Fintech545 Project Week6

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1 Question1

1.1 Basic Functions

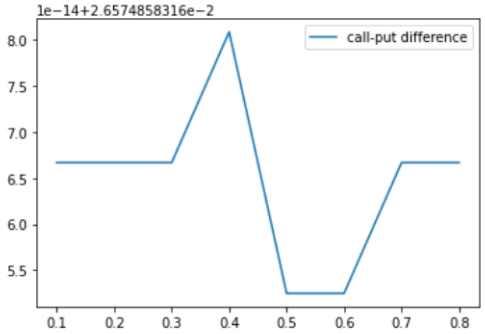
```
3]  def black_scholes(S,K,T,r,q,sigma,option_dir):
        d1 = (np.log(S/K)+(r-q+sigma**2/2)*T)/(sigma*np.sqrt(T))
        d2 = d1-sigma*np.sqrt(T)
        if option_dir == "Call":
            return S*np.exp(-q*T)*norm.cdf(d1)-K*np.exp(-r*T)*norm.cdf(d2)
        else:
            return K*np.exp(-r*T)*norm.cdf(-d2)-S*np.exp(-q*T)*norm.cdf(-d1)
```

```
def implied_volatility(S,K,T,r,q,opt_price,call=True):
    volatility = lambda x: black_scholes(S,K,T,r,q,x,call) - opt_price
    return fsolve(volatility, x0 = 0.5)[0]
```

Here are the BSM and implied volatility functions that I defined for this week's homework.

1.2 Value Results

```
def show_q1_results():
    sigmas = np.linspace(0.1, 0.8, 8)
    call_prices = []
    put_prices = []
    dif = []
    for sig in sigmas:
        call = black_scholes(S,K,T,r,q,sig,"Call")
        put = black_scholes(S,K,T,r,q,sig,"Put")
        call_prices.append(call)
        put_prices.append(put)
        dif.append(put-call)
    data = pd.DataFrame({'call':call_prices,'put':put_prices},index = sigmas)
    data1 = pd.DataFrame({'call-put difference':dif},index = sigmas)
    data1.plot()
    return data
```



	call	put
0.1	1.565266	1.591840
0.2	3.143538	3.170113
0.3	4.721369	4.747943
0.4	6.298521	6.325096
0.5	7.874768	7.901343
0.6	9.449882	9.476457
0.7	11.023638	11.050213
0.8	12.595810	12.622385

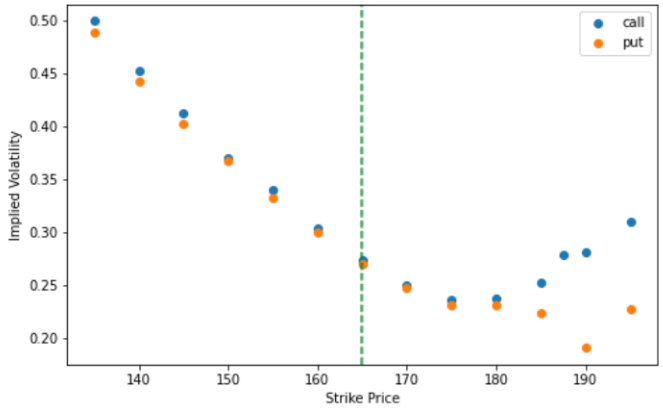
In this part, I calculate the option value for both put and call under different implied volatility, The results shows that, as the demand of options goes over supply, the price for both put and call option will rise, the the implied volatility will also rise, vice versa. Besides, I also plot the put-call spread of options under different volatility but find no concrete relation.

2 Question2

2.1 Results

AAPL

	Stock	Expiration	Type	Strike	Last Price	Implied vol
0	AAPL	3/18/2022	Call	135.0	30.175	0.499898
1	AAPL	3/18/2022	Call	140.0	25.300	0.452886
2	AAPL	3/18/2022	Call	145.0	20.525	0.412720
3	AAPL	3/18/2022	Call	150.0	15.850	0.369706
4	AAPL	3/18/2022	Call	155.0	11.525	0.340394
5	AAPL	3/18/2022	Call	160.0	7.525	0.304316
6	AAPL	3/18/2022	Call	165.0	4.225	0.273378
7	AAPL	3/18/2022	Call	170.0	1.935	0.249889
8	AAPL	3/18/2022	Call	175.0	0.715	0.235938
9	AAPL	3/18/2022	Call	180.0	0.260	0.237799
10	AAPL	3/18/2022	Call	185.0	0.115	0.252563
11	AAPL	3/18/2022	Call	187.5	0.120	0.278469



