Modelling Piezoelectric Devices as both Transmitters and Receivers

PHY312: Project Report

Bhavik Kumar MS18098
Kunal Verma MS18148

Contents

1 Introduction

Piezoelectricity is a physical phenomenon where an applied mechanical stress on certain solid materials leads to charge accumulation on their surface. The converse is also true, which means that an applied electrical voltage which leads to charge accumulation on the surface of such a material can generate a mechanical stress. Piezoelectric materials are available in the form of crystals, ceramics and polymers.

Piezoelectric materials couple electrical and mechanical effects and also thermal effects. They are thus inherently complicated, with a large number of parameters. Hence, the problem of Piezoelectricity is inherently a multi-physics problem which can be handled using the software COMSOL.

COMSOL is a multiphysics software which allows to combine problems related to multiple domains of physics to be solved in a single simulation. It is based on Finite Element Analysis (FEA). It has relatively fast calculations exploiting the symmetry of problems, and has a comprehensive Materials and Model Library.

The different interfaces of physics involved in the problem of piezoelectricity are-

- Pressure Acoustics
- Transient phenomenon
- Solid Mechanics
- Electrostatics

Due to the direct and converse piezoelectric effect, these devices are widely used to generate generate sound waves and detect acoustic signals as well in applications like Ultrasound. Since the same device can generate electricity due to pressure as well as generate pressure due to electricity, the same transducer can be used as the transmitter to generate signals and then used as the receiver to receive the echoes.

2 Theoretical Background

Let's study the case of 1D materials for simplicity. For linear dielectric materials with no piezoelectric properties, the electric displacement D is given by-

$$D = \epsilon E = \epsilon_0 E + P$$

where ϵ is the permittivity. But in a 1D model of piezoelectric materials, the polarization P caused by a strain S is given as

$$P = eS$$

where e is the Piezoelectric Stress Constant. This gives the form of the electric displacement to be-

$$\implies \boxed{D = \epsilon_S E + P = \epsilon_s E + eS} \tag{1}$$

where ϵ_S is the permittivity at constant strain, the Hooke's law relates stress T and strain S as $T = cS$, c being the stiffness. Since the stress for piezoelectric materials is given as $T = -eE$,

$$\implies \boxed{T = c_E S - eE} \tag{2}$$

where c_E is the stiffness at constant E. Together, equation (1) and (2) relate the electrical and mechanical properties of piezoelectric materials.

For actual 3D materials, the constitutive equations turn into tensor equations since T and S are second rank tensors.

3 Modelling on COMSOL

The way we'll model a piezoelectric device is by connecting a transducer to an external circuit using the *Terminal* feature.The *Terminal* feature can then be used with *Electrical Circuits* interface for providing and measuring the signals.

3.1 Model Setup

To begin with, we will set up a model of the piezoelectric device and the associated water domain to be azimuthally symmetric, with the cross section Figure given below.

The transducer being used is a Lead Zirconate Titanate disk (PZT-5H) with 2mm radius and 1mm thickness. It sends off a pressure signal into the water domain which extends out to infinity due to the PML (Perfectly Matched Layer) in the radial direction.

