Determining the media composition dependence of low-energy impact cratering characteristics in a dry, granular medium

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Abstract

Impact cratering depends on projectile-to-grain and grain-to-grain interactions during the very short time of impact. This study investigates the effects of different media composition, namely the ratio between beach sand and silica sand of the impacted medium, on crater diameter and depth. Pure silica sand, pure beach sand, and ratios of 1:2, 1:1 and 2:1 of silica:beach sand were tested, and a plastic ball was dropped from various heights for different media. The recorded crater diameters and depths indicate that impact cratering is a more complex process than previously thought mainly because of the increased randomness in grain-to-grain contacts and force chain distributions produced by mixing different granular materials. It seems that mixtures of smaller grains and larger grains create a quasi-alloy state where smaller grains fill in the gaps between larger grains to increase the number of grain-to-grain contacts and force chains, and hence increase the rigidity of the medium. An equal partitioning of silica sand and beach sand seem to maximize this effect, as the medium with a volume ratio of 1:1 silica:beach sand has the smallest scaling factor for crater diameter. Although the crater depths result did not follow the 1/3 to 1/4 scaling factor proposed by previous studies, the shallower depths with larger compositions of silica sand confirm that crater depth decreases as grain sizes increase. The data also suggest that the quasi-alloy state of mixed medium redirects the energy of the projectile from deeper penetrations instead to wider and shallower displacements of sand.

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1. Introduction

A granular medium is a macroscopic collection of distinct particles like sand. Since many economically influential products, ranging from coal to tree nuts, can be classified as granular materials, understanding the interactions characteristics of granular materials may have huge economic potentials. For this investigation, the reaction of the medium to an external disturbance due to a cratering projectile, a phenomenon called impact cratering, was examined.

The characteristics of impact cratering in a granular medium depend on projectile energy, energy dissipation in the impacted medium, and the medium's response mechanism to an externally unbalanced stress. The specific type of impact cratering investigated for this research was low-energy impact cratering in a granular medium, where the projectile's energy upon impact is relatively low compared to celestial high-energy impacts from asteroids and meteors (Kiefer, 2003). Low-energy impacts can also be distinguished from celestial high-energy impacts because celestial high-energy impacts are explosive (Pacheco-Vázquez and Ruiz-Suárez, 2011). That is, high-porosity projectiles, like meteors, disintegrate upon impact, and usually have enough thermal energy to melt and alter the properties of the impacted medium (Science Clarified). The low-energy impact craterings in this investigation were done with a plastic ball and thus are neither explosive nor capable of changing the properties of the impacted medium. To simplify the research, it was assumed that there were no cohesive forces between each grain (Duran, 2000).

1.1 Purpose

Although extensive previous research has been done on the cratering characteristics of granular material, only cratering properties with a quasi-homogeneous granular medium composed of grains with moderately similar grain diameters and properties were studied. This research attempts to examine whether the previously proposed scaling laws between projectile energy and crater parameters, namely crater diameter and depth, for an impacted medium composed of one type of granular material holds true for granular media composed of a mixture of different granular materials with different physical properties, namely silica sand and beach sand. If not, this research will propose a describable trend to explain the low-speed impact cratering characteristics in the medium composed of a mixture of granular materials.

1.2 Background and previous research

<u>Granular media properties</u>

When a granular medium is agitated through vibration or stress, grains come into contact with contiguous grains and collisions occur. Since grains stick to each other, the maximum amount of kinetic energy is lost through heat and sound via a completely inelastic collision (Claycomb, 2009).

The spatial distribution of physical contacts between each individual sand grain is theoretically calculable, but it requires exact information of the contact point positions and their intensities of previous contacts (Duran, 2000). However, the large number of grains and variations in surface properties, shapes, and sizes of the grains are why the solution to a problem involving analyses of single grain interactions is not approachable, and an empirical approach to the problem is more feasible (Duran, 2000). The inherently random collisions of grains cause the problem to be an indeterminate system (Duran, 2000).

