Fresnel Diffraction

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INTRODUCTION

Simpson's rule is a method of numerical integration. In this problem it is applied to a Fresnel diffraction from an aperture. Simpson's rule shows advantages over other basic types because it makes use of quadratic interpolation to approximate an integral closer to the actual value in fewer steps, it's the numerical approximation of definite integrals. The method was found by the mathematician Thomas Simpson.

The problem describes incident waves that travel parallel to the z-axis and are diffracted by an aperture. The diffraction pattern is observed on a screen a distance z away from the aperture. In the Fresnel approximation, the screen is close to the aperture in comparison to the size of the aperture, so the variation of the separation distance z is strongly visible in the scattering pattern on the screen. The Fresnel diffraction equation for near-field scattering is used to determine the diffraction pattern created by waves passing through an aperture when viewed from close to the aperture [1].

This exercise aimed to allow a better understanding of numerical integration, as well as develop the skill of plotting data in Python.

SECTION A

For this part of the exercise, Simpson's rule is coded to numerically integrate a function f(x) over defined limits.

$$\int_a^b f(x)dx \approx \frac{b-a}{6}\left(f(a) + 4f\left(\frac{b-a}{2}\right) + f(b)\right) \qquad \text{Eq 1}$$

A function which calculates the coefficient for each term in the Simpsons Rule is defined and then the results are summed to calculate the integral. First, a function SimpsonX is defined which takes fixed values for the upper and lower limit. Furthermore, a function totalsum is defined as the sum of the function evaluated at points a and b. Then, two arrays of coefficients of length N+1 are created, one used in a for loop running through all odd numbers, and the second used in a for loop running through all even numbers. The function was evaluated at dxi = a + ih within the loop. This is the area under the curve for the subinterval i. The resulting arrays were named totalsum for both different cases. The returning value for the defined function was then given by b−a/3∗totalsum. This is tested by evaluating sin(x) between 0 and π.

SECTION B

In this section, the function defined in section A is applied to the following integral.

$$X(x, y', z) = \int_{x_2'(y')}^{x_1'(y')} e^{\frac{ik}{2z}(x-x')^2} dx' \qquad \text{Eq 2}$$

The integral shown above could be described as an intermediate step to determine the 2-dimensional integral that corresponds to the Fresnel diffraction equation. It's a 1-dimensional integral integrated over x'. The result is then used to plot |X(x)|^2 against x using two arrays to plot the relationship formed by the Fresnel integrals.

When the value for N is changed, the shape of the function stays the same, but it does however start to approximate the fit better the larger the value. This means that the theoretical plot is not reflected as well for a small N as it is for a bigger one. When the value for z is changed, so is the graph. From a far-field diffraction we would expect a single slit diffraction, however this is not what we see from the Fresnel diffraction. However, the bigger the distance from the aperture, the more the graph looks like a single slit scattering.

