Mapping an Elliptical Trajectory of a Rocket in a Low Earth Orbit

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INTRODUCTION

This programme was made to determine the displacement of a rocket in an orbit. Assuming the problem is two dimensional, the force acting on the rocket is the gravitational force, and from Newton's second law,

$$m\frac{d\vec{v}}{dt} = F_G = -\frac{GMm}{r^3}\vec{r}$$

We can find the acceleration of the rocket in a circular orbit and extend it to an elliptical one,

$$\frac{d\vec{v}}{dt} = -\frac{GM}{r^3}\vec{r}$$

where,

$$\vec{r}^2 = x^2 + y^2$$

The 4th order Runge-Kutta evaluates a higher number of functions for each step of a function as opposed to the lower order ones. It goes through four evaluations of the differential at each step and was programmed using four equations of motion. Two of them calculate the x and the y coordinates for $\vec{r}$ and the other two calculate the components of the velocity [1].

$$f1(vx) = \frac{dx}{dt} = vx$$

$$f2(vy) = \frac{dx}{dt} = vy$$

$$f3(ax) = \frac{dx}{dt} = ax$$

$$f4(ax) = \frac{dx}{dt} = ay$$

ax and ay are the respective coordinates for the rocket's acceleration. We also need to consider the moon's gravitational field, so we add an extra term to the acceleration in the x direction, using $\vec{r}^2 = x^2 + y^2$,

$$\frac{dv}{dt} = -\frac{GM_{Earth}x}{(x^2+y^2)^{3/2}} - \frac{GM_{Moon}(x - D_{ME_x})}{(x - D_{ME_x})(x^2+y^2)^{3/2}}$$

where D_{EM_x} is the separation of the moon and the Earth in the x direction. The first term is the acceleration of the rocket due to the Earth's gravitational field and the second term is the same due to the Moon's gravitational field [1].

SECTION A

For the first part of the exercise, we had to simulate orbits of the rocket around the Earth. Figure 1 shows the elliptical trajectory taken by the rocket, which is placed in an orbit around planet Earth and given enough vertical speed to stay in it. The total energy is conserved, therefore figure 2 should show a straight line, which can be observed in the middle of the graph. This is because it's a combination of kinetic energy and potential energy, so it should stay constant. Figures 3 and 4 show the kinetic and potential energy. As the kinetic energy of the rocket increases, the potential energy decreases by the same amount which further verifies the conservation of the energy that was mentioned above. The initial parameters used for this part of the programme were $x = 6371 \times 10^3\ m$, $y = 0\ m$, $v_x = 0\ m/s$, $v_y = 9 \times 10^3\ m/s$.

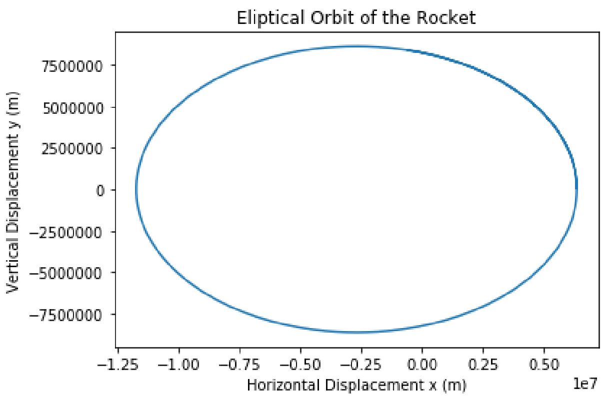

FIG 1. Displacement y plotted against displacement x. The displacement of a rocket follows the trajectory of an ellipse.

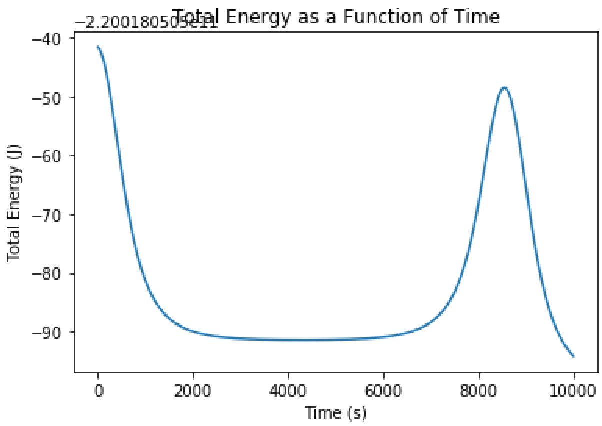

FIG 2. Total energy plotted against time.