In this part, I import the options of AAPL, then used the implied volatility function I defined above to calculate its volatility. After that, I plot the volatility with the price of options. In this plot, I can find that there is a smiling curve structure for the volatility spread under different strike prices. Which means that at-the-money options have lower implied volatility than out-of-the-money or in-the-money options. Volatility smile is not supported by BSM model, since BSM model supposed that volatility is a flat curve. I think this smile structure

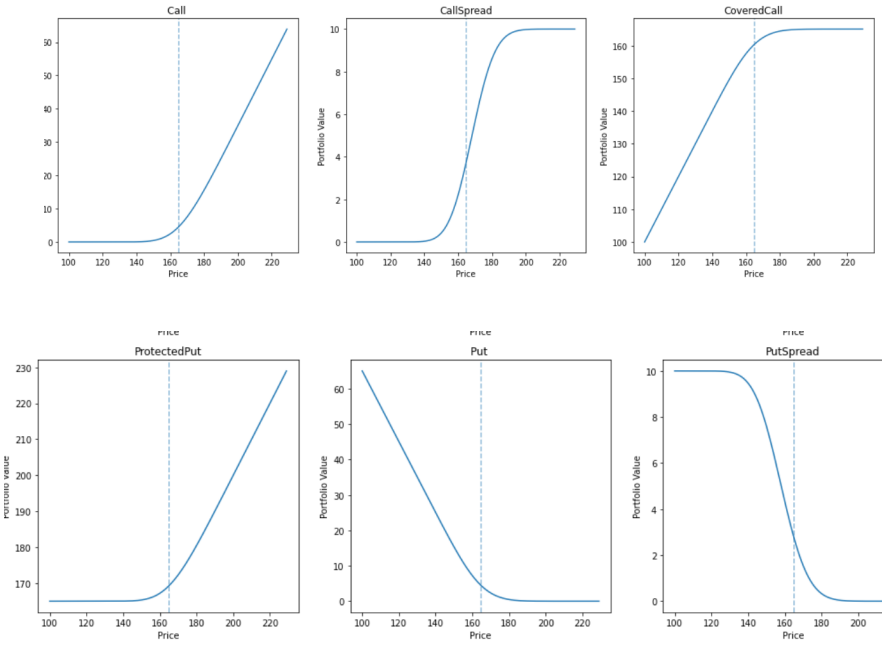
can be explained based on some behavioral economics theory like the Gambler's Fallacy which indicates that people will tend to overestimate the probability of some extreme cases to happen, only because it didn't happen for the past many time period.

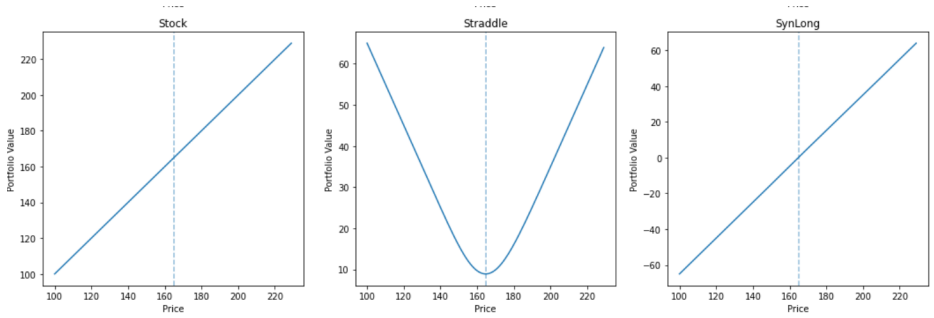
3 Question3

3.1 Different Synthetic Options

```
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:18: PerformanceWarning: DataFrame is highly fragmented.  This
```

Portfolio	Call	CallSpread	CoveredCall	ProtectedPut	Put	PutSpread	Stock	Straddle	SynLong
100	0.000000	0.000000	100.000000	165.006757	65.006757	9.998562	100.0	65.006757	-65.006757
101	0.000000	0.000000	101.000000	165.007062	64.007062	9.998562	101.0	64.007062	-64.007062
102	0.000000	0.000000	102.000000	165.007367	63.007367	9.998562	102.0	63.007367	-63.007367
103	0.000000	0.000000	103.000000	165.007672	62.007672	9.998562	103.0	62.007672	-62.007672
104	0.000000	0.000000	104.000000	165.007977	61.007977	9.998561	104.0	61.007977	-61.007977
...	...	...	...	...	...	...	...	...	...
225	59.955144	9.998563	165.044856	225.000003	0.000003	-0.000001	225.0	59.955147	59.955142
226	60.954836	9.998563	165.045164	226.000002	0.000002	-0.000001	226.0	60.954838	60.954834
227	61.954529	9.998563	165.045471	227.000001	0.000001	-0.000001	227.0	61.954530	61.954527
228	62.954222	9.998563	165.045778	228.000001	0.000001	-0.000001	228.0	62.954223	62.954221
229	63.953916	9.998563	165.046084	229.000001	0.000001	-0.000001	229.0	63.953917	63.953915