However, the stress from an external force is always distributed through force chains that were discovered by Dantu (Duran, 2000). Force chains are simply chains of contacts between grains through which the energy is dissipated, as shown by the white lines in Figure 1. Although force chain formations are still erratic because their formation still rely on the random contacts of individual grains, their general characteristics are predictable: the energy distribution of a vertical stress in the granular medium, much like the cratering projectile in this experiment, occurs not only in the vertical direction, but even more so in the horizontal direction, as seen in Figure 1 (Duran, 2000). In fact, the horizontal stress varies linearly with the vertical stress imposed by the impacting projectile, as illustrated by the Janssen/Rayleigh model (Duran, 2000). Increased horizontal stress means increased horizontal force acting on the container by the grains. Following Newton's 3rd law, an increase vertical stress on the medium means that the container also pushes the grains inwards with an increased force, effectively increasing the rigidity of the medium (Duran, 2000).

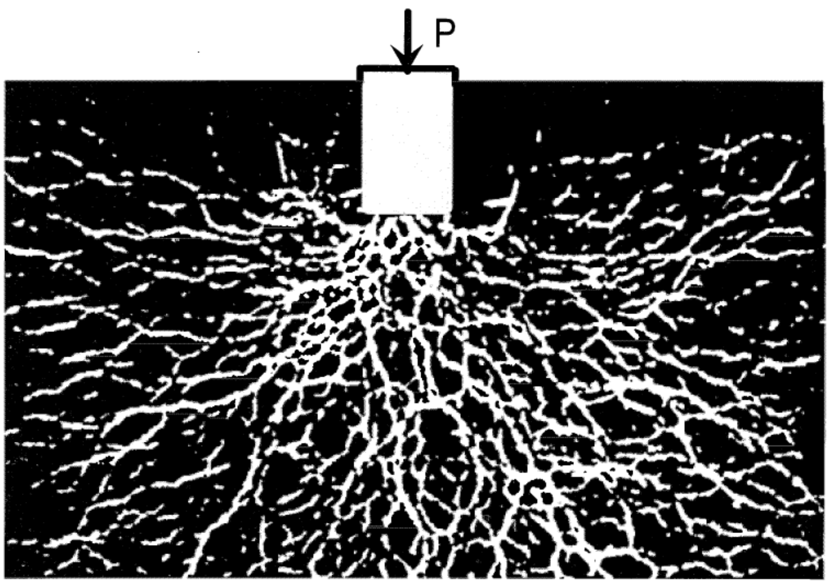

Figure 1 Dantu's experiment showing the force chains (white lines) of granular material (black areas) under stress (Duran, 2000)

Projectile interactions with granular media

When the projectile comes into contact with the granular media, it immediately gets influenced by velocity-dependent friction force, *f(v)*, and drag force, *N(v)*, exerted by the sand. Combined with the gravitational force, the net force on the projectile is

$$\Sigma F = f(v) + N(v) - mg \tag{1}$$

There are three stages for projectile interactions with the impacted medium to generate a crater: impact, penetration and collapse, respectively (Ciamarra et al, 2008). The impact phase consists of the initial interactions between projectile and the medium, resulting in a very high but decreasing acceleration in the opposite direction of motion (Ciamarra et al, 2008). Then, the penetration counter-intuitively consists of a relatively constant acceleration in the opposite direction of motion (Ciamarra et al, 2008). Finally, the projectile stops in the collapse phase (Ciamarra et al, 2008).