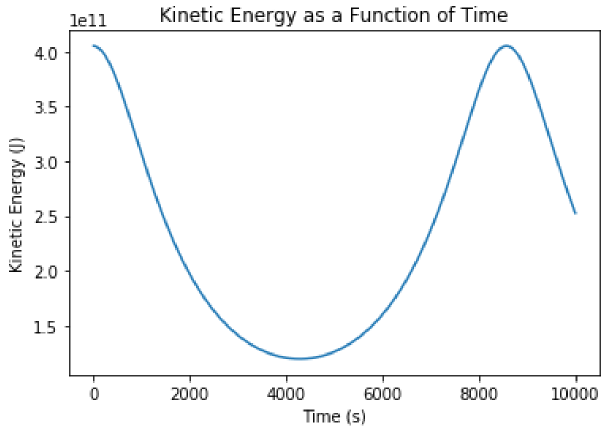

FIG 3. Kinetic energy plotted against time.

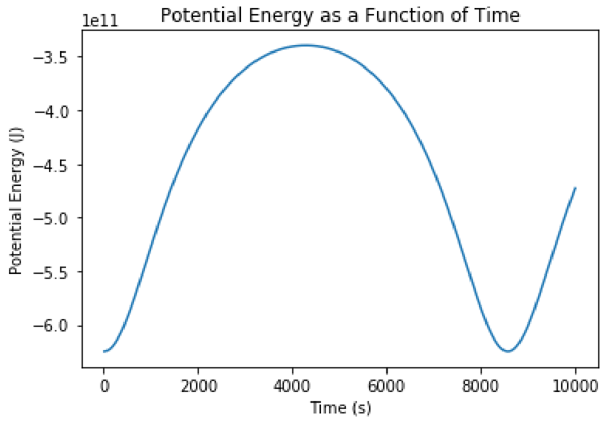

FIG 4. Potential energy plotted against time.

SECTION-B

For this part of the programme we had to simulate the motion of the rocket as it flies from the Earth to the Moon and back, using the same 4th order Range-Kutta method as section A. The orbit simulated should resemble an infinity sign, however looking at figure 5 it looks like the wrong parameters were chosen. The graphs below show planet Earth in blue, the Moon in red and the trajectory of the rocket in orange. Figure 5 is at parameters $x = 0\,m$, $y = 6371 \times 10^3\,m$, $v_x = 9 \times 10^3\,m/s$, and $v_y = 0\,m/s$. We can observe that the rocket orbits around the Earth. By increasing the horizontal velocity along the x axis, there should be a value between 10,000 m/s and 11,000 m/s where the rocket orbits around both the Earth and the Moon resembling the number 8. However, when the velocity is increased to its maximum possible value, $v_x = 11 \times 10^3\,m/s$, we can observe that the rocket doesn't get anywhere near the Moon.

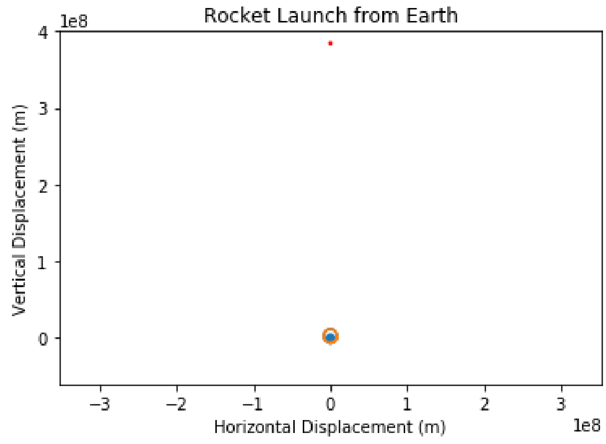

FIG 5. Vertical displacement plotted against horizontal displacement. Rocket launch from Earth (blue) and around the Moon (red).

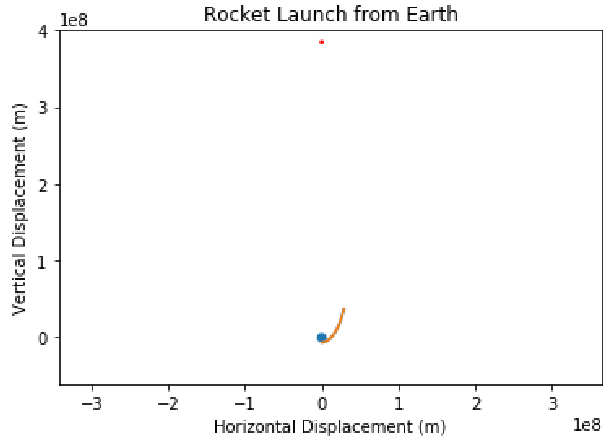

FIG 6. Vertical displacement plotted against horizontal displacement. Rocket launch from Earth (blue) and around the Moon (red).

CONCLUSION

The programme shows the effectiveness of the application of the 4th order Range-Kutta to solve ODEs. It perfectly represents physical systems like a rocket's orbit around the Earth and could be modified for other systems too.

REFERENCES

[1] Dr. Simon Hanna. *Level 5 Computational Physics and Level 6 PHYS30009 Intro to Computational Physics. Lecture 6.*