2 $L = \sum_{i=1}^{I} \omega_i \log\left[\frac{1}{1+\exp[-\phi^T x_i]}\right] + \sum_{i=1}^{I} (1-\omega_i)\log\left[\frac{\exp[-\phi^T x_i]}{1+\exp[-\phi^T x_i]}\right]$

We have to prove that : $\frac{\partial L}{\partial \phi} = -\sum_{i=1}^{I}\left(\frac{1}{1+\exp[-\phi^T x_i]} - \omega_i\right) x_i$

Sol^n Using $\log(A/B) = \log(A) - \log(B)$

$$L = \sum_{i=1}^{I}\left[\omega_i \cancel{\log 1}^{0} - \omega_i \log[1+\exp[-\phi^T x_i]]\right] + \sum_{i=1}^{I}\left[(1-\omega_i)[\log \exp[-\phi^T x_i] - \log\{1+\exp[-\phi^T x_i]\}]\right]$$

$$= \sum_{i=1}^{I}\left[-\cancel{\omega_i \log[1+\exp[-\phi^T x_i]]} + \log(\exp[-\phi^T x_i]) - \log[1+\exp[-\phi^T x_i]] - \omega_i \log \exp[-\phi^T x_i] + \cancel{\omega_i \log[1+\exp[-\phi^T x_i]]}\right]$$

$$= \sum_{i=1}^{I}\left[(1-\omega_i)\log(\exp(-\phi^T x_i)) - \log\{1+\exp[-\phi^T x_i]\}\right]$$

$$= \sum_{i=1}^{I}\left[-(1-\omega_i)(\phi^T x_i) - \log\{1+\exp[-\phi^T x_i]\}\right] \quad \text{—— (1)}$$

Taking derivative of (1) w.r.t. $d\phi$, we get

$$\frac{\partial L}{\partial \phi} = \sum_{i=1}^{I}\left[(1-\omega_i) - \frac{\exp[-\phi^T x_i]}{1+\exp[-\phi^T x_i]}\right](-x_i)$$

$$= \sum_{i=1}^{I}\left[\frac{1+\cancel{\exp[-\phi^T x_i]} - \omega_i - \omega_i \exp[-\phi^T x_i] - \cancel{\exp[-\phi^T x_i]}}{1+\exp[-\phi^T x_i]}\right](-x_i)$$

$$= \sum_{i=1}^{I}\left[\frac{1}{1+\exp[-\phi^T x_i]} - \frac{\omega_i \cancel{(1+\exp[-\phi^T x_i])}}{\cancel{(1+\exp(-\phi^T x_i))}}\right](-x_i)$$

$$\frac{\partial L}{\partial \phi} = \sum_{i=1}^{I}\left[\frac{1}{1+\exp[-\phi^T x_i]} - \omega_i\right](-x_i)$$ (Hence, Proved)