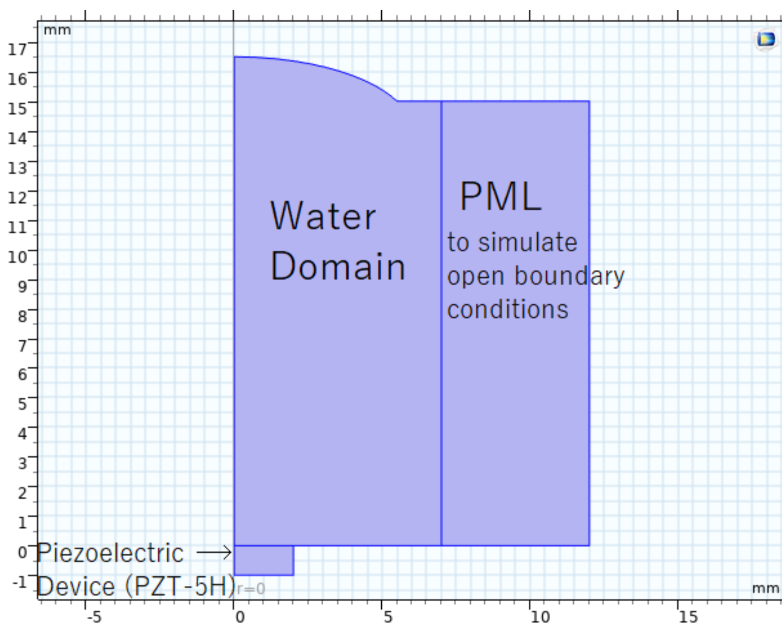
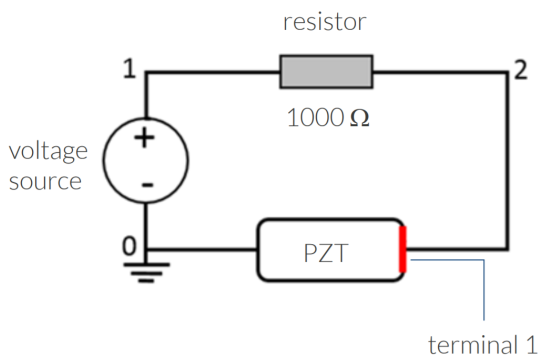


Figure 1: A 2D axisymmetric plot of the setup

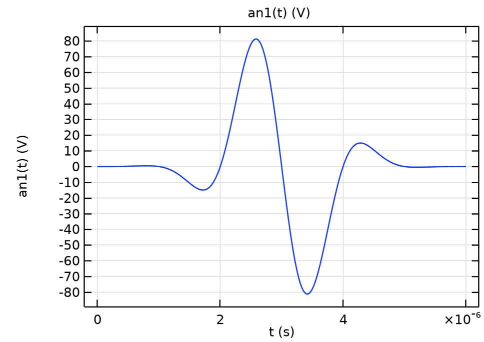
The required setup to send and receive acoustic waves using the Piezoelectric devices can be done using *Acoustic-Piezoelectric Interaction, Transient* interface, which combines *Pressure Acoustics, Transient and Piezoelectric Devices.*

When using the *Terminal*, an Electrical Circuit interface is added to the model to create an electric signal in the piezoelectric device and also receive them. The Electrical Circuit connected to the *Terminal* and a voltage $V(t)$ is sent as a signal. An illustration with a Gaussian-modulated input voltage is depicted in the Figure 2.

To connect the Transducer, the *Terminal* type is set to Circuit. The top surface of the



(a) Electrical Circuit for Transducer



(b) Gaussian-modulated sine wave

Figure 2

piezo device is connected to the one of the electrode and the other one is connected to Ground. Once the transducer is connected to the electrical circuit, we can plot the time-dependent plot of the Acoustic Pressure at the centerpoint of the top surface of the transducer.

3.2 Simulations

The simulations are done for a setup with the piezoelectric material *Lead Zirconate Titanate (PZT-5H)* with a *water domain* inside the chamber. We perform the simulations for the above materials with two different input voltage signals, and we will try to understand the effect of these voltage signals on the overall system. Some of the system parameters that we'll use later are summarised in Table 1.

Name	Expression	Value	Description
f_0	0.5 MHz	$5 \cdot 10^5$	Signal center frequency
T_0	$1/f_0$	$2 \cdot 10^{-6}$	Signal period
v_{pz}	1741 m/s	1741 m/s	Speed of shear waves in the piezoelectric material
v_{water}	1481 m/s	1481 m/s	Speed of pressue waves in water
λ_0	v_{water}/f_0	0.00296 m	Wavelength at f_0

Table 1: Relevant parameters of the simulation

The simulation uses the following mutiphysics packages:

- Pressure Acoustics and Transients

$$\frac{1}{\rho c^2}\frac{\partial^2 p}{\partial t^2} + \nabla \cdot \left(-\frac{1}{\rho}(\nabla p - \vec{q}_d)\right) = Q_m.$$

where we solve the wave equation for the acoustic pressure $p = p(\vec{x}, t)$. Here c is the speed of sound and and ρ denotes the equilibrium density, while $\vec{q}_d$ and Q_m are dipole and monopole sources, respectively.

