COMPUTER VISION AND IMAGE UNDERSTANDING
Vol. 64, No. 2, September, pp. 300–302, 1996
ARTICLE NO. 0060

NOTE

Fast Noise Variance Estimation

JOHN IMMERKÆR

Department of Information Technology, Odense University, Lindø Center for Applied Mathematics, Forskerparken 10, DK-5230 Odense M, Denmark

Received August 22, 1994; accepted May 9, 1995

The paper presents a fast and simple method for estimating the variance of additive zero mean Gaussian noise in an image. The method can also be used to give a local estimate of the noise variance in the situation in which the noise variance varies across the image. It requires only the use of a 3 × 3 mask followed by a summation over the image or a local neighborhood. A total of 14 integer operations per pixel is necessary. The method performs well for a large range of noise variance values. In highly textured images or regions, though, the noise estimator perceives thin lines as noise.

1. INTRODUCTION

Estimation of the amount of noise is important in many algorithms for image processing and analysis. It allows algorithms to adapt to the amount of noise instead of using fixed thresholds. A model for noisy images is to assume additive, zero mean noise, given by

$$I(x, y) = f(x, y) + n(x, y),$$

where f is the ideal image, n is the noise, and I is the observed image. The image has width W and height H, and each pixel has an integer value $0, \dots, 255$. The goal is to estimate the standard deviation σ_n of the noise n. An essential problem in noise estimation is to measure deviations of I from f that may contain structure like edges and texture.

For a description and comparison of earlier methods, the reader should refer to Olsen [1], who has evaluated six methods for estimating the amount of noise in images.

This paper presents a fast algorithm for estimating the noise variance in images. The proposed method uses a zero mean operator, which is almost insensitive to image structure. The variance of the output from the operator is an estimate of the noise variance.

2. NOISE VARIANCE ESTIMATION

Since image structures like edges have strong second order differential components, a noise estimator should be insensitive to the Laplacian of an image. This suggests using the difference between two masks L_1 and L_2, each approximating the Laplacian of an image. The elements of L_1 and L_2 are

$$L_1 = \begin{array}{|c|c|c|}\hline 0 & 1 & 0 \\ \hline 1 & -4 & 1 \\ \hline 0 & 1 & 0 \\ \hline\end{array} \qquad L_2 = \frac{1}{2}\begin{array}{|c|c|c|}\hline 1 & 0 & 1 \\ \hline 0 & -4 & 0 \\ \hline 1 & 0 & 1 \\ \hline\end{array}$$

The noise estimation operator N is the mask operation using the mask

$$N = 2(L_2 - L_1) = \begin{array}{|c|c|c|}\hline 1 & -2 & 1 \\ \hline -2 & 4 & -2 \\ \hline 1 & -2 & 1 \\ \hline\end{array} \tag{1}$$

which has zero mean and variance $(4^2 + 4 \cdot (-2)^2 + 4 \cdot 1^2)\sigma_n^2 = 36\sigma_n^2$ assuming that the noise at each pixel has standard deviation σ_n. Let $I(x, y) * N$ denotes the value of applying the mask N at position (x, y) in the image I.

Computing the variance of the output of the N operator applied to the image I, will give an estimate of $36\sigma_n^2$ at each pixel, which can be averaged over the image I or local neighborhoods to give an estimate of the noise variance σ_n^2. The variance of the noise in I can then be computed as

$$\sigma_n^2 = \frac{1}{36(W-2)(H-2)} \sum_{\text{image } I} (I(x, y) * N)^2 \tag{2}$$

but it has the disadvantage that it uses one multiplication per pixel.

