EE5531 Miniproject 2

Kai Ye ye000064@umn.edu

December/17/2020

1 Wiener Deconvolution

1. Show that $R_{VU}(\tau) = g(-\tau) * R_V(\tau)$.

Solution:

$$U_t = \int_{-\infty}^{\infty} g(\alpha) V_{t-\alpha} d\alpha + W_t$$

$$\begin{aligned}
R_{VU}(\tau) = E[V_{t+\tau}U_t] &= E[V_{t+\tau} \int_{-\infty}^{\infty} (g(\alpha)V_{t-\alpha}d\alpha + W_t)] \\
&= E[V_{t+\tau} \int_{-\infty}^{\infty} g(\alpha)V_{t-\alpha}d\alpha] + E[V_{t+\tau}W_t] \\
&= \int_{-\infty}^{\infty} g(\alpha)E[V_{t+\tau}V_{t-\alpha}]d\alpha + 0 \\
&= \int_{-\infty}^{\infty} g(\alpha)R_V(\tau+\alpha)d\alpha \\
&= g(-\tau) * R_V(\tau)
\end{aligned}$$

2. Show that $R_U(\tau) = g(-\tau) * g(\tau) * R_V(\tau) + R_W(\tau)$.

Solution:

$$\begin{aligned}
R_U(\tau) = E[U_{t+\tau}U_t] &= E[\int_{-\infty}^{\infty} (g(\alpha)V_{t+\tau-\alpha}d\alpha + W_{t+\tau}) \int_{-\infty}^{\infty} (g(\alpha)V_{t-\alpha}d\alpha + W_t)] \\
&= E[\int_{-\infty}^{\infty} g(\alpha)V_{t+\tau-\alpha}d\alpha \int_{-\infty}^{\infty} g(\alpha)V_{t-\alpha}d\alpha] + E[W_{t+\tau}W_t] + 0 \\
&= g(-\tau) * g(\tau) * E[V_{t+\tau}V_t] + R_W(\tau) \\
&= g(-\tau) * g(\tau) * R_V(\tau) + R_W(\tau)
\end{aligned}$$

2 Discrete Time Wiener Filtering

No task for part 2.

3 Applications in Digital Image Processing

1. Use the Wiener filter formulation to estimate the signal, assuming that $\sigma^2 = 0$. Submit a printout of the estimated image. **Note that the noise is not actually zero here; you're only assuming that it is zero when forming the estimate. Your filter, then, will also apply to the noise that is in the blurred image**.

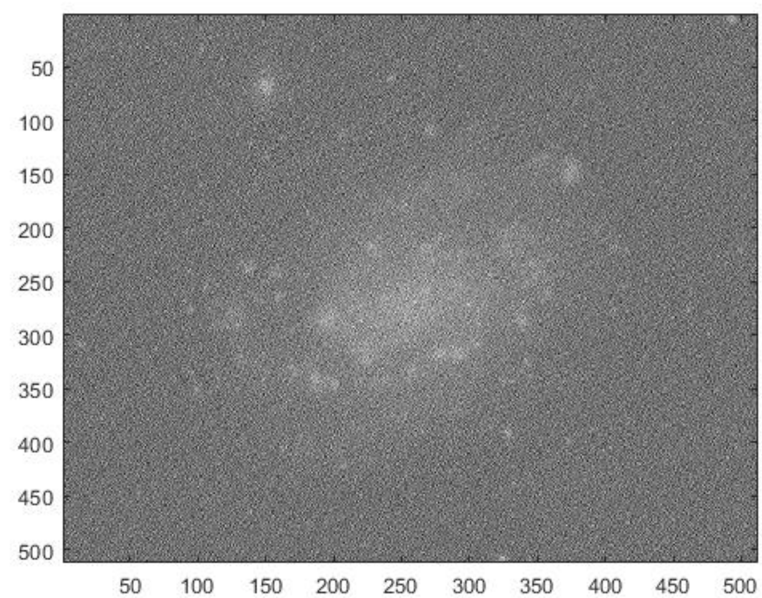

Figure 1: Estimated image with $\sigma = 0$

2. What happened in this case? Examine the form of the Wiener filter (in the frequency domain) when $\sigma^2 = 0$. What happens, in particular, at frequencies that are severely attenuated (i.e., are near zero) by the aberration $g(n1, n2)$? Can we hope to recover frequency components that have been set to zero? Submit a written explanation of your answer to this question.