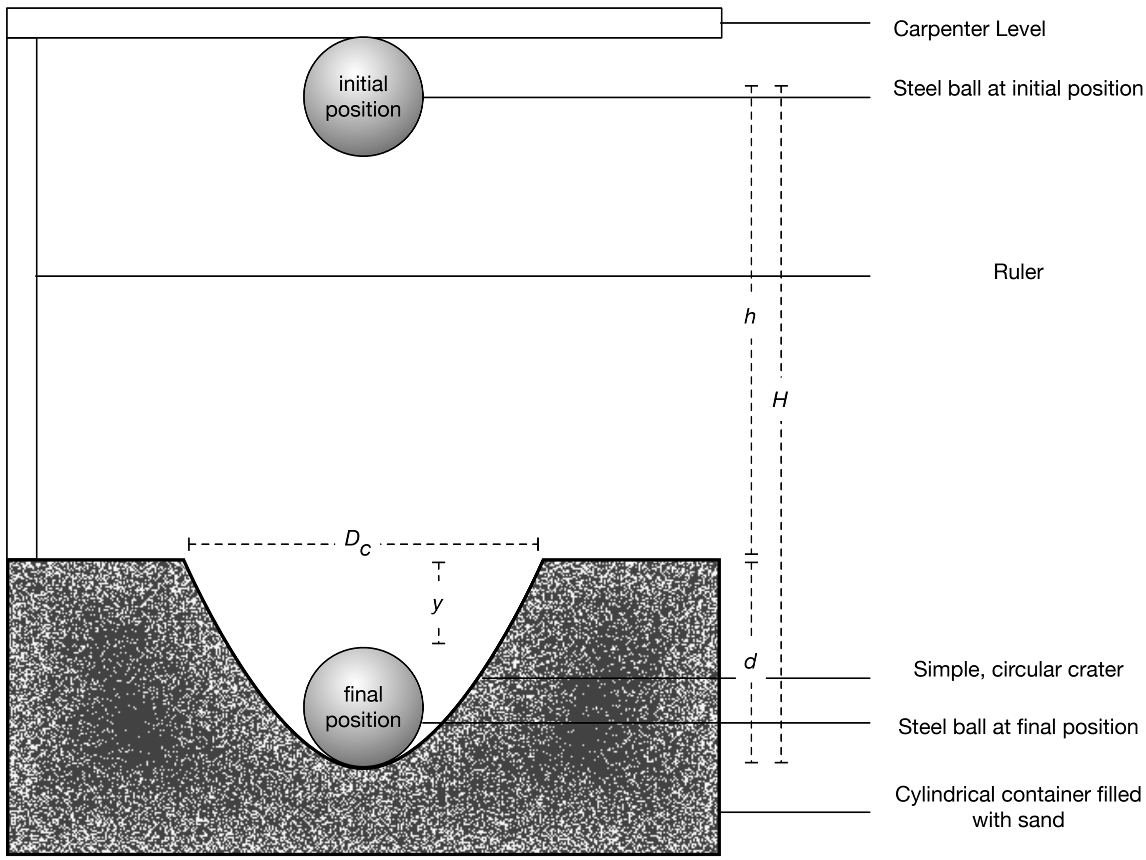

Figure 2 Experimental design showing the different variables and crater parameters

By using simple conservation of energy, relations between the total energy of the projectile and the properties of the formed crater can be derived. The work done by the medium to stop the projectile's motion is the total mechanical energy *mgH*.

$$W = mgH \tag{2}$$
$$where\ H{=}h{+}d.$$

Though friction is relatively small compared to the massive normal forces created by the force chains, it does have an affect on the penetration depth of the projectile. But in an ideal system without friction and the total force is the restoring force of the medium, Hertz's law, stating that the deformation by a distance x of a perfect-lattice of spheres under an influence of a vertical load F is $F{=}kx^{3/2}$, can be used to demonstrate that the depth of the crater does indeed decrease non-linearly (Duran, 2000).
Equation (3) shows the work done by granular medium that follows Hertz's law can be calculated using the general form of work.

$$W = \int_a^b F(x)dx = \int_0^d (kx^{\frac{3}{2}})dx$$

$$W = \frac{2k}{5} d^{\frac{5}{2}} \tag{3}$$

Equating equation (3) with equation (2),

$$mgH = \frac{2k}{5} d^{\frac{5}{2}}$$

$$d^{\frac{5}{2}} = \left(\frac{5}{2k}\right)(mgH)$$

Since k is a constant,

$$d \propto (mgH)^{\frac{2}{5}} \tag{4}$$

A new theoretical relation, equation (4), between the total mechanical energy of the projectile and crater depth is proposed.
Since craters are hyperbolic in nature and since the crater depth (d) is proportional to crater diameter (D_c), the volume and hence the mass of the displaced medium of an impact is proportional to D_c^3 and the force required to move the mass would also be proportional to D_C^3 (de Vet and de Bruyn, 2007). Equation (5) shows that when the total mechanical energy of the projectile goes into displacing the mass proportional to D_c^3 by a distance proportional to D_c, the scaling factor is 1/4 for the relationship between crater diameter and projectile energy (Uehara et al. 2003).

$$(D_c)^4 \propto (mgH)$$

$$D_c \propto (mgH)^{1/4} \tag{5}$$

Equation (5) is another model of the ideal case, where all the mechanical energy goes into cratering the crater.
In reality, force chains create sound and heat, and the container absorbs some energy as stored elastic energy. In fact, most distribution of energy dissipation occurs from the internal friction from grain-to-grain contact (Tsimring and Volfson, 2005). As a result, equation (4) and equation (5) are expected to break down as force chains become more numerous.