1077-3142/96 $18.00

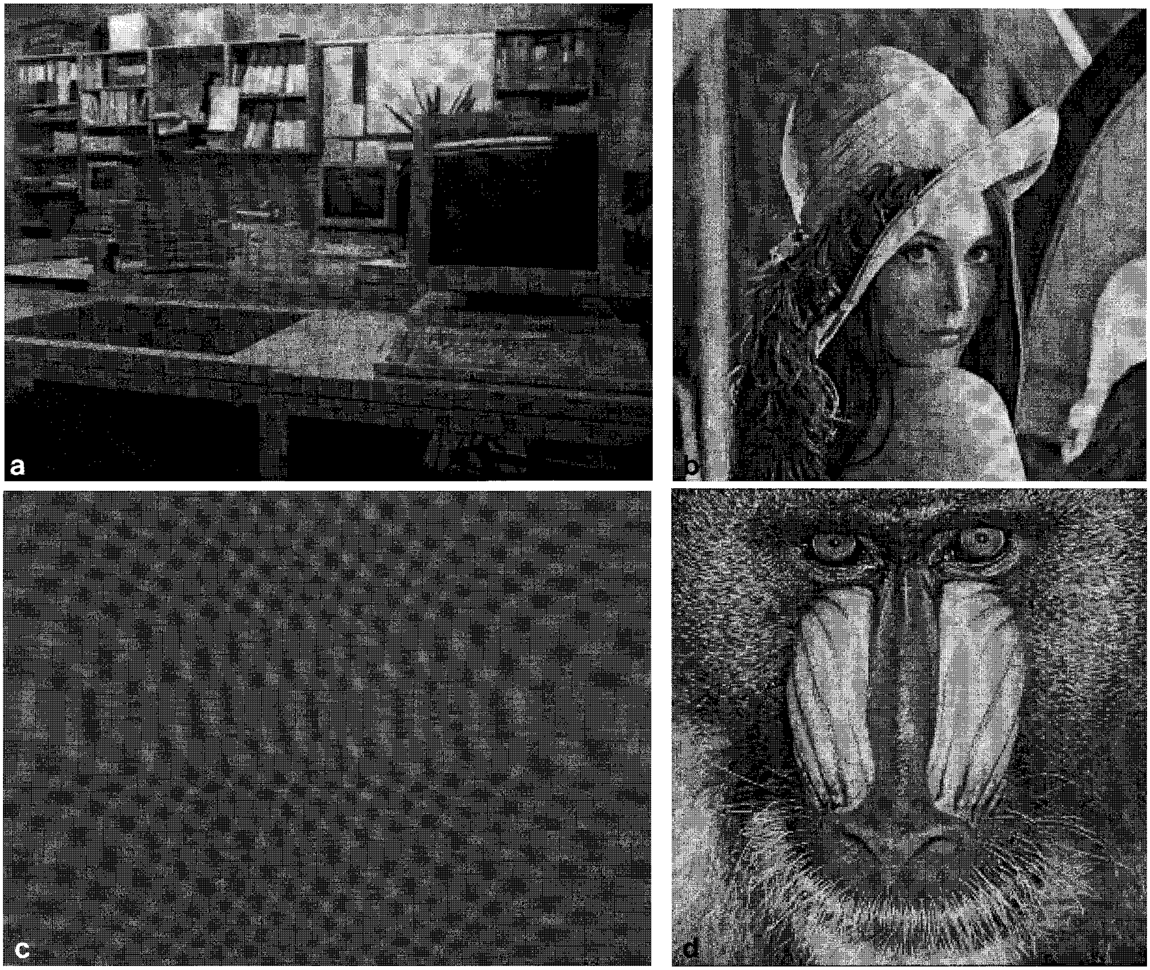

FIG. 1. Images used in the experiments. (a) Laboratory, (b) Lena, (c) Gray, (d) Mandrill.

The variance can also be computed using the absolute deviation. Assuming a Gaussian distribution with zero mean and variance σ^2, then the absolute deviation is

$$\int_{-\infty}^{\infty} |t| \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-t^2}{2\sigma^2}\right) dt = \sqrt{\frac{2}{\pi}}\,\sigma$$

giving

$$\sigma = \sqrt{\frac{\pi}{2}} \int_{-\infty}^{\infty} |t| \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(\frac{-t^2}{2\sigma^2}\right) dt.$$

It follows that σ_n can be computed as

$$\sigma_n = \sqrt{\frac{\pi}{2}} \frac{1}{6(W-2)(H-2)} \sum_{\text{image } I} |I(x,y) * N|, \qquad (3)$$

where the summation can be performed without multiplication.

3. EXPERIMENTS

The algorithm above has been implemented in the language C on a SUN Sparc workstation. The images used to test the method are shown in Fig. 1. Gaussian distributed noise with zero mean and one of the standard deviation values (σ_a): 0 (no noise), 1, 2, 5, 10, 20, 50 were added to the images and rounded to nearest integer. For each value of σ_a, ten noisy images were generated.

