

Q1. You flip a coin 100 times and it lands on heads 44 times. You then use the same coin and do another 100 flips. This time it lands on heads 49 times. You repeat this experiment a total of 10 times and get the following results for the number of heads.

{44;49;52;62;53;48;54;49;46;51}

Compute the mean and variance of this data set.

**N=10, Sum = 508, Mean =  $508/10 = 50.8$**

$X_i$	$X_i - X_{\text{bar}}$	$(X_i - X_{\text{bar}})^2$
44	-6.8	46.24
49	-1.8	3.24
52	1.2	1.44
62	11.2	125.44
53	2.2	4.84
48	-2.8	7.84
54	3.2	10.24
49	-1.8	3.24
46	-4.8	23.04
51	0.2	0.04
<b><math>\sum X_i = 508</math></b>		<b><math>\sum (X_i - X_{\text{bar}})^2 = 225.6</math></b>

$$\text{Variance, } \sigma^2 = \sum (X_i - X_{\text{bar}})^2 / n$$

$$= 225.6/10 = 22.56$$

Q2: Complete the table below to calculate the standard deviation of

{57; 53; 58; 65; 48; 50; 66; 51}

**N=8, Sum = 448, Mean =  $448/8 = 56$**

$X_i$	$X_i - X_{\text{bar}}$	$(X_i - X_{\text{bar}})^2$
57	1	1
53	-3	9
58	2	4
65	9	81
48	-8	64
50	-6	36
66	10	100
51	-5	25
<b><math>\sum X_i = 448</math></b>		<b><math>\sum (X_i - X_{\text{bar}})^2 = 320</math></b>

- Calculate the variance using the completed table.

$$\text{Variance, } \sigma^2 = \sum (X_i - X_{\text{bar}})^2 / n$$

$$= 320/8 = 40$$

- Then calculate the standard deviation.

$$\text{Standard deviation, } \sigma = \sqrt{\sum (X_i - X_{\text{bar}})^2 / n}$$

$$= \sqrt{40} = 6.325$$

Q3. You grow 20 crystals from a solution and measure the length of each crystal in millimeters. Here is your data:

9, 2, 5, 4, 12, 7, 8, 11, 9, 3, 7, 4, 12, 5, 4, 10, 9, 6, 9, 4

Calculate the sample standard deviation of the length of the crystals.

**N=20, Sum = 140, Mean = 140/20 = 7**

$X_i$	$X_i - X_{\text{bar}}$	$(X_i - X_{\text{bar}})^2$
9	2	4
2	-5	25
5	-2	4
4	-3	9
12	5	25
7	0	0
8	1	1
11	4	16
9	2	4
3	-4	16
7	0	0
4	-3	9
12	5	25
5	-2	4
4	-3	9
10	3	9
9	2	4
6	-1	1
9	2	4
4	-3	9
<b><math>\sum X_i = 140</math></b>		<b><math>\sum (X_i - X_{\text{bar}})^2 = 178</math></b>

$$\text{Standard deviation, } \sigma = \sqrt{\sum (X_i - X_{\text{bar}})^2 / (n-1)} \text{ (for sample)}$$

$$= \sqrt{178 / (20-1)} = 3.06$$

Q4. A realtor tells you that the average cost of houses in a town is \$176,000. You want to know how much the prices of the houses may vary from this average. What measurement do you need?

- (A) standard deviation
- (B) interquartile range
- (C) variance
- (D) percentile
- (E) Choice (A) or (C)

**Solution:** - (E) Choice (A) or (C). Because, both the standard deviation and variance describe the variation in the data with respect to the average.

Q5: You take a random sample of ten car owners and ask them, “To the nearest year, how old is your current car?” Their responses are as follows: 0 years, 1 year, 2 years, 4 years, 8 years, 3 years, 10 years, 17 years, 2 years, and 7 years. To the nearest year, what is the standard deviation of this sample?

**N=10, Sum = 54, Mean = 54/10 = 5.4**

$X_i$	$X_i - X_{\text{bar}}$	$(X_i - X_{\text{bar}})^2$
0	-5.4	29.16
1	-4.4	19.36
2	-3.4	11.56
4	-1.4	1.96
8	2.6	6.76
3	-2.4	5.76
10	4.6	21.16
17	11.6	134.56
2	-3.4	11.56
7	1.6	2.56
$\sum X_i = 54$		$\sum (X_i - X_{\text{bar}})^2 = 244.4$

$$\begin{aligned} \text{Standard deviation, } \sigma &= \sqrt{\sum (X_i - X_{\text{bar}})^2 / (n-1)} \text{ (for sample)} \\ &= \sqrt{244.4 / (10-1)} = 5.21 \end{aligned}$$

Q6: Two companies pay their employees the same average salary of \$42,000 per year. The salary data in Ace Corp. has a standard deviation of \$10,000, whereas Magna Company salary data has a standard deviation of \$30,000. What does this mean?

**Solution:** - The standard deviation measures on average how spread out the data is and higher the standard deviation, more the variability. The standard deviation for Magna Company is greater than Ace Corp. This interprets that there is more variation in salaries in Magna Company than in Ace Corp.