- Solid Mechanics
$$\nabla \cdot \sigma + \vec{F} = \rho \frac{\partial^2 \vec{u}}{\partial t^2}.$$
where σ is the stress tensor, $\vec{F}$ is the force per unit volume, ρ is the mass density, and $\vec{u}(\vec{x}, t)$ is the displacement vector.

- Electrostatics
$$\nabla \cdot \vec{D} = \rho_f \qquad \vec{E} = -\nabla V.$$
which is the standard Gauss' Law equations for electrostatics where $D = \epsilon_0 \vec{E} + \vec{P}$ is the electric displacement.

- Electrical Circuits which uses the standard Ohm's Law for calculations.

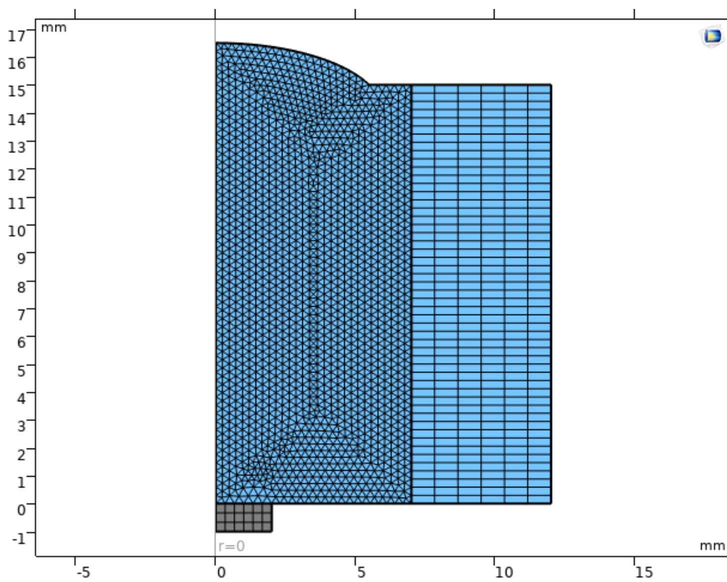


Figure 3: Mesh structure of the system

Using the above multiphysics packages, we will perform the simulations to use the Piezoelectric material as both a transmitter and a receiver starting with generating an initial voltage signal in the piezoelectric material, which will lead to generation of stress and radiates a sound pulse to the water domain above. When the sound waves reach the rigid walls on the top, they are reflected back and picked up by the transducer. This process repeats many times till the acoustic energy is damped out and attenuates to very small signals. We'll use two different input voltage signals:

1. $V_1(t) = 100\, e^{-(t-1.5T_0)^2/(T_0/2)^2} \sin(2\pi f_0 t)$
2. $V_2(t) = 100\, e^{-(t-1.5T_0)^2/(T_0/2)^2}$

4 Results of multiphysics simulations

Let us now have a look at the simulation results for the input voltage signals $V_1(t)$, the Gaussian modulated sine wave, and $V_2(t)$, the Gaussian pulse.

4.1 Results for $V_1(t)$

Let's start with the input voltage signal $\boxed{V_1(t) = 100\, e^{-(t-1.5T_0)^2/(T_0/2)^2} \sin(2\pi f_0 t)}$, the Gaussian modulated sine wave. All the relevant parameters such as T_0 and f_0 are stated in the Table 1.

Our main focus is as follows. Since the input voltage induces a stress in the piezoelectric material PZT-5H, sound waves are generated as a result of this induced stress which travel inside the water domain. Our aim is to study the evolution of the pressure at the centre point of PZT-5H, and the evolution of terminal voltage at the Terminal. This can be achieved by using point probes and global variable probes in COMSOL. Other properties to investigate are the Stress inside the piezoelectric material, the electric potential of the piezoelectric material, and the acoustic pressure inside the water domain throughout the process.

Since these phenomenon occur at the time scale of a few microseconds in actual materials, we will run the simulations for $T = 6.5 \cdot 10^{-5}$ seconds $= 65\ \mu$ seconds .

