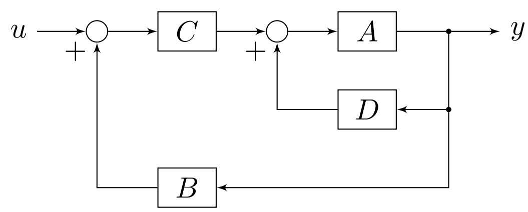
ME 132 — Homework # 2

Issued: September 10, 2021 — Due: September 19, 2021

1 *Sensitivity Analysis*

Consider the block diagram shown below. Compute the sensitivity S_a of the output y with respect to changes in the parameter A. Also find the sensitivity S_c of the output y with respect to changes in the parameter C.



2 *Model Properties*

Consider the following properties a model may have: **M**emoryless, **L**inear, **T**ime-invariant, **C**ausal.

For each of the following models $\mathcal{M}$, indicate which of the preceding properties the system possesses.

(a) $\mathcal{M}\{u(t)\} = e^{u(t)}$

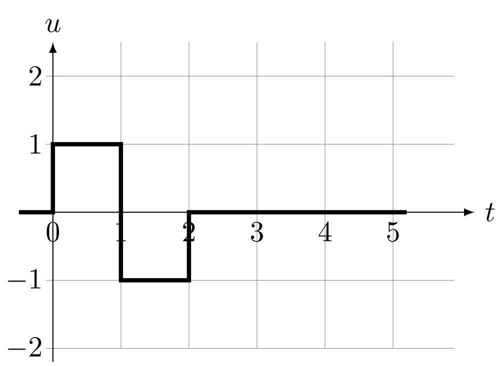
(b) $\mathcal{M}\{u(t)\} = au(t) + b$

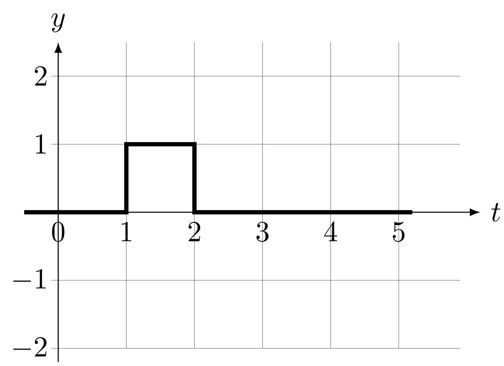
(c) $\mathcal{M}\{u(t)\} = u(-t)$

(d) $\mathcal{M}\{u(t)\} = \int_{t=-\infty}^{\infty} u(t)\, dt$

3 *Properties of Models*

The input u shown below is applied to some LTI causal system $\mathcal{M}$. The resulting output y is plotted below. Assume that the initial conditions are zero.





(a) Find and sketch the unit step response of $\mathcal{M}$.

(b) Does you answer require causality?

(c) Does your answer require linearity?

(d) Does your answer require time-invariance?

4 *Parameter Estimation*

In this problem, we will explore how to estimate parameters in a model from data. We consider again a ball thrown in the air that we treated in Homework 1. The ball has mass $m = 0.3\ Kg$. It is released at an angle of $\theta(0) = 65^o$ relative to horizontal, with initial speed of 35 m/s. Because of wind resistance, the ball experiences a force of $c * v^2$ where v is the speed of the ball. This force is in the direction opposite to the velocity vector. We derived the model for the motion of the ball:

$$\begin{aligned}
\ddot{x} &= -\frac{cv\dot{x}}{m} \\
\ddot{y} &= -\frac{cv\dot{y}}{m} - g \\
v &= \sqrt{\dot{x}^2 + \dot{y}^2}
\end{aligned}$$

We have a noisy sensor that records the trajectory of the ball for 7 seconds. This data is stored as a matrix with three columns (t, x^d, y^d) in the excel file *TrajectoryData.xls*. We will use the measured data to estimate the unknown parameter c in the model.

(a) Plot the observed trajectory $x(t)$ versus $y(t)$.

(b) Fix c. Suppose we simulated the trajectory of the ball for 7 seconds. Let $(x^c(t), y^c(t))$ be the simulated trajectory. How would you calculate how close this simulated trajectory is to the measured trajectory? In other words, find some function

$$J(x^c, y^c, x^d, y^d)$$

that is small if the trajectories are close to each other.

(c) Write a MATLAB program that simulates the model for various values of c in the interval $c \in [0\ 0.1]$. Make sure your simulation calculates the trajectory at the same time values specified in the measured data set. For each value of c, calculate $J(c)$. Plot $J(c)$ versus c.

(d) What is your best estimate of the drag coefficient c?