Previously proposed scaling laws

It is widely established and agreed that, for low-energy impact cratering in a dry granular medium, the relation between total projectile height ($h+d = H$) and crater depth (d) is $d \propto H^{1/3}$ (Uehara et al., 2003). A few others have proposed the relation to be $d \propto H^{1/4}$, but that is concerned with larger energy impacts and denser projectiles (Walsh et al., 2003). It is also widely agreed that the relation between projectile height and crater diameter (D_c) is well accepted to be $D_c \propto H^{1/4}$ (Uehara et al., 2003). The impact cratering of high porosity projectiles that disintegrated upon impact, like a celestial impact, showed the relation between projectile height and the crater depth to be $d \propto H^{1/4}$ (Pacheco-Vázquez and Ruiz-Suárez, 2011).
Intuitively, the scaling factors are all less than one, as the collisions between grains are inelastic and energy is effectively lost via friction. However, the magnitude of this loss of

energy is counter-intuitive and quite large, as the difference in crater diameter between dropping a ball from heights differing by 4 orders of magnitude (i.e. 10000m and 1m), when extrapolating with the model in [15] that $D_c \propto H^{1/4}$, only vary by 1 order of magnitude (i.e. 10m and 1m).

Note that since *mg* is simply a constant and does not affect the scaling factors, all previously mentioned scaling relationships in the form $x \propto H^y$ can be rewritten as $x \propto (mgH)^y$, which is the form of the derived models in equation (4) and equation (5).

Equation (5) coincides strikingly well with empirical models from previous studies, despite not considering the energy dissipation through friction.

But although equation (4) doesn't completely agree with the previously proposed scaling laws that the crater depth is proportional to $H^{1/4}$ and $H^{1/3}$, which takes into consideration the factor of friction, equation (4) is reasonably close (Uehara et al., 2003). Equation (4) also has an intuitively higher scaling factor than the empirical models as friction is expected to lower the scaling factor.

1.3 Hypothesis

The interaction between idealized granular materials was already erratic (Duran, 2000). The addition of different types of granular materials may provide another layer of randomness of contacts between contiguous grains. Hence, the force chain distributions may also dramatically change. Thus, the scaling laws proposed by preceding research are likely to break down and be altered significantly when the impacting medium contains multiple types of grains. However, since the total mechanical energy, *mgH*, of the impacting projectile will still be dissipated via heat and sound created by friction, a logarithmic relation between crater diameter, D_c, and crater depth, *d*, is expected (Uehara et al. 2007).

$$D_c = \alpha E^p \qquad \text{(Hypothesis.1)}$$

where D_c is the crater diameter,
α is the constant of proportionality,
E is the total mechanical energy of projectile equivalent to mgH,
p is a scaling factor where p<1

$$d = \beta E^q \qquad \text{(Hypothesis.2)}$$

where h_b is crater depth,
β is the constant of proportionality,
E is the total mechanical energy of projectile and is equivalent to mgH,
q is a scaling factor where q<1

In addition to studying the variations in scaling factors, p and q, between different media compositions, the constants of proportionality, α and β, should vary between trials because the ratio of ball density to grain density to the ¼ power is proportional to the crater diameter, as shown in Equation 6 (Uehara et al., 2003).

$$D_c \propto (\frac{\rho_{Ball}}{\rho_{Grain}})^{1/4} \tag{6}$$

And the ratio of ball density to grain density to the ½ power is proportional to the crater depth, as shown in Equation 7 (Uehara et al., 2003).

$$d \propto (\frac{\rho_B}{\rho_G})^{1/2} \tag{7}$$

As the volume ratio of denser grains' composition to the less dense grains' composition in the impacted medium increases, the overall density of the impacted medium should also increase, suggesting that α and β should decrease.

2. Experimental method and materials

2.1 *Materials*

A steel cylindrical container with a diameter of 14.15±0.05cm and a height of 6.568±0.004cm was used to hold the granular medium. It is important that both the distance of the ball from the sidewall of the cylinder and the height of the cylinder are larger than the diameter of the sphere because a smaller distance between the sphere and the sidewall of the cylindrical container causes shallower penetrations than normal. Counter-intuitively, a shallower filling height actually increases the cratering depth by a noticeable amount (Nelson et al., 2008).