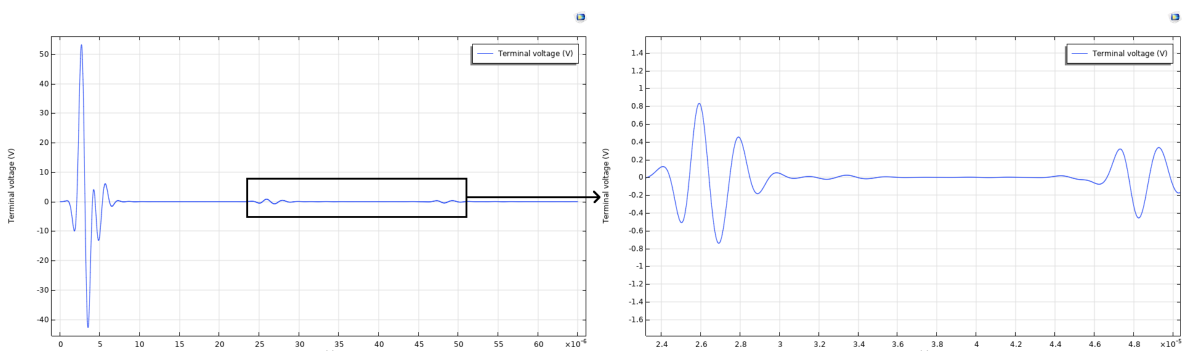


Figure 4: Voltage signals at the Terminal

The graph in Fig. 4 shows the time evolution of the Terminal Voltage in the system. The first pulse is due to the input electric potential on the piezoelectric material, while the other two slightly damped out Terminal Voltage signals are a result of acoustic pressure waves hitting back the piezoelectric material again after reflection which causes stress and hence an induced electric potential to be generated at the Terminal. The second graph on the right shows the received damped out voltage signals when zoomed in and we do indeed see that they carry a similar waveform.

