ME 132 — Homework # 3

Issued: September 17, 2021 Due: September 27, 2021

Suggested Readings:

1 Chapter 2, *Astrom & Murray*.

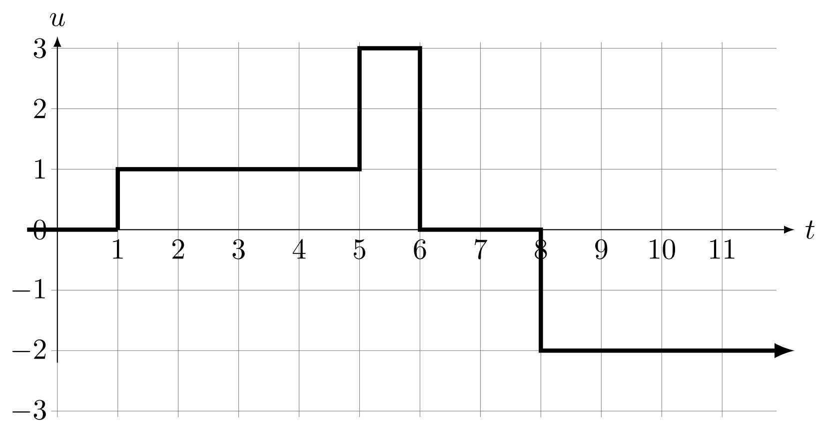
Problems to be turned in:

1 *First Order Systems: Time Domain*

Suppose a system is described by the differential equation

$$\dot{y} + y = u$$

where u is given in the graph below and $y(0) = 1$.



(a) Is this system stable?

(b) What is the time constant T for this system?

(c) What is the DC/steady-state gain from u to y?

(d) Sketch the response $y(t)$, $0 \leq t \leq 11$, for the input u shown above.

2 *First Order Systems: Frequency Domain*

Suppose a system is described by the differential equation

$$\dot{y} + y = -2u$$

where $y(0) = 1$.

(a) Find the transfer function $G(s)$ from u to y.

(b) Plot the frequency response of this system for $\omega = 10^{-2}$ to 10^{2} `rad/sec`.

Magnitude plot: $20 \log_{10} |G(j\omega)|$ versus $\log_{10} \omega$

Phase plot: $\angle G(j\omega)$ versus $\log_{10} \omega$.

(c) Suppose the input is $u(t) = 3\cos(2t)$ for $t \geq 0$. Find the steady-state response.

3 *Controller Design*

Consider a plant

$$\dot{y} - 3y = u$$

with $y(0) = -1$. In this problem, we want to design a proportional controller:

$$u = -k_1 y + k_2 r$$

The objective is for $y(t)$ to track the set-point r.

(a) Is the plant stable?

(b) Find the differential equation describing the closed-loop feedback system.

(c) For what values of k_1 is the closed-loop system stable?

(d) Find the DC gain of the closed-loop system from r to y in terms of k_1 and k_2. What should this DC gain be for the tracking objective?

(e) Choose k_1 and k_2 to satisfy the following requirements:

- $y(t) \to r$ in steady state.
- $|u(t)| \leq 6$ for all $t \geq 0$, when $r = 2$.
- The 95% settling time as small as possible. This must be smaller than 6 `sec`.

(f) Plot y versus t and u versus t to verify that your controller satisfies the requirements.

4 *DC Gain*

Consider the nonlinear input-output differential equation

$$\ddot{y} + y + \cos(\dot{y}) = u^3$$

We are told that this system is stable. Suppose we apply the input

$$u(t) = \begin{cases} 0 & 0 \leq t \leq 7 \\ 2 & 7 < t \end{cases}$$

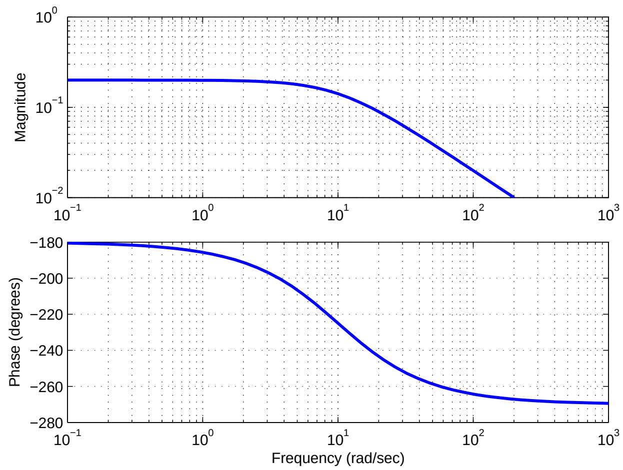
What is the steady state value of y?

5 *Modeling of First-order Systems*

We want to build a model for some physical system. We believe that the system can be modeled as first-order system

$$\dot{y} + ay = bu$$

We need to find the constants a and b. For this, we have measured the frequency response of the system. This is plotted below. For your convenience the magnitude plot is in absolute units, not in decibels.



Estimate the constants a and b. Explain your reasoning clearly in two or three sentences.

6 *Transfer Functions*

Consider the block diagram shown below. All the blocks are single-input single-output transfer functions. We can write the closed-loop system as

$$y = A(s)r + B(s)d$$

Find the transfer functions $A(s)$ and $B(s)$ in terms of P, K and G.