A 18.125±0.002g plastic spherical marble of diameter 24.08±0.01mm was used for this research. Silica sand of grain size 0.70±0.02mm to 1.27±0.02mm and beach sand of grain size 0.20±0.02mm to 0.58±0.02mm were used in this experiment. To ensure the uniformity of the grain size of beach sand, a wire mesh composed of squares with side lengths 0.80±0.02mm was used to filter out any big pieces of stone in the beach sand. However, the store-bought silica aquarium gravel was not filtered due to its larger grain sizes and irregularity in shape. It is important that the grains are sufficiently large, as cohesive forces directly exert influence on crater formation when the diameter of each grain is less than 0.090mm (Zheng et. al, 2004). The ratio of the projectile's volume to the average target grain volume in the granular mixture was fairly large to minimize faint and irregular craters (Gütter et. al 2012).

2.2 *Experimental method*

<u>Preparing the medium</u>:

Five different media with varying compositions of silica sand and beach sand were used. Namely, the different media were pure beach sand, pure silica sand, and media of 1:2, 1:1, and 2:1 volume/volume ratios of beach sand to silica sand composition. They were prepared by mixing measured volumes of each type of sand with the aforementioned ratios.

For the experiment, a steel wire mesh attached with three strings and composed of rectangles with side lengths 0.65±0.05cm was placed at the bottom of the cylindrical container. Then, the granular medium was poured slowly into the container and was mixed with a dry and clean spoon until the grains were evenly mixed. For the media characteristics and hence cratering characteristics to be reproducible, the media had a quasi-similar packing density and flow history (Douglas J. Durian, Department of Physics and Astronomy, University of Pennsylvania, personal communication). To ensure both, the mixture was poured into the container until it overflowed. Then, the wire mesh was pulled out evenly and slowly from the bottom of the cylinder via the three strings attached. This procedure evened out the packing density by loosening areas of compacted medium from the inevitable, uneven spreading and mixing. Finally, a straight ruler was used to slowly and carefully remove sand grains above the highest point of the container. The finished surface was clean, clear and smooth for a consistent and reproducible medium surface.

<u>Dropping the projectile</u>:
The plastic sphere was dropped from a known height above the medium, h, found from the ruler and secured by the carpenter level, such that it had no angular momentum (Figure 2). The sphere impacted the surface of the prepared medium at an impact angle of 90° so that it forms only simple circular craters (Zheng et al., 2004). The diameter was measured using a caliper at three different points on the circular crater rim. The distance from the medium's surface to the highest point of the sphere in its final position, y, was measured and added to the diameter of the sphere to determine the total depth of the crater.
The ball was dropped from heights ranging from 60.0±0.2cm to 100.0±0.2cm at 5.0±0.2cm intervals.

3. Results

Over 350 data points consisting of crater diameters, crater depths, and total height of projectile, were collected over this investigation. A representative table of values for 1:1 silica:beach sand composition is listed in the appendix (A-1) for demonstration.
Using the raw data points, a log-log graph was used to determine the scaling factor and the constant of proportionality for the relationships between projectile and crater depth, and between projectile energy and crater diameter. Due to the precise caliper, the calculated uncertainty[1] was extremely small and does not take into account the subjectivity in crater diameter measurements since craters do not usually have a well-defined crater rim. Manually measuring the crater diameter is inevitably less precise and accurate than the caliper. Thus, a more accurate uncertainty[2] was determined to be around 0.75%.

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[1] $\delta \ln(x) = \frac{\delta x}{x} \ln(x)$ is the theoretical uncertainty of the log-log plot.

[2] Calculated by averaging the differences in D_c measurements (M1, M2, M3) for each trial.