Q7: Thirty farmers were asked how many farm workers they hire during a typical harvest season. Their responses were:

4, 5, 6, 5, 3, 2, 8, 0, 4, 6, 7, 8, 4, 5, 7, 9, 8, 6, 7, 5, 5, 4, 2, 1, 9, 3, 3, 4, 6, 4

Calculate Standard Deviation. (Hint: - Covert the data into Frequency Table)

$X_i$	$f_i$	$X_i \times f_i$	$X_i - X_{\text{bar}}$	$(X_i - X_{\text{bar}})^2$	$(X_i - X_{\text{bar}})^2 \times f_i$
0	1	0	-5	25	25
1	1	1	-4	16	16
2	2	4	-3	9	18
3	3	9	-2	4	12
4	6	24	-1	1	6
5	5	25	0	0	0
6	4	24	1	1	4
7	3	21	2	4	12
8	3	24	3	9	27
9	2	18	4	16	32
	$\Sigma f_i = 30$	$\Sigma (X_i \times f_i) = 150$		$\Sigma (X_i - X_{\text{bar}})^2 = 150$	$\Sigma (X_i - X_{\text{bar}})^2 \times f_i = 152$

$$\Sigma (X_i \times f_i) = 150, \Sigma f_i = 30$$

$$\text{Mean} = \Sigma (X_i \times f_i) / \Sigma f_i = 150 / 30 = 5$$

$$\begin{aligned} \text{Standard deviation, } \sigma &= \sqrt{\Sigma ((X_i - X_{\text{bar}})^2 \times f_i) / \Sigma f_i} \\ &= \sqrt{152 / 30} = 2.25 \end{aligned}$$

Q8: 220 students were asked the number of hours per week they spent watching television. With this information, calculate the mean and standard deviation of hours spent watching television by the 220 students.

Hours	Number of students
10 to 14	2
15 to 19	12
20 to 24	23
25 to 29	60
30 to 34	77
35 to 39	38
40 to 44	8

Hours	$X_i$	$f_i$	$X_i \times f_i$	$X_i - X_{\text{bar}}$	$(X_i - X_{\text{bar}})^2$	$(X_i - X_{\text{bar}})^2 \times f_i$
10-14	12	2	224	-17.82	317.55	635.1
15-19	17	12	204	-12.82	164.35	1972.2
20-24	22	23	506	-7.82	61.15	1406.45
25-29	27	60	1620	-2.82	7.95	477
30-34	32	77	2464	2.18	4.75	365.75
35-39	37	38	1406	7.18	51.55	1958.9
40-44	42	8	336	12.18	148.35	1186.8
		$\sum f_i = 220$	$\sum (X_i \times f_i) = 6560$		$\sum (X_i - X_{\text{bar}})^2 = 755.65$	$\sum (X_i - X_{\text{bar}})^2 \times f_i = 8002.2$

$$\sum (X_i \times f_i) = 6560, \sum f_i = 220$$

$$\text{Mean} = \sum (X_i \times f_i) / \sum f_i = 6560 / 220 = 29.82$$

$$\begin{aligned} \text{Standard deviation, } \sigma &= \sqrt{\sum ((X_i - X_{\text{bar}})^2 \times f_i) / \sum f_i} \\ &= \sqrt{8002.2 / 220} = 6.03 \end{aligned}$$

Q9:- Marks scored by Section A and section B of class XI in a mathematics test are given below:

Section A	28	36	34	30	48	22	35	19	27	41
Section B	28	29	33	32	33	34	30	33	34	34

Which section has more variation?

FOR SECTION-A: -  $N=10$ , Sum = 320, Mean =  $320/10 = 32$

FOR SECTION-B: -  $N=10$ , Sum = 320, Mean =  $320/10 = 32$

SECTION-A			SECTION-B		
$X_i$	$X_i - X_{\text{bar}}$	$(X_i - X_{\text{bar}})^2$	$X_i$	$X_i - X_{\text{bar}}$	$(X_i - X_{\text{bar}})^2$
28	-4	16	28	-4	16
36	4	16	29	-3	9
34	2	4	33	1	1
30	-2	4	32	0	0
48	16	256	33	1	1
22	-10	100	34	2	4
35	3	9	30	-2	4
19	-13	169	33	1	1
27	-5	25	34	2	4
41	9	81	34	2	4
$\sum X_i = 320$		$\sum (X_i - X_{\text{bar}})^2 = 680$	$\sum X_i = 320$		$\sum (X_i - X_{\text{bar}})^2 = 44$

FOR SECTION-A: - Standard deviation,  $\sigma = \sqrt{\sum (X_i - X_{\text{bar}})^2 / n}$

$$= \sqrt{680/10} = 8.246$$

FOR SECTION-B: - Standard deviation,  $\sigma = \sqrt{\sum (X_i - X_{\text{bar}})^2 / n}$

$$= \sqrt{44/10} = 2.098$$

Here, for Section-A the standard deviation is higher than the Section-B. This interprets that Section-A has more variability in the marks scored in a mathematics test than Section-B.

Q10:- For instance, 5 friends just measured their height in centimeters. Using the example below, find the mean, variance, and standard deviation.

Name	Height (cm)
Corin	157.48 cm
Jen	165.099 cm
Raffy	172.72 cm
Jessie	152.4 cm
Kat	167.64 cm

N=5, Sum = 815.339

Mean height= 815.339/5 = 163.0678 cm

Name	$X_i$	$X_i - X_{\text{bar}}$	$(X_i - X_{\text{bar}})^2$
Corin	157.48	-5.5878	31.22
Jen	165.099	2.0312	4.13
Raffy	172.72	9.6522	93.16
Jessie	152.4	-10.6678	113.8
Kat	167.64	4.5722	20.91
	$\sum X_i = 815.339$		$\sum (X_i - X_{\text{bar}})^2 = 263.22$

Variance,  $\sigma^2 = \sum (X_i - X_{\text{bar}})^2 / n = 263.22/5 = 52.644$

Standard deviation,  $\sigma = \sqrt{\sum (X_i - X_{\text{bar}})^2 / n} = \sqrt{52.644} = 7.256$