In this part, here are several synthetic options, the ways to build them are listed below:
1.Call Spread: A bull call spread consists of one long call with a lower strike price and one short call with a higher strike price in the same underlying asset and expiration date.
2.Put Spread: A put spread consists of 2 puts in different direction but in the same underlying asset and expiration date.Maximum profit and maximum risk are all limited for call and put spread.
3.Covered Call: A covered call is a two-part strategy in which stock is purchased and calls are sold. Maximum profit = strike price - stock price + premium. Maximum risk: unlimited.
4.Protected Put: A Protected Put is a two-part strategy in which stock is purchased and puts are bought. Maximum profit: stock price - premium paid which is unlimited. Maximum risk = stock price - strike price - premium
5.Straddle: simultaneously buying both a put option and a call option for the underlying security with the same strike price and the same expiration date. This strategy expects the underlying asset has a high fluctuation.Maximum profit: unlimited. Maximum risk: 2 * premium paid.
6.Synlong: A strategy that repeats the stock payoff.

3.2 Simulate stock price series

	0	1	2	3	4	5	6	7	8	9
0	162.435533	165.248521	160.810588	166.902162	164.353140	167.947377	166.005565	163.743239	166.484436	163.401748
1	166.432084	164.461563	166.292697	166.650184	166.409542	165.928883	164.154347	162.950827	164.606150	165.950683
2	165.536934	165.621669	166.614745	161.873546	164.964602	163.544961	166.418360	162.589061	170.564784	166.675680
3	165.583267	162.770590	163.424953	163.342408	162.022223	163.301368	165.584048	162.687396	168.069699	159.072126
4	160.382889	169.246551	160.505721	165.003444	171.009482	169.389457	164.836706	163.942309	167.265859	163.885336
...	...	...	...	...	...	...	...	...	...	...
1995	167.348282	168.445125	164.082795	162.173188	165.850588	160.361177	165.117028	164.720055	162.997937	165.628547
1996	167.584319	165.711656	167.218156	166.904694	166.337498	166.721319	166.424172	165.425267	165.850370	163.855083
1997	162.415947	167.128583	164.905502	164.483154	169.781169	162.054867	170.343940	161.836505	162.322618	169.142327
1998	162.359532	164.542137	165.870578	164.965974	167.421814	161.751311	164.151187	164.159752	168.182235	165.838326
1999	162.211764	165.467108	163.160805	167.292771	168.126730	165.348398	164.205460	167.339321	163.341988	165.858597

	Portfolio	Call	CallSpread	CoveredCall	ProtectedPut	Put	PutSpread	Stock	Straddle	SynLong
171.74019427732406	7.762123	6.265096	163.978072	172.630650	0.890455	0.750085	171.740194	8.652578	6.871667
170.7802912180562	7.016782	5.813390	163.763510	171.872304	1.092013	0.914073	170.780291	8.108794	5.924769
160.94213211603858	1.639861	1.587632	159.302271	166.479456	5.537324	4.131875	160.942132	7.177185	-3.897463
164.6673706359085	3.148816	2.937322	161.518555	167.962742	3.295371	2.600418	164.667371	6.444187	-0.146555
159.6571527733964	1.265060	1.234918	158.392093	166.121138	6.463985	4.710947	159.657153	7.729045	-5.198925
...	...	...	...	...	...	...	...	...	...	...
152.61958129187303	0.210885	0.210139	152.408696	165.168813	12.549232	7.695321	152.619581	12.760117	-12.338347
161.63919170335794	1.873203	1.803867	159.765988	166.705217	5.066025	3.825178	161.639192	6.939229	-3.192822
179.80951184301654	14.951881	9.042349	164.857631	179.923232	0.113720	0.098719	179.809512	15.065600	14.838161
156.17794339090986	0.566939	0.561304	155.611005	165.475857	9.297914	6.280190	156.177943	9.864853	-8.730975
164.12847035724124	2.889902	2.714162	161.238568	167.705089	3.576619	2.802585	164.128470	6.466521	-0.686716

2000 rows × 9 columns

In this part, I simulate the stock price series and got the price 10 days from now. Then, I used this price to calculate the value of each portfolios.