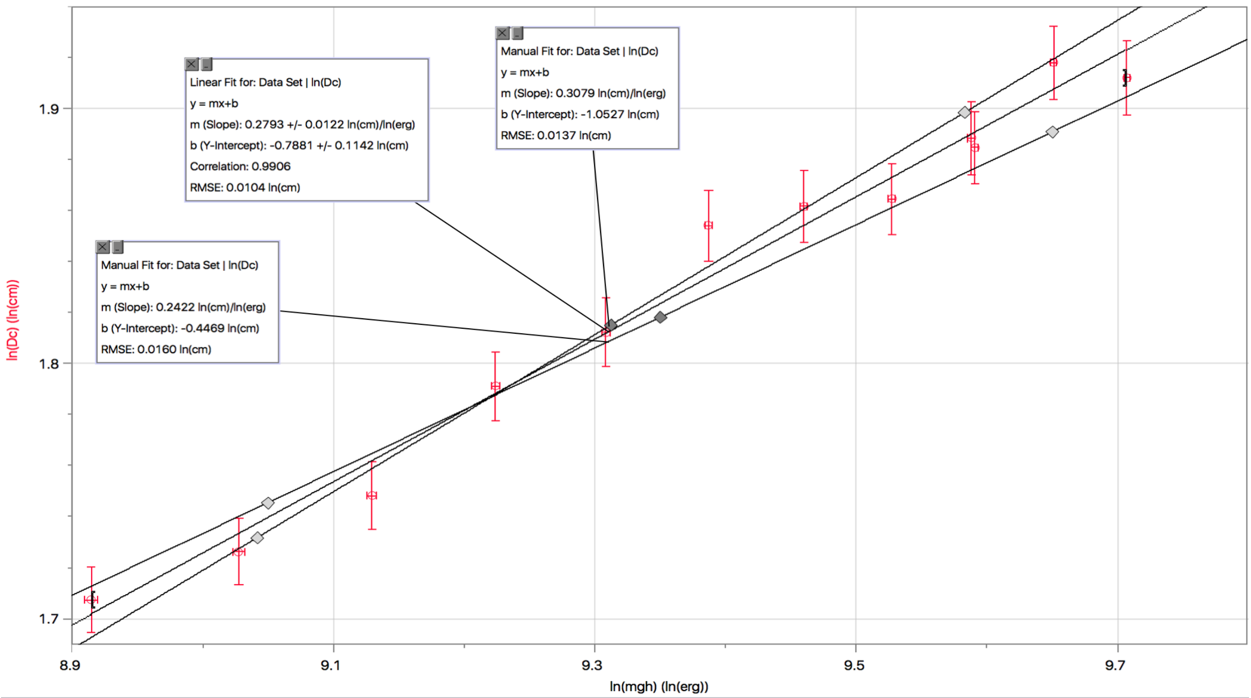

Figure 3 A representative graph of ln(mgH) plotted against ln(D_c) for experiments conducted in a 2:1 medium. The CGS unit erg is used for gravitational potential energy = mgH. Dimensional analysis: [g][cms^{-2}][cm]=[erg]. 1 erg = 10^{-7} J.

Figure 3 shows a clear positive linear relationship between ln(mgH) and ln(D_c) and is representative of the graphs for other medium compositions. From the error bars, the uncertainty of the slope is calculated to be (-1.0527+0.4469)/2=-0.303 and the uncertainty of the y-intercept is calculated to be (1.0532-0.4925)/2=0.28035. Thus, the slope is 0.28±0.03 and the y-intercept is -0.8±0.3.

Figure 4 shows the slope (m) and y-intercept (b) of log-log plots for all the collected datasets.

Medium type	Pure sand	silica:sand=2:1	silica:sand=1:1	silica:sand=1:2	Pure silica
Crater diameter	m = 0.24±0.04 b = -0.4±0.4	m = 0.28±0.03 b = -0.8±0.2	m = 0.17±0.04 b = 0.19±0.4	m = 0.21±0.06 b = -0.2±0.5	m = 0.26±0.07 b = -0.5±0.7
Crater depth	m = 2.0±0.1 b = -19±1	m = 3.5±0.9 b = -35±8	m = 1.4±0.3 b = -14±3	m = 1.6±0.3 b = -16±2	m = 5±5 b = -53±49

Figure 4 Table of slope (m) and y-intercept (b) values for ln(mgH) plotted against ln(D_c) and ln(d) in different media. Henceforth, the ratio x:y refers to silica:beach sand volume ratio. Units are not included as they are not relevant for discussion.

Crater diameter:

Besides the 1:1 medium, the y-intercepts are consistently negative. In Figure 4, higher slopes correlate with lower y-intercepts; there seems to be an inverse relationship between the slopes and y-intercepts for the crater diameter. Besides the 2:1 medium, Figure 4 shows that medium composed of a mixture of both types of sand gave lower slopes and larger y-intercepts. The lowest slope and y-intercept occur when the mixture is 1:1.

Crater depth:
The crater depth data seemed to be very randomly distributed, mostly reinforced by the large uncertainties. There is definitely a decrease in slope for the mixed media. Lower slopes correlate with higher y-intercepts. The maximum slope and minimum y-intercept occur when the mixture is made of pure silica sand.

