SOLUTION-1:

Given, $f(x) = \begin{cases} cx^2 & 0 < x < 3 \\ 0 & otherwise \end{cases}$

Since it is a pdf, so we have, $\int_0^3 \mathrm{f(x)dx} = 1$

⇨ $\int_0^3 \mathrm{cx^2.dx} = 1$

⇨ $\left(\frac{cx^3}{3}\right)|_0^3 = 1$

⇨ $c\frac{27}{3} = 1$

⇨ $c = \frac{1}{9}$

To find P(1 < X < 2), we have, $\int_1^2 \frac{1}{9}\mathrm{x^2.dx}$

$= \frac{1}{9}.\frac{x^3}{3}|_1^2 = \frac{1}{9}.\frac{8}{3} - \frac{1}{9}.\frac{1}{3} = \frac{7}{27}$

SOLUTION-2:

Let E be the event of tossing a fair coin 100 times.
Let p be the probability of getting heads and (1-p) is the probability of getting tails.
p = $\frac{1}{2}$ and 1-p = $\frac{1}{2}$
Here, n be the number of tosses. So, n=100
Since, it is a binomial distribution so the mean will be np

⇨ μ = np

⇨ μ = 100 x $\frac{1}{2}$ = 50

Variance, σ^2 = np(1-p)

⇨ σ^2 = 100 x $\frac{1}{2}$ x $\frac{1}{2}$ =25

Standard deviation, σ = √ np(1-p)
= √25 = 5

SOLUTION-3:

Given, Mean, μ = 151 and Standard deviation, σ = 15

Since the weights are normally distributed, z = $\frac{X-\mu}{\sigma}$
Students weight,

(a) **between 120 and 155 lbs** :-

for X = 120, Z = $\frac{X-\mu}{\sigma} = \frac{120-151}{15}$ = -2.067

for X = 155, Z = $\frac{X-\mu}{\sigma} = \frac{155-151}{15}$ = 0.267

P(120< x <155) = P(-2.067 < z <0.267)

$= \int_{-2.07}^{0.27} f(z).dz$

$= \int_{0}^{0.27} f(z).dz + \int_{0}^{2.07} f(z).dz$

= 0.1026 + 0.4803 = 0.5829

Now, the number of students = 500

So, the number of students weighing between 120 lbs and 155 lbs

= 0.5829 x 500 = **291**

(b) **more than 185 lbs** :-

for X = 185, Z = $\frac{X-\mu}{\sigma} = \frac{185-151}{15}$ = 2.267

P(x >185) = P(z >2.267) = 1- P(z <2.267)

$= 1 - \int_{0}^{2.27} f(z).dz)$

= 1 - 0.9881 = 0.0119

Now, the number of students = 500

So, the number of students weighing more than 180 lbs

= 0.0119 x 500 = **6(approx.)**

SOLUTION-4:

Given, the probability that an individual will suffer a bad reaction from injection of a given serum,

p = 0.001 and number of individuals, n = 2000 individuals.

The given data is in Poisson distribution, so, μ = n x p =0.001 x 2000 = 2

Probability that individuals will suffer a bad reaction,

(a) exactly 3:

P(x = 3) = $\frac{e^{-\mu}\,\mu^{x}}{x!}$

$= \frac{e^{-2}\,2^{3}}{3!} = \frac{8}{6e^{2}}$ = 0.1805

(b) more than 2:

$P(x > 2) = 1 - P(x \le 2)$

$= 1 - [P(x = 0) + P(x = 1) + P(x = 2)]$

$= 1 - [\frac{e^{-\mu}\mu^x}{x!} + \frac{e^{-\mu}\mu^x}{x!} + \frac{e^{-\mu}\mu^x}{x!}]$

$= 1 - [\frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{-2}2^2}{2!}]$

$= 1 - [\frac{1}{e^2} + \frac{2}{e^2} + \frac{2}{e^2}] = 1 - 0.6768 = 0.3232$

SOLUTION-5:

Given, Mean weight of 500 ball bearings, μ = 5.02 oz.

Standard deviation, $\sigma_x = \frac{\sigma}{\sqrt{n}}(\sqrt{\frac{N-n}{N-1}}) = \frac{0.3}{\sqrt{100}}(\sqrt{\frac{500-100}{500-1}}) = 0.027$

Sample of 100 has a combined weight of 510 oz.
So, mean weight = 510/100 = 5.1 oz.

Now, Z = = $\frac{X-\mu}{\sigma} = \frac{5.1-5.02}{0.027} = 2.96$

The probability of taking random sample with combined weight that exceeds 510 oz. is :-
$P(X>5.1) = P(Z > 2.96) = 1- P(Z <2.96)$

$= 1 - \int_0^{2.96} f(z).dz)$

= 1 - 0.99846 = 0.00154

SOLUTION-6:

(a) Let x and y be the precipitation for next 2 years.

Mean of x+y, μ(x+y) = μ(x) +μ(y) = 12.08 +12.08 = 24.16

Standard deviation of x+y, $\sigma(x+y) = \sqrt{[(3.1)^2 + (3.1)^2]} = 4.38$

Now, Z = = $\frac{X-\mu}{\sigma} = \frac{25-24.16}{4.38} = 0.1917$

The probability that precipitation totals for next 2 years will exceed 25 :-

P(x+y>25) = P(z > 0.19) = 1- P(Z <0.19)

$= 1 - \int_0^{0.19} f(z).dz)$

= 1 - 0.57535 = 0.4247

(b) Let y be the precipitation for 3rd year.

Mean of x-y, μ(x-y) = μ(x) - μ(y) = 12.08 -12.08 = 0

Standard deviation of x-y, σ(x-y) = √[(-1)^2 x (3.1)^2] = 4.38

We have to find the probability that next year's precipitation will exceed that of the following year by more than 3 inches.

So, Z = = $\frac{X-\mu}{\sigma}$ = $\frac{3-0}{4.38}$ = 0.685

P(x-y>3) = P(z > 0.69) = 1- P(Z <0.69)

$= 1 - \int_0^{0.69} f(z).dz)$

= 1 - 0.7549 = 0.2451

SOLUTION-7:

Let, sample size be n
Given, standard deviation, σ = 0.3

95 percent confidence interval estimate is within $(X_{bar} - 1.96\frac{0.3}{\sqrt{n}}, X_{bar} + 1.96\frac{0.3}{\sqrt{n}})$

We are 95 percent certain that X_{bar} is within 0.1 of μ,

- ⇨ $1.96\frac{0.3}{\sqrt{n}}$ ≤ 0.1
- ⇨ $\frac{0.588}{\sqrt{n}}$ ≤ 0.1
- ⇨ $\sqrt{n} \geq 5.88$
- ⇨ n ≥ 34.57 =**35(approx.)**