3.3 Final results: ProfitLost, VaR, ES

	Mean	VaR	ES
Straddle	0.324467	2.442145	2.452667
SynLong	0.127025	13.682455	16.666597
CallSpread	-0.147681	3.636945	3.711153
PutSpread	0.292053	2.679557	2.740726
Stock	0.250977	13.461481	16.427392
Call	0.225746	4.356602	4.431042
Put	0.098721	4.261091	4.331759
CoveredCall	0.025230	9.104879	11.996350
ProtectedPut	0.349698	4.135627	4.191836

	Straddle	SynLong	CallSpread	PutSpread	Stock	Call	Put	CoveredCall	ProtectedPut
0	-0.247422	6.771667	2.485096	-2.049915	6.890194	3.262123	-3.509545	3.628072	3.380650
1	-0.791206	5.824769	2.033390	-1.885927	5.930291	2.516782	-3.307987	3.413510	2.622304
2	-1.722815	-3.997463	-2.192368	1.331875	-3.907868	-2.860139	1.137324	-1.047729	-2.770544
3	-2.455813	-0.246555	-0.842678	-0.199582	-0.182629	-1.351184	-1.104629	1.168555	-1.287258
4	-1.170955	-5.298925	-2.545082	1.910947	-5.192847	-3.234940	2.063985	-1.957907	-3.128862
...	...	...	...	...	...	...	...	...	...	...
1995	3.860117	-12.438347	-3.569861	4.895321	-12.230419	-4.289115	8.149232	-7.941304	-4.081187
1996	-1.960771	-3.292822	-1.976133	1.025178	-3.210808	-2.626797	0.666025	-0.584012	-2.544783
1997	6.165600	14.738161	5.262349	-2.701281	14.959512	10.451881	-4.286280	4.507631	10.673232
1998	0.964853	-8.830975	-3.218696	3.480190	-8.672057	-3.933061	4.897914	-4.738995	-3.774143
1999	-2.433479	-0.786716	-1.065838	0.002585	-0.721530	-1.610098	-0.823381	0.888568	-1.544911

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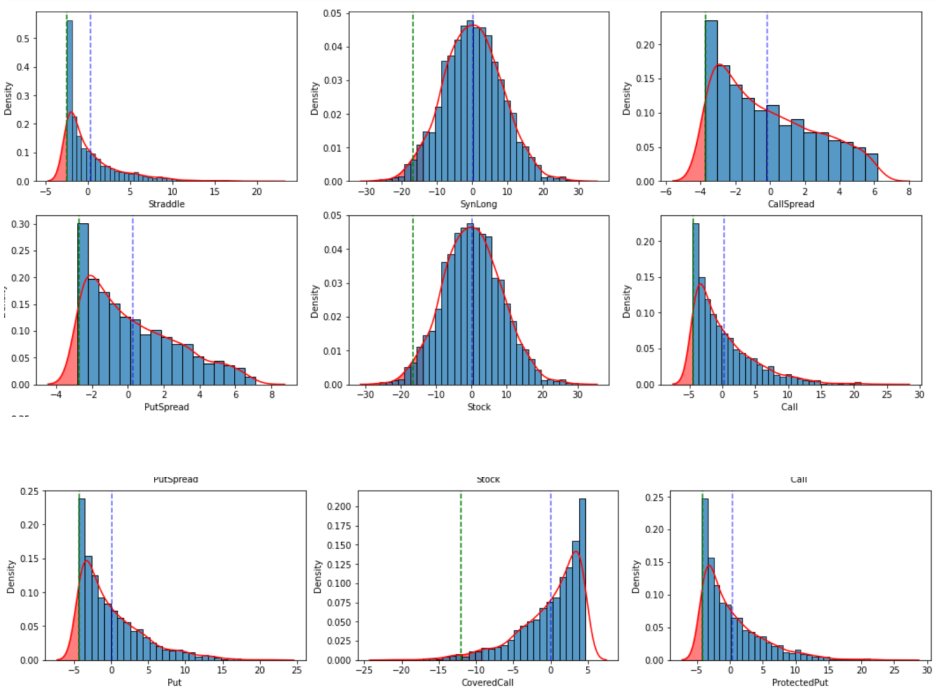

In this part, I plot and show the final results of profit and loss distribution and also the ES and VaR. The blue line represents the mean and the green line represents the ES, the red zone indicates VaR.

1.Call/Put Strategy: Highly similar distribution, positive average payoff with similar VaR and ES

2.Covered call/Protected put: Highly similar distribution,average return close to 0 and has the lowest risk

3.Straddle: Have high density to loss a small amount, can be used to limit loss in bad market condition, the average return is positive with relative low ES and VaR

4.SynLong. The most risky portfolio, extremely high VaR and ES, which is a repetition of the underlying asset payoff.