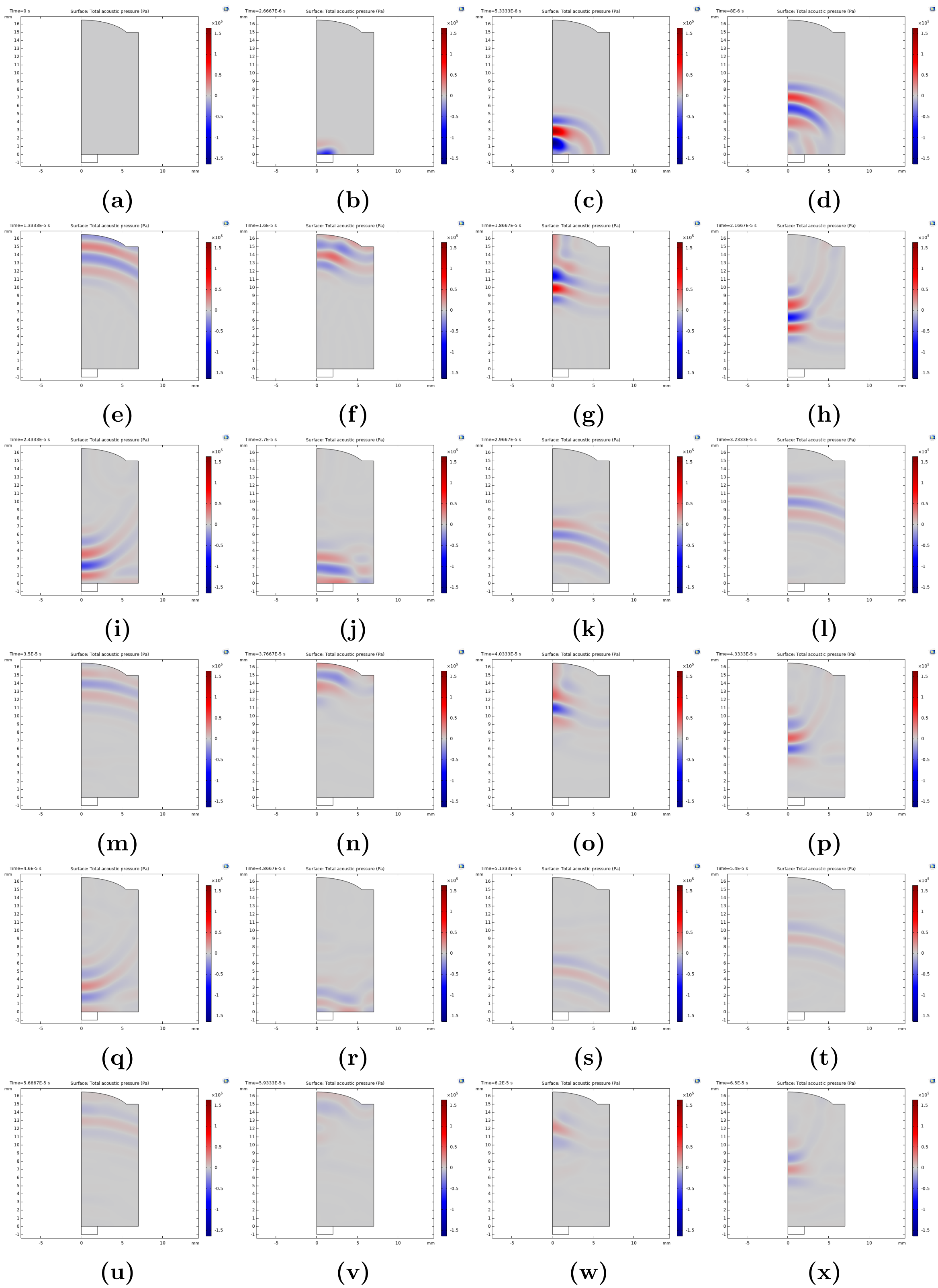


Figure 5: The time evolution of Acoustic Pressure waves inside the water domain of the setup (time moves forward from Fig. 5a to Fig. 5x). The GIF is attached here.

Figure 5 on the last page shows the variation of the time dependent acoustic pressure in the bulk of the water domain of the setup. It is clear from the heatmap of the acoustic waves that the pressure begins to damp out after reflection from the top of the water domain and progressively keeps damping. Figure 6 on the other hand shows the total acoustic pressure at the *center point of the top surface of the PZT-5H transducer*. The waves appearing at 25 μs and 50 μs are the first and the second echo of the original pressure signal.

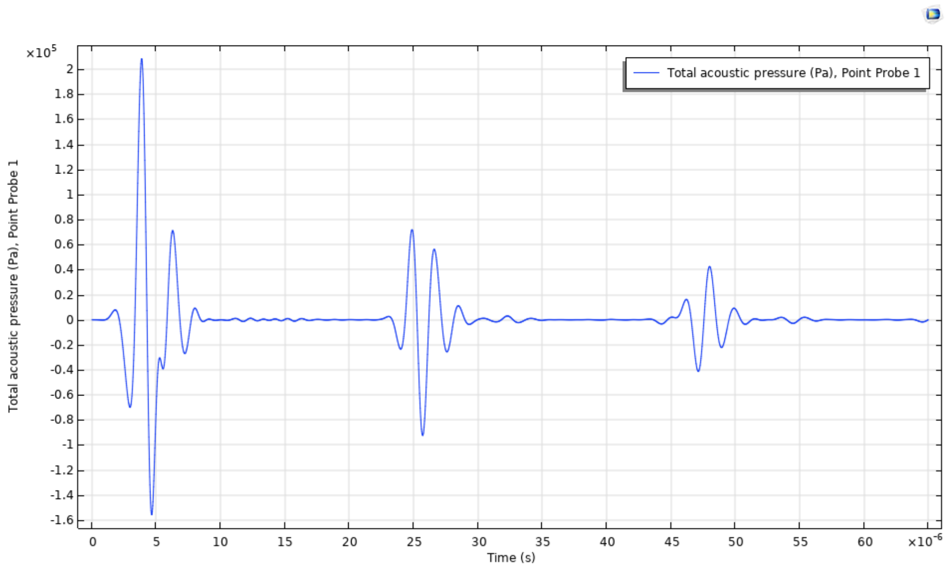
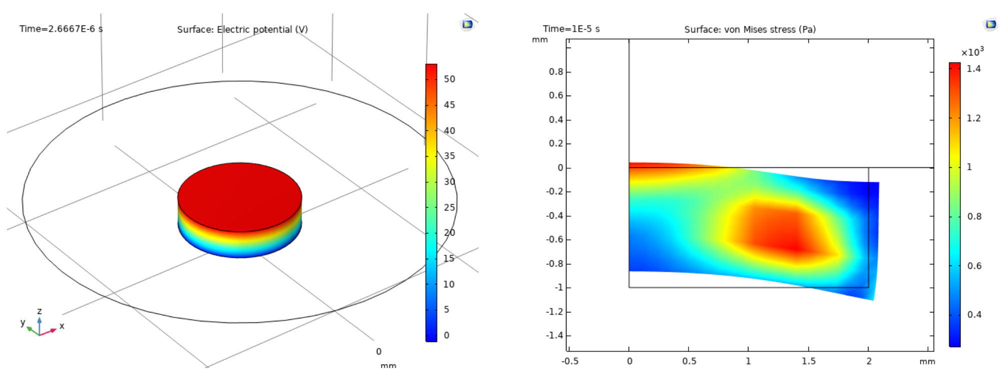


