ME 132 — Homework # 6

Issued: October 15, 2021 — Due: October 24, 2021

1 *PI position control for a DC Motor*

In this problem we will investigate position control of a DC motor. Use the following simplified motor model. You do not have to understand details of how this model was derived.

$$\begin{aligned}\frac{d}{dt}\theta(t) &= \omega(t) \\ \frac{d}{dt}\omega(t) &= -0.97\omega(t) + 1.9u(t) - 1600 * T_L(t)\end{aligned}$$

where

$\theta(t)$	angular position of the motor
$\omega(t)$	angular velocity of the motor
$T_L(t)$	load torque
$u(t)$	voltage applied to motor (control signal)

and all initial conditions are zero.

(a) Design a PI controller for angular position control. Plot simulations with the load torque T_L set to zero, and the reference command

$$r = \begin{cases} 0 & \text{for } 0 \le t \le 2 \\ 0.5 & \text{for } t > 2 \end{cases}$$

Do this for 6 values of K_I, evenly spaced from 0.0 to 1. In these simulations, set the proportional gain $K_P = 1$. Also hand in you the `simulink` block diagram.

(b) Repeat part (a), but with

$$T_L(t) = \begin{cases} 0 & t < 6 \\ 0.0003 & t \ge 6 \end{cases}$$

Comment on the effect (good and bad) of the integral-control action.

(c) Approximately at what value of K_I does the system exhibit unstable behavior?

(d) What is the characteristic equation of the closed-loop system? This should be a 3rd order polynomial in s.

(e) Write a MATLAB program to plot (in the complex plane) the roots of the characteristic equation as K_I varies from 0.01 to 1. In other words, your program will generate the *root-locus* of the closed-loop system poles as K_I varies.
Hint: The command `roots` computes the roots of a polynomial. For example, the roots of

$$\chi(s) = 2s^4 - 7s^3 + 14.1s^2 + s - 9 = 0$$

can be found and plotted as red crosses using

```
>> foo = roots([2 -7 14.1 1 -9]);
>> plot(foo,'rx');
```

Note that `foo` is a 4×1 complex matrix containing the 4 roots of $\chi(s)$.

(f) Are your answers to parts (c) and (e) in agreement?

2 *Integral Control for the Cruise-control Problem*

Consider the cruise control model treated in class and in an earlier homework. Recall that the car model is

$$m\dot{v} = -\alpha v + Eu - Gw$$

Use the values $\alpha = 60, E = 40, m = 1000, G = 100$, where all are SI units, w is measured as a percent grade of the roadway. In this problem, you will simulate the closed-loop response using an integral controller

$$\begin{aligned} u &= K_p(v_{des} - v) + K_i z \\ \dot{z} &= v_{des} - v \end{aligned}$$

Use the initial condition $v(0) = 20$ meters/sec. Try various values of K_p and K_i and simulate the response for 80 seconds to the following command and disturbance:

$$v_{des}(t) = \begin{cases} 50 & \text{for } 0 \le t < 5 \\ 55 & \text{for } 5 \le t < 15 \\ 60 & \text{for } 15 \le t < 25 \\ 60 - \frac{4}{3}(t - 25) & \text{for } 25 \le t < 45 \\ 45 & \text{for } 45 \le t \end{cases} \quad \text{miles-per-hour}$$

$$w(t) = \begin{cases} 0 & \text{for } 0 \le t < 35 \\ 2.6 & \text{for } 35 \le t < 45 \quad \text{(small hill)} \\ 0 & \text{for } 45 \le t < 55 \\ 10 & \text{for } 55 \le t < 70 \quad \text{(large hill)} \\ 0 & \text{for } 70 \le t \end{cases} \quad \text{percent grade}$$

3 *PI control with Anti-windup*

Consider the angular position control problem for the DC motor with zero load torque. Use the model and initial conditions from Problem 1.

Suppose we design a PI controller with

$$u(t) = K_P(r - \theta(t)) + K_I \int_0^t (r - \theta(\tau))d\tau$$

where $K_P = 1$ and $K_I = 0.5$. Use the reference command r to be a unit step (in `radians`) at time $t = 2$ seconds.

(a) Plot using SIMULINK of the response $\theta(t)$ and the control signal $u(t)$ for $0 \le t \le 20$.

(b) Now suppose the actuator saturates at ± 0.15 volts. Repeat part (a) above.

(c) Design an integrator anti-windup scheme as described in class. Do you notice an improvement?