4. Discussion

Clearly, the relationship between crater depth and projectile energy is completely divergent from previous research and the theoretical relationship in equation (4). The absolute uncertainties increase as the ratio of silica: beach increases. This suggests that silica sand's irregular shape and larger size may have rendered projectile-sand interactions much more random. But although no palpable scaling relationship between projectile energy and crater depth can be derived, the very low y-intercept values of pure silica and the 2:1 medium demonstrate a fairly intuitive concept that greater compositions of larger silica sands decrease cratering depth (Gütter et. al 2012).
The relationships between projectile energy and crater diameter are plausible and can be derived. As outlined in Hypothesis 1 and Hypothesis 2, the constant of proportionality, α and β, [3] is e^b, and the scaling factor, p and q, [4] is simply m.

Medium type	Equations
Pure silica	$D_c=(0.7\pm0.3)\bullet E^{(0.24\pm0.04)}$
Silica:sand=2:1	$D_c=(0.45\pm0.09)\bullet E^{(0.28\pm0.03)}$
Silica:sand=1:1	$D_c=(1.2\pm0.5)\bullet E^{(0.17\pm0.04)}$
Silica:sand=1:2	$D_c=(0.8\pm0.4)\bullet E^{(0.21\pm0.06)}$
Pure sand	$D_c=(0.6\pm0.4)\bullet E^{(0.26\pm0.07)}$

Figure 5 Projectile energy and crater diameter relationships for different media

As expected, both pure sand and pure silica follow the predicted ¼ scaling law proposed by previous research (Uehara et al., 2003). Since the scaling factor is at a minimum for the 1:1 medium, the craters in the 1:1 medium have the smallest diameters with the same impact energy compared to the other tested media. This may be explained by the formation of a quasi-alloy mixture of granular materials. In other words, this is very similar to interstitial alloys, where smaller atoms, like carbon, are positioned between larger atoms, like iron, to "fill in the gaps" for a more rigid structure, namely steel. Likewise, the smaller beach sand grains fill in the gaps between the larger silica sand and create more rigid and more numerous force chains that significantly increase the rigidity of the medium, hence decreasing the cratering diameter.
Furthermore, the less dense silica sand has a higher constant of proportionality than the denser beach sand, because denser media require more force to be displaced than less dense medium (Uehara et al, 2003). If beach sand does fill in the gaps between silica sand, the density of the medium should increase and the constant of proportionality should decrease. However, the maximum constant of proportionality occurs at 1:1, when the density should be the greatest. This

[3] Uncertainty for the constant of proportionality is $e^b \cdot \delta b$
[4] Uncertainty for the scaling factor is δm

may be because the increased rigidity from the quasi-alloy state of the medium changes the energy distribution altogether; the mixture may direct the projectile energy from penetrating deeper to instead displacing more sand from the surface, creating the larger diameter and a smaller penetration depth. This explanation is supported by the result that the minimum scaling factor for crater depth, and hence the smallest penetration depth, occurs at 1:1 in Figure 4.

5. Possible errors and mitigation

Although the manual wire mesh fluidization of granular media provided similar packing density and flow history, a better method, namely dry air-fluidization of media, should be adopted. In dry-air fluidization, the sand should be placed into a container where dry air initially blows at a high rate and decreases in a slow and controlled manner to reduce large gas bubbles in the medium, allowing for the creation of a consistent, reproducible and completely homogenous medium (Ambroso et al. 2005). This method produces media with properties closer to that of the theoretical ideal by preventing any environmental interference from humidity and condensation on the granular materials (Duran, 2000).

Also, more controlled granular materials may produce a more reproducible and clearer relationship between media composition and cratering characteristics. While there is a difference between average sizes of silica sand and beach sand by a factor of two to three, the variances of grain sizes for each medium is not negligible. Although beach sand was filtered by a thin mesh to discard any sand with irregular shape or sizes, silica sand was not filtered due to the large size and irregular shape. As a result, the randomness of contacts within the media is further increased due to the randomness in size and shape for the silica sand and beach sand. To address this random error, spherical glass beads with approximately the same size should be used in future research (Tiwari et al, 2014). However, this will compromise the "reality" of the research, as uniformly shaped and sized sand are unprecedented in nature. But, it will allow better control of a controlled variable and better define the crater characteristics with lower uncertainties and randomness in data.