Figure 6: Plot of the acoustic pressure at the center point of the top surface of the transducer for the input signal $V_1(t)$.

We also show the preview of the electric potential at $T = 2.6\,\mu$s (Fig. 7a) and the von Mises stress inside the piezoelectric material at $T = 10\,\mu$s (Fig. 7b) in the Figure 7 below. The animated GIFs are attached in the captions of the figures.



(a) Electric Potential. (GIF here) **(b)** von Mises Stress. (GIF here)

Figure 7

4.2 Results for $V_2(t)$

Let us now analyze the setup with the input voltage signal $\boxed{V_2(t) = 100\, e^{-(t-1.5T_0)^2/(T_0/2)^2}}$, the Gaussian pulse.

We'll start with the time dependence graph of Terminal Voltage in the system, shown in Fig. 8 below. We can see the plot of the transmitted and received voltage signals at the terminals, and a zoomed in view of the received signals.

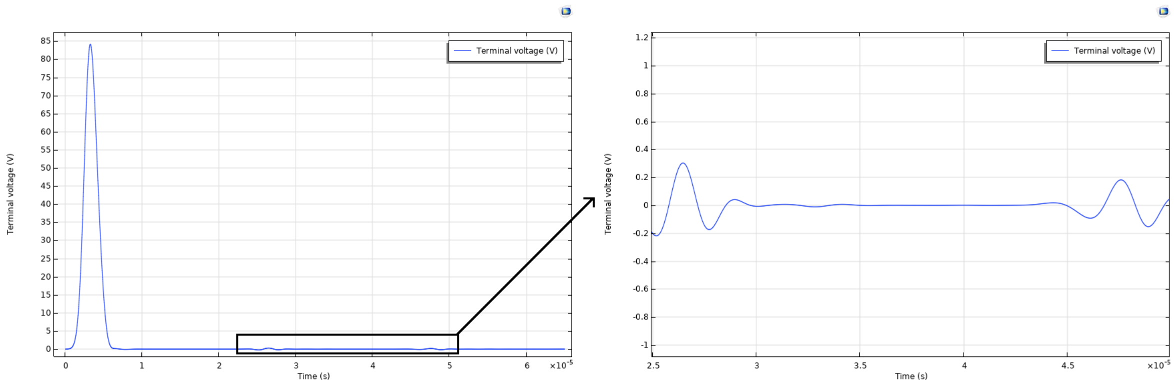


Figure 8: Voltage signals at the Terminal. The graph at the right shows the zoomed in received signals.

Similarly, we can also plot the time dependent total acoustic pressure receib ved at the center point of the top surface of PZT-5H.