5 *Complex Arithmetic*

(a) Let a be real. Show that

$$\left| \frac{j\omega - a}{j\omega + a} \right| = 1 \quad \text{for all } \omega \in \mathbb{R}$$

(b) Plot the log-magnitude and phase of the complex number

$$H(j\omega) = \frac{a}{j\omega + b}$$

versus $\log \omega$ as ω varies from 10^-1 to 10^2 in the following cases.

- $\diamond$ $a = 1, b = 3$
- $\diamond$ $a = 1, b = -3$
- $\diamond$ $a = -1, b = 3$
- $\diamond$ $a = -1, b = -3$

(c) Plot the log-magnitude and phase of the complex number

$$H(j\omega) = \exp(-j\omega T)$$

versus $\log \omega$ as ω varies from 10^-2 to 10^0 rad/sec in the following cases.

- $\diamond$ $T = 1$
- $\diamond$ $T = 10$

(d) What is the equation of a circle in the complex plane of radius R and centered at z_o?

6 *Simulation*

Consider the two input-output differential equation models:

$$\begin{aligned} \dot{y} + y &= u \\ \ddot{y} + 11\dot{y} + 10y &= 10u \end{aligned}$$

Simulate the responses for both models to the following inputs defined for $0 \le t < 10$. Set all initial conditions to zero.

(a) $u(t) = 1$

(b) $u(t) = \sin(t)$

(c) $u(t) = \cos(5 * t)$

(d) $u(t) = 25\cos(10 * t)$

(e) $u(t) = 25\cos(20 * t)$

(f) $u(t) = \begin{cases} t & \text{for } 0 \le t < 3 \\ 3 & \text{for } 3 \le t < 5 \\ -3 & \text{for } 5 \le t < 7 \\ 2t - 17 & \text{for } 7 \le t < 10 \end{cases}$

In each case, plot both the model outputs together. You will notice that the outputs look very similar. Thus, the two models are "close" even though that may not be directly apparent from the differential equations. Based on your answers above, and by trying other simulations, for what class of inputs do the two models produce similar outputs?

7 *Cruise Control for a Car*

In this problem, I want you to simulate the behavior of the controller using a more complex and realistic model for the plant. This will reveal to you the limitations on control design imposed by modeling errors.

A more detailed model of the car (as developed in class) is

$$m\dot{y} = -\alpha y + Eu - Fd \tag{1}$$

Use the values $\alpha = 60, E = 720, m = 1000, F = 180$, where all units are MKS. The input $u(t)$ is an electrical signal limited to 5 `volts` that drives the throttle fuel valve actuator, and $d(t)$ is the disturbance due to hills in the roadway measured in percent grade.

(a) What is the maximum speed the car can attain on a flat surface?

(b) If you were to simplify the car model to the form (as in class)

$$y(t) = Gu(t) - Hd(t)$$

what would the most appropriate choices G^o and H^o for the constants G and H be?

(c) Now lets use a feedback controller of the form

$$u(t) = K_1 r(t) - K_2 y(t)$$

where $r(t)$ is the command. We have to design the constants K_1 and K_2. As done in lecture, we already have one design equation

$$\frac{G^o K_1}{1 + G^o K_2} = 1$$

Explain where this equation comes from.

(d) Now simulate the closed loop behavior of the cruise control system with the detailed model (Equation 1) for 70 seconds with the disturbance and command as:

$$r(t) = \begin{cases} 50 & \text{for } \; 0 \le t < 5 \\ 55 & \text{for } \; 5 \le t < 15 \\ 60 & \text{for } \; 15 \le t < 25 \\ 60 - \frac{4}{3}(t-25) & \text{for } \; 25 \le t < 45 \\ 45 & \text{for } \; 45 \le t \end{cases} \qquad \textit{miles-per-hour}$$

$$d(t) = \begin{cases} 0 & \text{for } \; 0 \le t < 35 \\ 1.3 & \text{for } \; 35 \le t < 45 \quad \text{(small hill)} \\ 0 & \text{for } \; 45 \le t < 55 \\ 5.0 & \text{for } \; 55 \le t < 70 \quad \text{(large hill)} \\ 0 & \text{for } \; 70 \le t \end{cases} \qquad \textit{percent grade}$$

Note that the units for the command above are in `miles/hour`. Use the initial condition $y(0) = 20$ `meters/sec`. Do this for the following designs

$\diamond$ $K_2 = 0.05$ $\quad$ $\diamond$ $K_2 = 0.1$ $\quad$ $\diamond$ $K_2 = 1$

and plot the car speed in all three cases together on the same graph.

(e) What are the advantages of choosing larger values of K_2?

(f) The car will not actually behave as simulated with $K_2 = 1$. Why?
Hint: look at the acceleration plots.