6. Conclusion and future research

Pure silica, pure beach sand, and ratios of 1:2, 1:1 and 2:1 of silica:beach sand were tested to determine how the scaling factor and the constant of proportionality of projectile energy-crater diameter relationship and projectile energy-crater depth relationship are influenced by different compositions of the impacted medium. The relationship between projectile mechanical energy and crater depth had unexpected results, deviating greatly from the proposed 1/3 to 1/4 scaling factor from previous research. Future research with more consistent materials, like uniform glass beads, can generate more plausible data. The relationship between projectile energy and crater diameter of pure beach sand and pure silica followed the 1/4 scaling factor proposed by previous research (Uehara et al, 2003). The constant of proportionality of the denser beach sand was smaller than the constant of proportionality of the less dense silica, also corroborating previous research (Uehara et al, 2003). However, unexpectedly, the medium with equal volume partitions of silica sand and beach sand (1:1 medium) had the lowest scaling factor. This result implies that a mixture of silica sand and beach sand create a quasi-alloy medium, just like how carbon and iron create a more rigid material steel, which has more contact points, more force chains and thus more rigidity. Also, the 1:1 medium had the highest constant of proportionality. This suggests that the quasi-alloy medium redirects the projectile energy from penetrating deeper to creating larger and shallower craters, which is substantiated by the minimum scaling factor for crater depth for the 1:1 medium. This research illuminates an important concept: stronger and more rigid granular media can be created by mixing large and small grains in equal volume proportions, at least for the two granular materials used in this research. Of course, this is subject to change with different media mixtures and with the addition of a 3rd medium, a subject of a future research. In the future, more rigorous research should be done to show more palpable trends to minimize randomness in grain-grain contacts. As well, more compositions of media should be tested to develop a relationship between the scaling factors and constants of proportionality against media composition. This heuristic research illustrates that complex mechanisms of media composition do affect, in both intuitive and counter-intuitive ways, the cratering properties of granular media by altering force chains interactions in media.

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A-1 Data Sample

A-1 Data for impacted medium of 1:1 silica:beach sand composition

Trial Number	height(h)	ln(mgh)	Δln(mgh)	Dc (±0.02mm)			Avg Dc	ln(Dc)	Δln(Dc)	depth (d)	ln(d)	Δln(d)
	±0.2cm	ln(erg)	(1/mgh)*Δmgh	M1	M2	M3	±0.002cm	ln(cm)	(1/Dc)* ΔDc	±0.004cm	ln(cm)	(1/h)*Δh
1	92.628	9.708	0.002	65.74	65.59	65.60	6.564	2.2730	0.0002	0.628	0.82	0.09
2	87.541	9.652	0.002	66.05	64.89	65.00	6.531	2.2671	0.0002	0.541	0.82	0.09
3	82.523	9.593	0.003	64.71	63.70	65.27	6.456	2.2610	0.0002	0.523	0.82	0.09
4	77.549	9.531	0.003	63.77	63.76	64.73	6.409	2.2545	0.0002	0.549	0.81	0.09
5	72.574	9.464	0.003	63.84	62.28	63.58	6.323	2.2475	0.0002	0.574	0.81	0.09
6	67.498	9.392	0.003	63.58	63.38	61.82	6.293	2.2398	0.0002	0.498	0.81	0.09
7	62.373	9.313	0.003	62.64	61.77	61.43	6.195	2.2314	0.0002	0.373	0.80	0.09
8	57.275	9.228	0.004	60.91	61.51	60.79	6.107	2.2222	0.0002	0.275	0.80	0.09
9	52.214	9.135	0.004	58.43	59.29	60.03	5.925	2.2121	0.0002	0.214	0.79	0.09
10	82.576	9.593	0.003	64.35	63.83	63.83	6.400	2.2611	0.0002	0.576	0.82	0.09
11	87.35	9.650	0.002	66.90	64.00	64.55	6.515	2.2669	0.0002	0.35	0.82	0.09
12	92.544	9.707	0.002	66.61	67.56	66.25	6.681	2.2729	0.0002	0.544	0.82	0.09
13	87.621	9.653	0.002	65.26	65.56	65.42	6.541	2.2672	0.0002	0.621	0.82	0.09
14	87.574	9.652	0.002	66.83	65.71	65.07	6.587	2.2672	0.0002	0.574	0.82	0.09