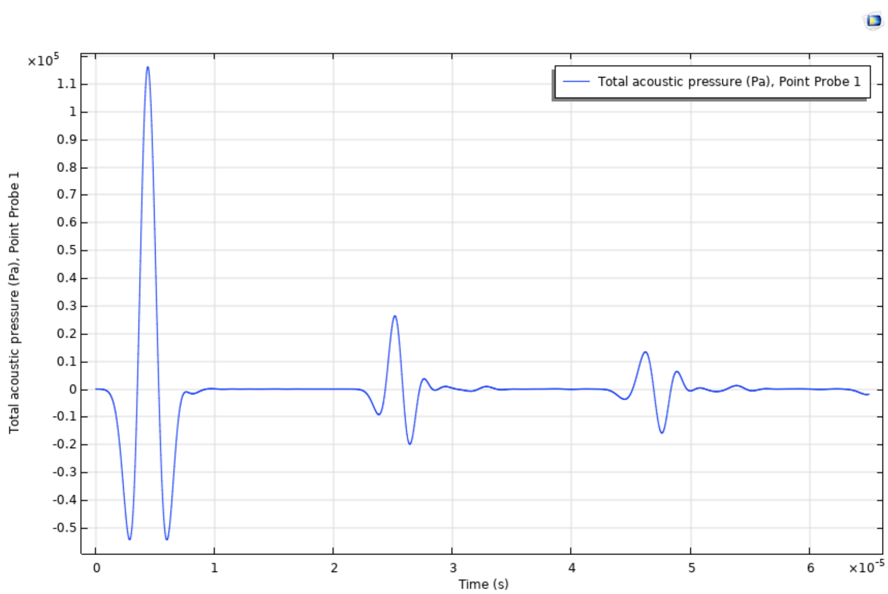
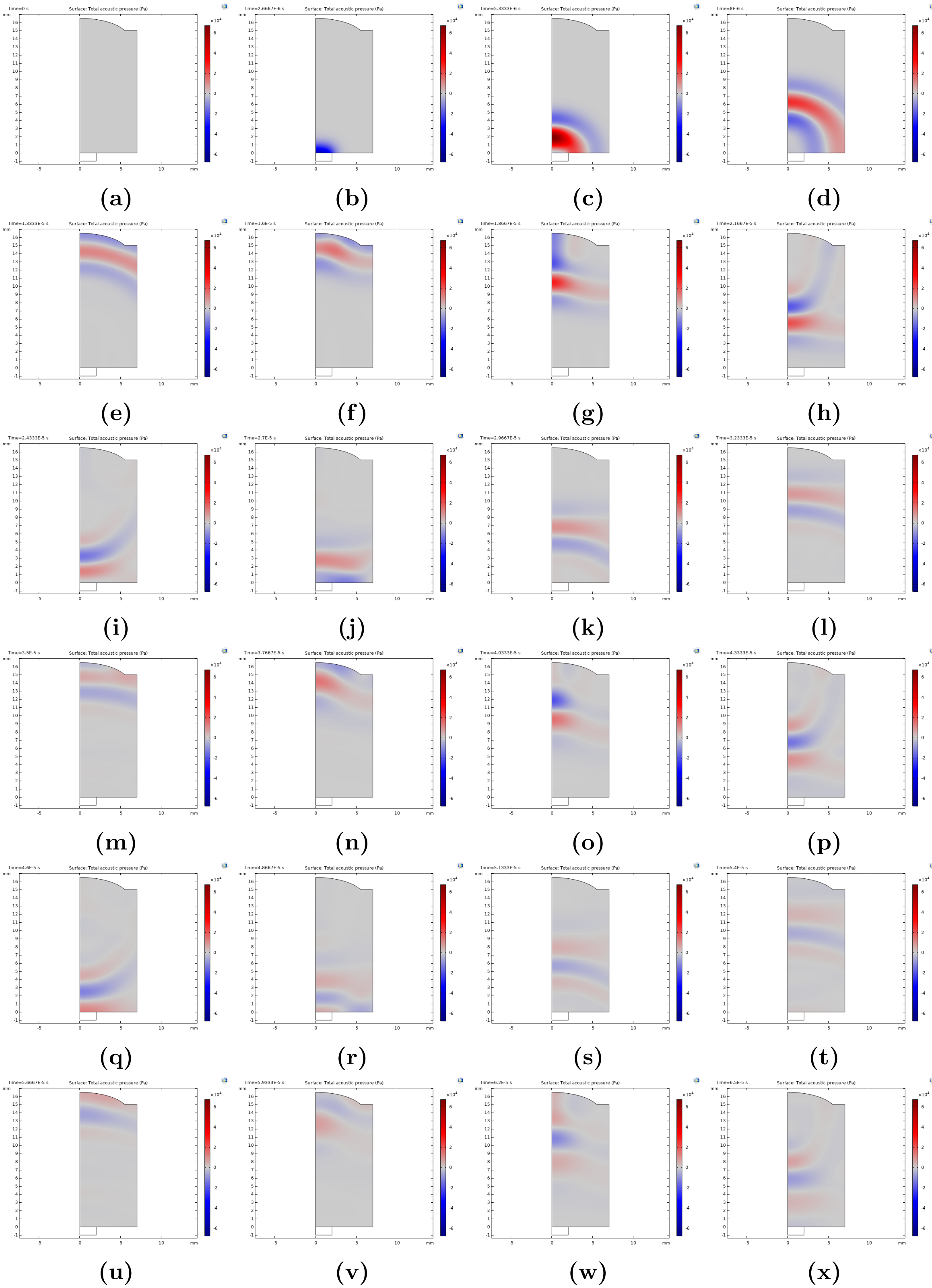