Solution: When $\sigma^2 = 0$, $R_W(\tau) = \sigma^2\delta(\tau) = 0$, $S_W(f, \nu) = \sigma^2 = 0$:

$$H(f,\nu) = \frac{G^*(f,\nu)S_V(f,\nu)}{|G(f,\nu)|^2 S_V(f,\nu) + S_W(f,\nu)} = \frac{G^*(f,\nu)S_V(f,\nu)}{|G(f,\nu)|^2 S_V(f,\nu)} = \frac{G^*(f,\nu)}{|G(f,\nu)|^2}$$

After checking with the form of the Wiener filter in the frequency domain, the values are near zero at the four corners where the original parts of the zero-padded impulse response locate. As shown above by equation, the filter is now just a normalized conjugate of G.

When we apply this filter which ignores noise on the blurred image with noise, (i.e. $H(f,\nu) * U(f,\nu)$), the frequency components that have been set to zero can be recovered. Hence the image estimated by $\sigma^2 = 0$ is still blurred.

3. Use the Wiener filter formulation to estimate the signal, assuming that $\sigma^2 = 0.00001$, 0.0001, 0.001, 0.01, 0.1. Submit a printout of the estimated image in each of these 5 cases.

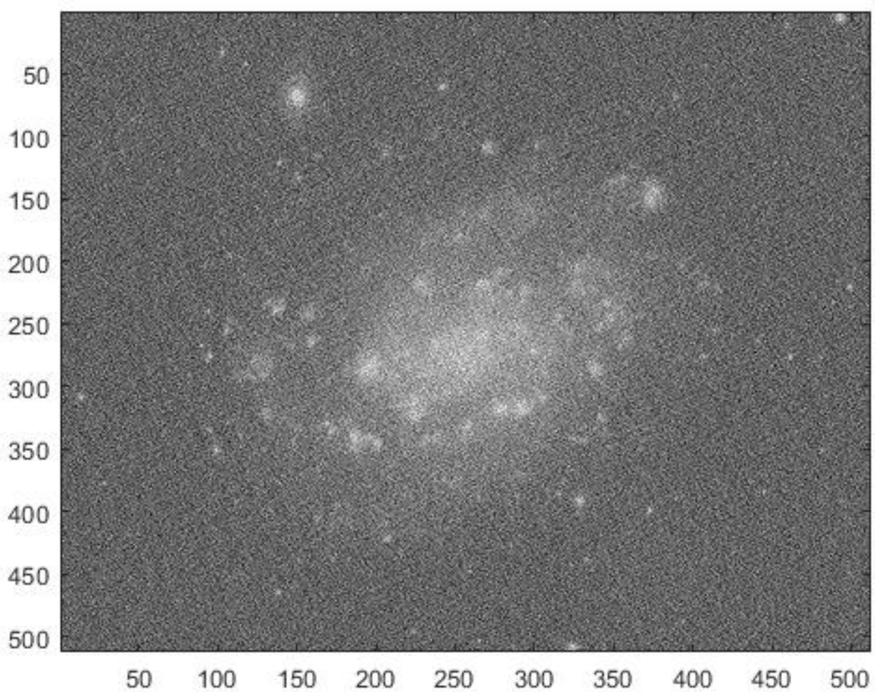

Figure 2: Estimated image with $\sigma = 0.00001$

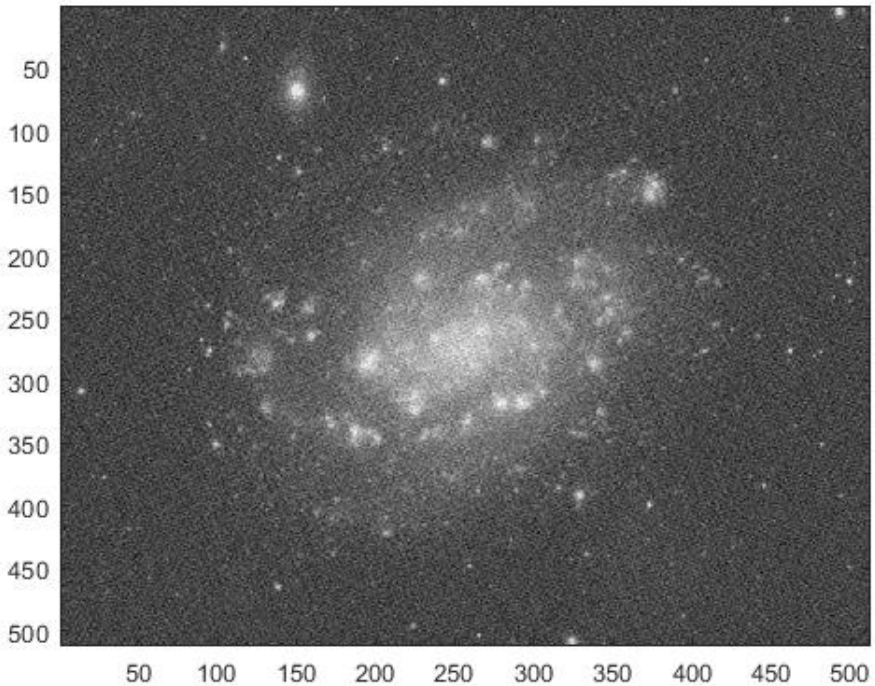

Figure 3: Estimated image with $\sigma = 0.0001$

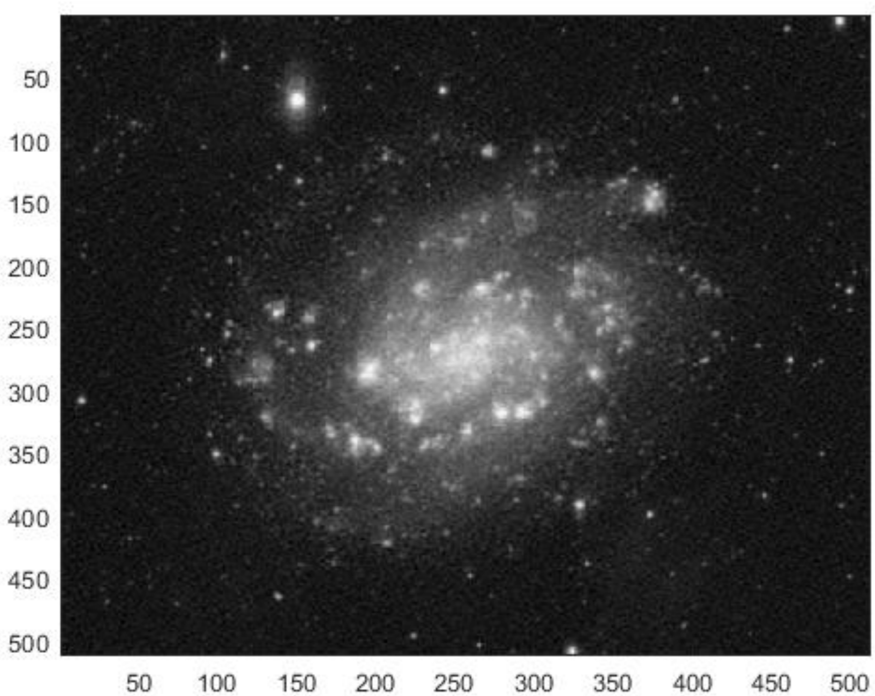

Figure 4: Estimated image with $\sigma = 0.001$

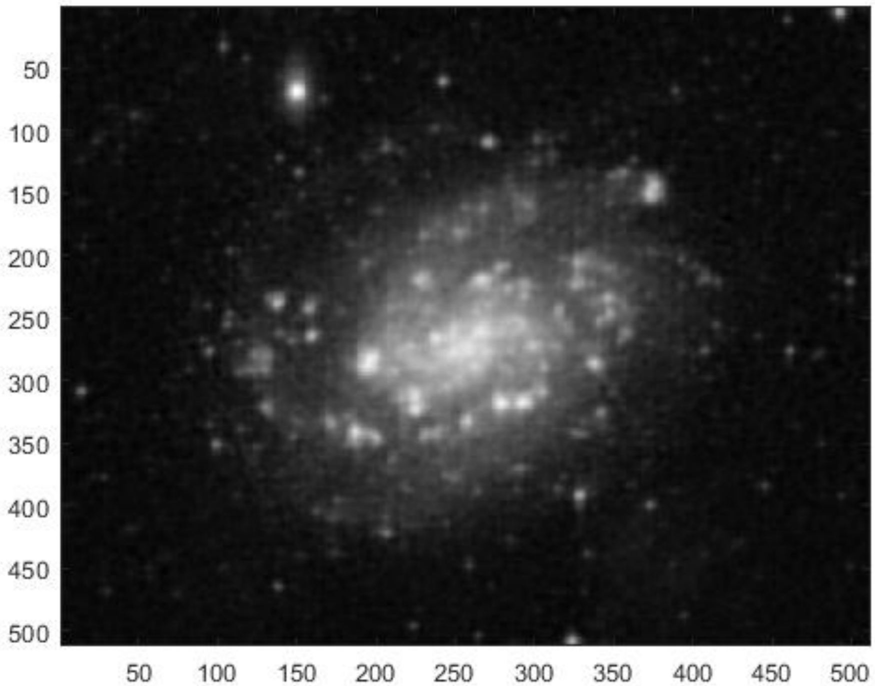

Figure 5: Estimated image with $\sigma = 0.01$

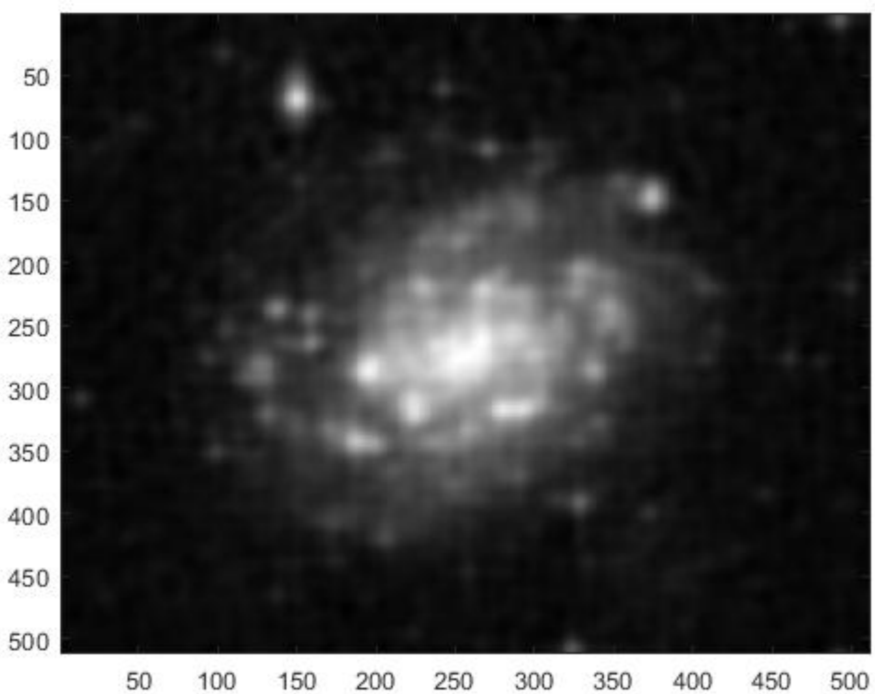

Figure 6: Estimated image with $\sigma = 0.1$

Continued

4. Which of the estimates looks the cleanest? Submit a written explanation of your answer to this question.

Solution: As shown above from previous two questions, comparing the six figures, the estimated figure with $\sigma = 0.001$ looks the cleanest, which means the variance of the white noise is around 0.001. Also, I noticed that it's at the same level of the frequency grid I chose (N=1000, i.e. $f = 0.001$). Recall that the power spectral density is represented as:

$$\hat{S_V}(k,l) = \frac{1}{N_1 N_2} \left| \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \hat{v}_{n_1,n_2} e^{-j2\pi(n_1 \frac{k}{N_1} + n_2 \frac{l}{N_2})} \right|^2$$

