

Let $P = \text{Property}$, $L = \text{LoI}$

Let $N = \text{Freq}$, $X = \text{Severity}$

Let $S = \sum X$

Hence total loss $Z = E[N_P] E[X_P] + E[N_L] E[X_L]$
+ Unknown Interaction

Rearranging,

$$(Z - E[S_P] - E[S_L]) = \text{Unknown Interaction Term}$$

Let $Y = Z - E[S_P] - E[S_L]$,

$\underbrace{\quad}_{\text{actual total loss}} \quad \underbrace{\quad}_{\text{GLM/Tweedie Prediction}}$
LoI/Property

Then $Y = \text{Interaction affected by correlated freq \& correlated severities}$
 $\ln(Y) = \text{sum of correlated effects}$

$$\ln(Y) = 1 + \beta_1 (N_P \cdot N_L) + \beta_2 (X_P \cdot X_L) + \beta_3 (X_P \cdot X_L \cdot N_P \cdot N_L)$$

• RHS = 1 + β_1 (freq interaction) + β_2 (severity int.) + β_3 (Total int.)

• Achieves $\ln(Y) = 1$ when any of N or L is zero

Final Model

$$Z = e^{\mu_{N_P}} e^{\mu_{X_P}} + e^{\mu_{N_L}} e^{\mu_{X_L}} + e^{1 + \beta_1 \text{freq} + \beta_2 \text{sev} + \beta_3 \text{All}}$$

Pure premium:

↳ To split interaction, just multiply by ratio $P:L$