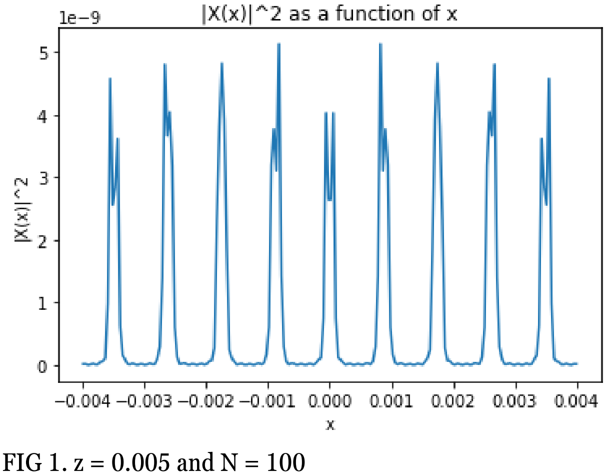

FIG 1. z = 0.005 and N = 100

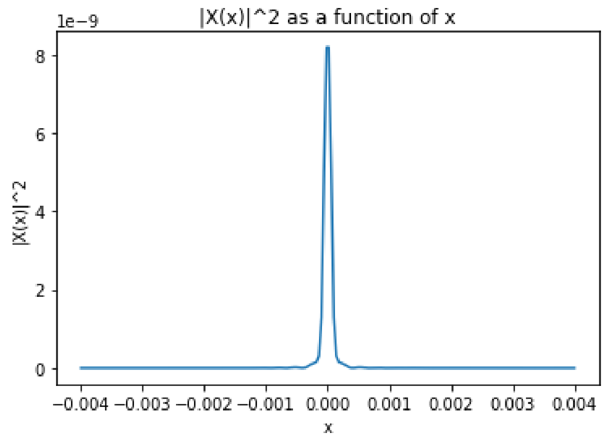

FIG 2. z = 0.01 and N = 1000

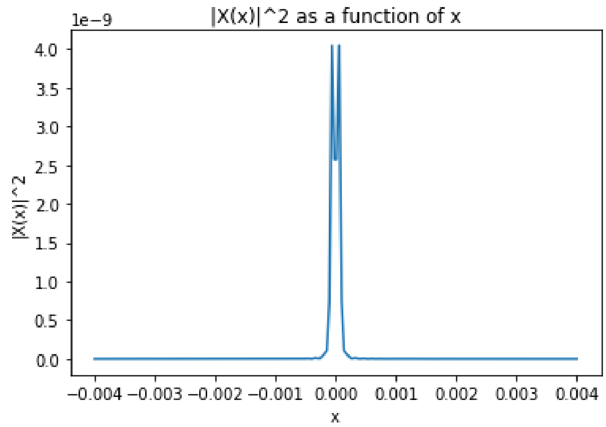

FIG 3. z = 0.005 and N = 1000

SECTION C

For this section, we used the same process as section B to create an electric field intensity plot for x and y, like a 2-dimensional version of Simpson's rule (equation 2). When this equation is evaluated, it allows the 2-dimensional Fresnel equation to be reduced to a 1-dimensional one. The aim of this section is to find the 2-dimensional intensity and create an image of the diffraction pattern with it. The electric field at a point is determined using the Fresnel equation below.

$$E(x,y,z) = \frac{kE_0}{2\pi z}\int_{y_2'}^{y_1'} X(x,y',z)e^{\frac{ik}{2z}(y-y')^2}dy' \qquad \text{Eq 3}$$

We know the wavenumber k is 2pi/lambda, which means

$$I(x,y,z) = \epsilon_0 c E(x,y,z)E^*(x,y,z) \qquad \text{Eq 4}$$

The exponent is defined as a new function called SimpsonY. The process is basically the previous 1-dimensional integral between the x' limits with an incrementing y' value. Also, the resulting values are squared and assigned to a 2-dimensional array of their corresponding x and y values. This array is then plotted on an x and y axis with the intensity as a contour value.

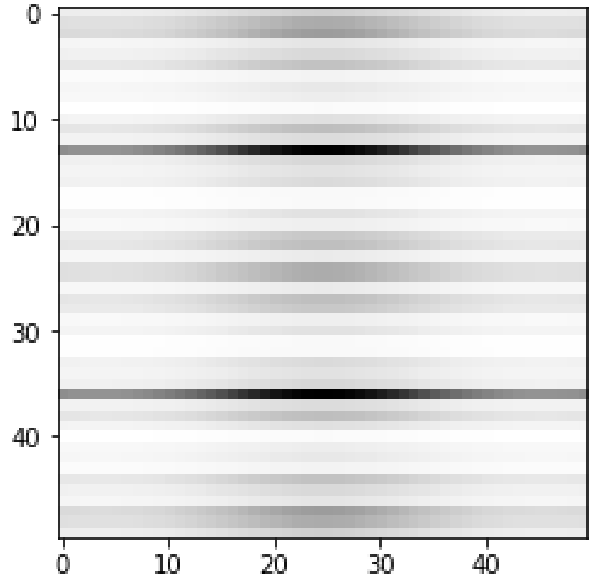

FIG 4. z = 0.005 and N = 50

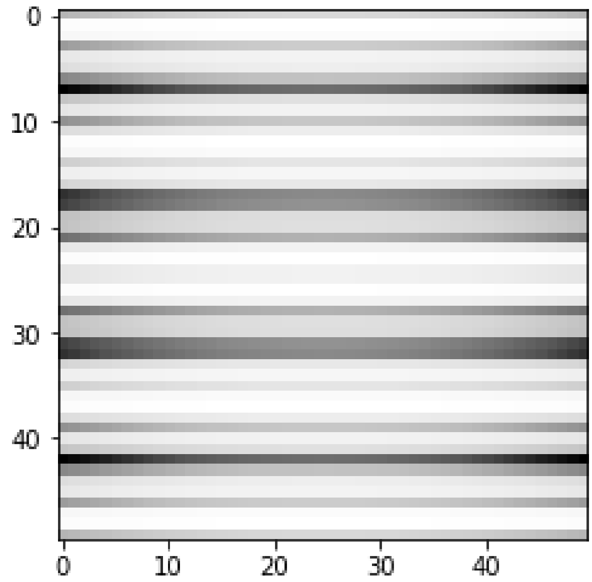

FIG 5. z = 0.01 and N = 50

CONCLUSION

The code seemed to give good results for sections A and B, however a bit confusing in section C. The patterns don't seem to be clear. This could be due to choosing an aperture and screen size that is not appropriate. The code for this section is quite slow, which means there is room for improvement, but the Fresnel diffraction can still be interpreted from our results.

REFERENCES

[1] Dr Simon Hanna. *Computational Physics Exercise 3*. University of Bristol, 2019/2020.