Figure 9: Plot of the acoustic pressure at the center point of the top surface of the transducer for the input signal $V_2(t)$.



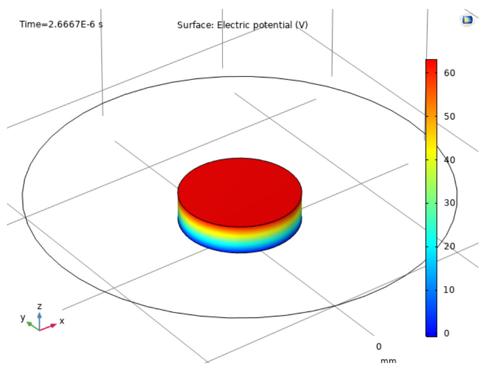
(a) (b) (c) (d) (e) (f) (g) (h) (i) (j) (k) (l) (m) (n) (o) (p) (q) (r) (s) (t) (u) (v) (w) (x)

Figure 10: The time evolution of Acoustic Pressure waves inside the water domain of the setup (time moves forward from Fig. 10a to Fig. 10x). The GIF is attached here.

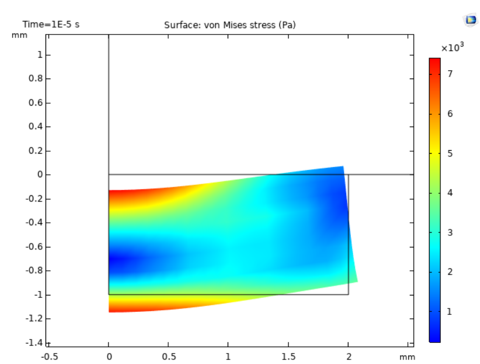
Figure 9 shows the time dependent behaviour of total acoustic pressure at the point of interest. As was observed previously, the waves appearing at 25 μs and 50 μs are the first and second echoes of the original pressure signal. Figure 10 on the other hand shows the travelling acoustic pressure waves inside the water domain.

The generation of pressure waves is a consequence of the development of the stress and the electric potential on the piezoelectric material. When the input voltage signal is applied, that leads to accumulation of charge on the surface of the transducer and also leads to generation of an electric potential. This causes the piezoelecric material to deform under the stress and therefore leads to the acoustic pressure waves.

The preview of the electric potential at $T = 2.6\ \mu$s (Fig 11a) and the von Mises stress inside the piezoelectric material at $T = 10\ \mu$s (Fig. 11b) in the Figure 11 below. The animated GIFs are attached in the caption of the figures.



(a) Electric Potential. (GIF here)



(b) von Mises Stress. (GIF here)

Figure 11

5 Conclusions

Thus, we were able to perform simulations on the application of a piezoelectric transducer as both a transmitter and receiver. Briefly, the application of a voltage signal lead to generation of stress in the piezoelectric material which led it to behave like a transmitter by transmitting an acoustic pressure wave. When the wave comes back after reflection from the top of the water domain, the piezoelectric material deforms due to the wave and induces a voltage, hence acting like a receiver. We studied the time dependent behaviour of electric potential and the von Mises stress on the PZ material, studied the propogation of acoustic pressure waves together with voltage signals induced at the terminal.

References

[1] S. Cochran, *Piezoelectricity and basic configurations for piezoelectric ultrasonic transducers.* (link)

[2] COMSOL Blog, *How to Model Piezoelectric Devices as Both Transmitters and Receivers.* (link)

[3] Introduction to COMSOL Multiphysics, YouTube playlist by *Teacheetah.* (link)