Tables 1 and 2 show the results of using Eq. (2) and (3) to estimate the amount of noise in the laboratory image. Both tables contain, for each value of σ_a, the mean (mean(σ_e) and mean(σ_e^2)) and the standard deviation (std(σ_e) and std(σ_e^2)) of the estimated noise variance (σ_e and σ_e^2). A comparison of the two tables shows that the mean values differ at most by 0.06 ($\sigma_a = 0$), so there is no significant difference between using Eq. (2) or (3) to estimate the amount of noise in an image. The standard deviations are small compared to the mean values.

TABLE 1

Estimation of Noise Variance in the Laboratory Image Using Eq. (2): Mean (mean(σ_e) and mean(σ_e^2)) and Standard Deviation (std(σ_e) and std(σ_e^2)) of the Estimated Noise as a Function of the Added Noise (σ_a)

σ_a	mean(σ_e)	std(σ_e)	mean(σ_e^2)	std(σ_e^2)
0	1.0677	—	1.1399	—
1	1.4921	0.0023	2.2263	0.0070
2	2.2866	0.0057	5.2285	0.0262
5	5.1239	0.0130	26.2549	0.1332
10	10.0646	0.0284	101.2979	0.5722
20	20.0380	0.0551	401.5260	2.2062
50	50.0293	0.1377	2502.9469	13.7728

Assuming that the noise distributions are independent, the estimate to correct an error in the manuscript of the noise variance is the sum of the variance of the noise in the original image and of the added noise, i.e., $\sigma_e^2 \approx \sigma_n^2 + \sigma_a^2$. Table 3 shows, for each value of σ_a, the mean of the estimate of $\sigma_n^2 \approx \sigma_e^2 - \sigma_a^2$, which should ideally be constant within each column. The method performs well for σ_a up to 20 and begins to degrade at a σ_a value of 50. In the Mandrill image the fine lines of the hair are perceived as noise, and give a too high estimate of σ_e. In regions without hair the noise variance σ_n^2 is around 15, which is small compared to the values 83, . . . , 110 for the Mandrill image in Table 3. The true value of $\sigma_n = 3.8$ would be estimated to about 9 or 10—a too pessimistic value.

TABLE 2

Estimation of Noise Variance in the Laboratory Image Using Eq. (3): Mean (mean(σ_e) and mean(σ_e^2)) and Standard Deviation (std(σ_e) and std(σ_e^2)) of the Estimated Noise as a Function of the Added Noise (σ_a)

σ_a	mean(σ_e)	std(σ_e)	mean(σ_e^2)	std(σ_e^2)
0	1.0397	—	1.0809	—
1	1.4820	0.0028	2.1964	0.0083
2	2.2840	0.0058	5.2167	0.0267
5	5.1245	0.0139	26.2609	0.1423
10	10.0654	0.0294	101.3134	0.5925
20	20.0394	0.0572	401.5804	2.2924
50	50.0310	0.1419	2503.1254	14.1981

TABLE 3

Estimate of $\sigma_n^2 \approx$ mean(σ_e^2) $-$ σ_a^2 for the Images in Fig. 1

σ_a	Laboratory	Lena	Gray	Mandrill
0	1.0809	7.2610	0.0000	83.9218
1	1.1964	7.5149	0.0797	84.4523
2	1.2167	7.8147	0.0795	85.5642
5	1.2609	8.4910	0.1040	90.5673
10	1.3134	8.9027	0.1346	97.9548
20	1.5804	9.0178	0.3639	105.4341
50	3.1254	8.4199	1.8030	110.3674

4. CONCLUSION

This paper presents a fast and simple method for estimating the variance of additive zero mean Gaussian noise in an image. The method can also be used to give a local estimate of the noise variance $\sigma_n^2(x, y)$, i.e., the noise variance may vary across the image.

It requires only the use of a 3×3 mask followed by a summation over the image or a local neighborhood. The 3×3 mask is separable, and can be computed using six read-pixel and two write-pixel operations. A summation in a rectangular image or neighborhood requires four read-pixel and two write-pixel operations. A final floating point multiplication by a constant may be required to get the noise variance σ_n^2. A total of 14 integer operations per pixel is necessary.

The method performs well for a large range of noise variance values. In textured images or regions, though, the noise estimator perceives thin lines as noise.

ACKNOWLEDGMENTS

The author wishes to acknowledge the financial support of the Danish Research Council's special program in Information Technology (PIFT) and Odense Steel Shipyard.

REFERENCES

1. S. I. Olsen, Estimation of noise in images: An evaluation. *Comput. Vision Graphics Image Process. Graphic Models Image Process.* **55**(4), 1993, 319–323.