Choosing appropriate grid of frequencies would also be helpful for the recovery of blurred images.

4 Extension!

1. Explain why different types of motion blur may be affecting different portions of this image. Think about all of the kinematic forces that are at play in this scene.

Solution: The observation is modeled by $U_t = \int_{-\infty}^{\infty} g(\alpha) V_{t-\alpha} d\alpha + W_t$, where $g(t)$ is the (real) impulse response of a known deterministic system and W_t is the noise. As shown in Figure 7 and by our common sense, the pictures of fireworks usually include three sets of trajectories: the trajectory of ascent, the explosion center with diffusing trajectory, and the falling trajectory of scattered spark. These different trajectories represent different motions thus they represent different $g(t)$ functions with different noises.

I tried with processing with the whole image but the result is not good. I used Figure 8as the clean picture to estimate the original power spectral density and the best result I got is shown as Figure 9. I take a trajectory with one falling scattered spark shown in Figure 10 as the impulse response. The clean image and the transient response are not very accurate, hence the final estimation of whole image is scracely satisfactory.

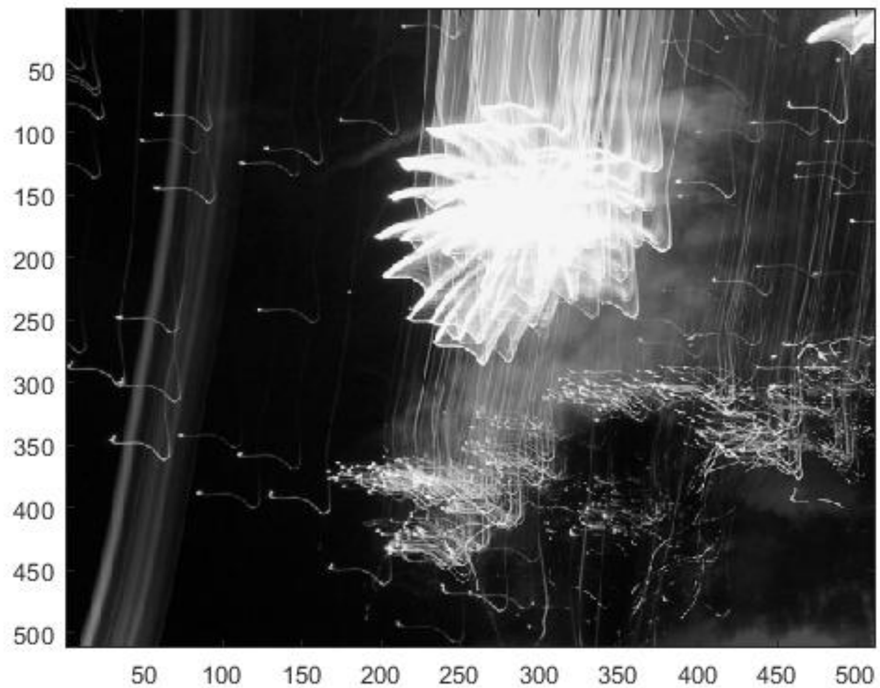

Figure 7: Blurred image of firework

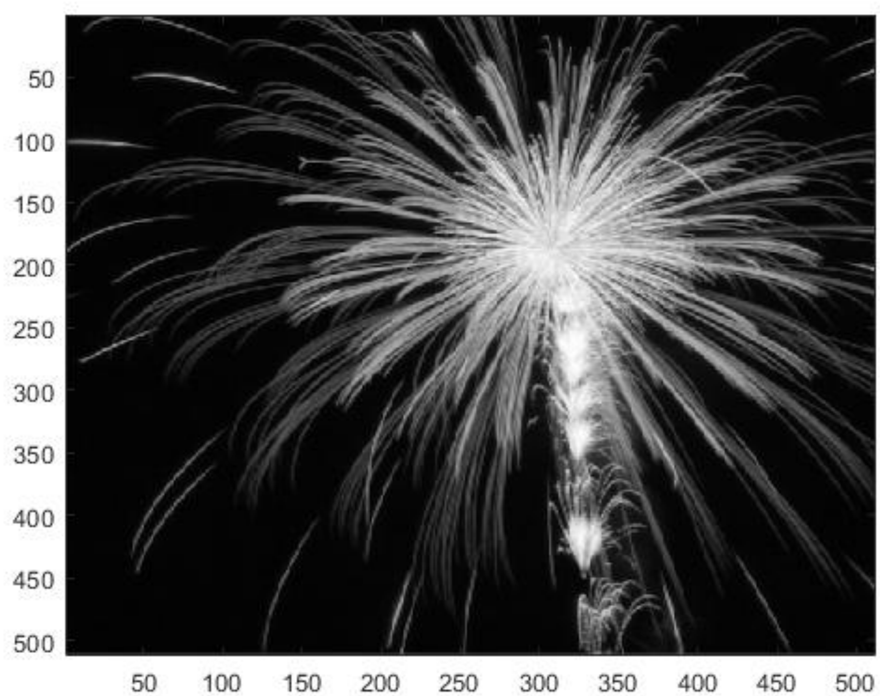

