnL Attribution

Calculation methods:

Then the change carries on

Etc.

图表, 瀑布图

描述已自动生成

Then we try to reverse the order, using the same algorithm in the above:

图表, 瀑布图

描述已自动生成

The difference in each section can be explained as follows: the sequential PnL attribution requires prioritize the time change. This is because when it comes to option pricing, factors such as risk-free rate, dividend yield, volatility all depend on the time change. If the PnL attribution does not account for time changes before calculating these factor, the attribution result for these factors will be biased. In addition, stock price is dependent on volatility. Therefore if we calculate volatility pnl attribute before price, we will have a biased pnl attribute on stock price.

Q2: VaR and ES

For , Assume loss is distributed , given that by definition needs to satisfy

Then in distribution

For , by definition:

Evaluating at the limit:

Set , then the limit changes to

The last equal is in 0/0 form at the limit, so we Applying L'Hôpital's rule

The is in 0/0 form at the limit, we apply L'Hôpital's rule again

Hence

Suppose loss are pareto distributed, pdf:

cdf:

Hence:

Therefore,

Evaluate at the limit:

The ES/VaR ratio is larger for pareto compared to normal distribution. This indicates that if the loss has a pareto distribution, then it has a larger tail risk than a loss that has normal distribution, i.e. more fat tail distributed.

Q3: Hedging an equity portfolio

Similarly

Correlation:

VaR:

Weighting of three assets:

Variance of portfolio

Delta hedge:

First we calculate the portfolio sensitivity to the index, i.e. the portfolio beta:

Therefore, using the delta approximation of the option,

Hence one dollar change in the index can be approximate to -0.25 dollar change in the option. Using this option to hedge the portfolio directional risk on index, we have

But we cannot hold fractional options. therefore we need to hold minimum of 4 options.

New Portfolio 10 days VAR:

Pricing of the option: we need to calculate the risk-free return first. Assuming the implied volatility of the option is the underlying volatility, and assume our hedging put option have an expiry of one year, i.e. .

Notice that there is only risk-free rate unknown in the above equation. Plug in the numbers, we have:

Therefore, using Black-Scholes pricing, we have

Where ,

Assuming

Where 1.06980257

Therefore, the new portfolio VaR

Return of the put option:

Option VaR dollar amount

New 10 days VaR

Value pullback

Q4: Netting for correlated Brownian Motion

Assume two contracts are not nettable：

If two contracts are nettable:

# max(ab)=1/2 something