ME 132 — Homework # 7

Issued: October 23, 2021 — Due: October 31, 2021

1 *Model Approximation*

We have studied the behavior of the first-order differential equation

$$\dot{v}(t) = -\frac{1}{\tau}v(t) + \frac{1}{\tau}u(t) \tag{1}$$

which has a time-constant of τ, and a steady-state gain (to step inputs) of 1 (check both of these facts). Hence, if τ is "small," the output v of system follows u quite closely. For "slowly-varying" inputs u, the behavior is approximately $v(t) \approx u(t)$.

(a) Find the transfer function of the system (1).

(b) Find the transfer function of the following cascade connection.

$$\begin{aligned} y^{(2)} + 3y^{(1)} + 10y &= v^{(1)} - 5v \\ 0.01v^{(1)} + v &= u \end{aligned}$$

(c) How would you approximate the behavior of the system in part (b) above as a second order system? Plot using MATLAB the step responses of the original system and your approximation.

(d) How would you approximate the fourth order system with transfer function

$$H(s) = \frac{3s^2 + 11s}{(s+100)(s+25)(s^2+2s+4)} \quad ?$$

2 *Gain and Time-delay Margins*

We have a nominal plant model $P^o(s)$ for which a controller $C(s)$ has been designed. In each of the following cases, find the gain margin and the time-delay margin.

(a) $C(s) = \dfrac{2.4s+1}{s}, \quad P^o(s) = \dfrac{1}{s-1}$

(b) $C(s) = \dfrac{0.4s+1}{s}, \quad P^o(s) = \dfrac{1}{s+1}$

(c) $C(s) = \dfrac{10(s+3)}{s}, \quad P^o(s) = \dfrac{-0.5(s^2-2500)}{(s-3)(s^2+50s+1000)}$

3 *Time Delay Margin*

Consider the controller $C(s) = \frac{4}{s}$ that has bee designed for the nominal plant $P^o(s) = 2$ in a standard feedback system with negative feedback.

(a) Find the closed-loop transfer function from r to y. Is it stable?

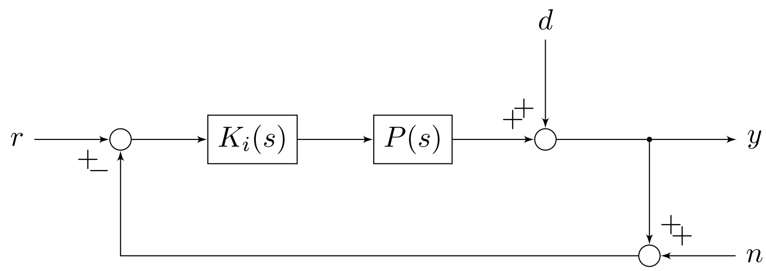
(b) Find the steady-state gain from $r \to y$.

(c) What is the time-constant of the closed-loop system.

(d) What is the time-delay margin? Denote it by T_d.

(e) Verify your answers with Simulink. Using the following time delays, plot the unit step response of the closed-loop system. $0, \frac{1}{10}T_d, \frac{3}{10}T_d, \frac{6}{10}T_d, \frac{9}{10}T_d, \frac{9.9}{10}T_d$. You should notice oscillations of increasing magnitudes.

4 *Loop Gain*



Three companies have designed a controller for an atomic force microscope.
The feedback system is shown above.
Here, r is the reference, d is the disturbance, y is the tip position, and n is sensor noise.

The reference is constant.
The sensor noise is predominant in the bandwidth 10^6 to 10^{10} radians/sec.
The disturbances are predominant in the bandwidth 10^2 to 10^4 radians/sec.

The frequency responses plots of the *loop-gain* $L_i(s) = P(s)K_i(s)$ for each of the three controllers $i = 1, 2, 3$ are shown below.

Which design would you choose? Explain your answer.