Figure 8: Clean firework image with similar power spectral density

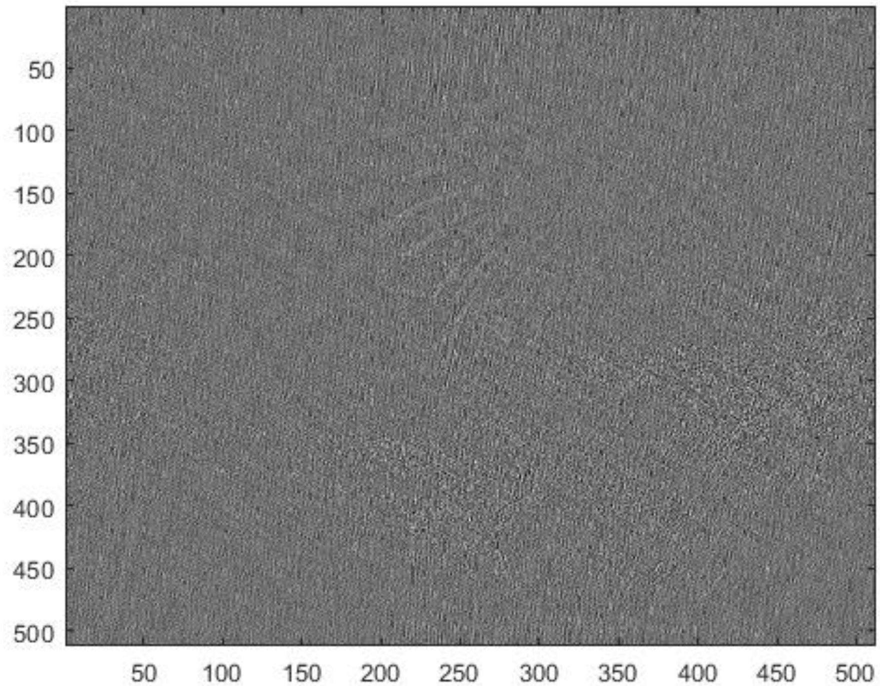

Figure 9: Estimated firework image with $\sigma = 0.001$

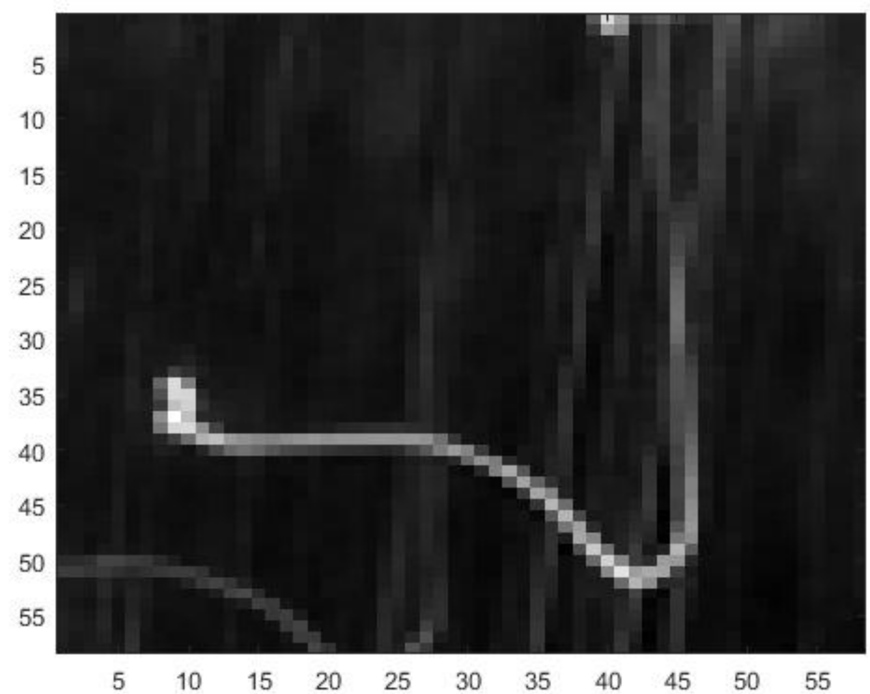

Figure 10: Impulse response from a single bright spot

2. Apply an approach analogous to that from the previous part to deblur a smaller (but still reasonably large) subimage of this image.

Solution: My result is shown below as Figure 11, which is the left part of the original image. The actual clean image should be several light spots as shown in the estimated results.

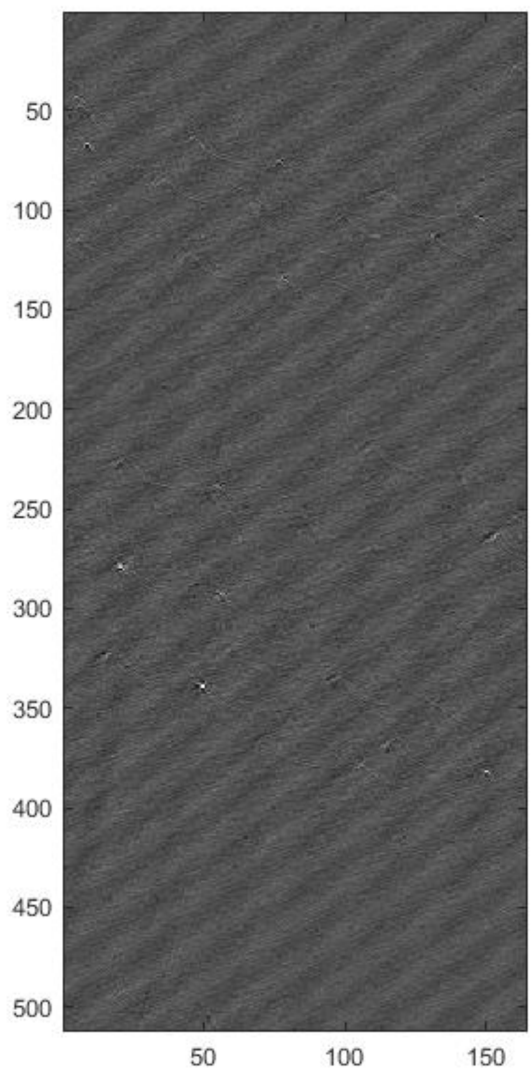

Figure 11: Estimated left part of the firework with $\sigma = 0.01$