

1. We have a medicine that is being manufactured and each pill is supposed to have 14 milligrams of the active ingredient. What are our null and alternative hypotheses?

The null hypothesis is- $H_0: \mu=14$

The alternate hypothesis is- $H_a: \mu \neq 14$

2. The school principal wants to test if it is true what teachers say – that high school juniors use the computer an average 3.2 hours a day. What are our null and alternative hypotheses?

The null hypothesis is- $H_0: \mu=3.2$

The alternate hypothesis is- $H_a: \mu \neq 3.2$

3. Sacramento County high school seniors have an average SAT score of 1,020. From a random sample of 144 Sacramento High School students we find the average SAT score to be 1,100 with a standard deviation of 144. We want to know if these high school students are representative of the overall population. What are our hypotheses?

The null hypothesis is- $H_0: \mu=1020$

The alternate hypothesis is- $H_a: \mu \neq 1020$

4. A researcher claims that black horses are, on average, more than 30 lbs heavier than white horses, which average 1100 lbs. What is the null hypothesis, and what kind of test is this?

Given, Weight of white horses=1100 lbs

Weight of black horses is 30 lbs more than black horses i.e. $1100+30=1130$ lbs

The null hypothesis is- $H_0: \mu \leq 1130$

The alternate hypothesis is- $H_a: \mu > 1130$

The kind of test is: - One-tailed test or Right-tailed test.

5. A package of gum claims that the flavor lasts more than 39 minutes. What would be the null hypothesis of a test to determine the validity of the claim? What sort of test is this?

The null hypothesis is- $H_0: \mu \leq 39$

The alternate hypothesis is- $H_a: \mu > 39$

The kind of test is: - One-tailed test or Right-tailed test.

6. The school nurse thinks the average height of 7th graders has increased. The average height of a 7th grader five years ago was 145 cm with a standard deviation of 20 cm. She takes a random sample of 200 students and finds that the average height of her sample is 147 cm. Are 7th graders now taller than they were before? Conduct a single-tailed hypothesis test using a .05 significance level to evaluate the null and alternative hypotheses.

Following are the hypotheses:

$H_0: \mu \leq 145$ (null hypothesis) & $H_a: \mu > 145$ (alternate hypothesis)

Given, Sample size, $n = 200$

Sample mean, $X_{\text{bar}} = 147$

Standard deviation, $\sigma = 20$

Level of significance, $\alpha = 0.05$

Z-value at $\alpha = 0.05 = 1.64$

$$Z_{\text{critical}} = (X_{\text{bar}} - \mu) / (\sigma / \sqrt{n})$$

$$= 147 - 145 / (20 / \sqrt{200})$$

$$= \sqrt{2} = 1.414 (\text{approx.})$$

As, $Z_{\text{critical}} < Z_{0.5}$

So, we fail to reject the null hypothesis, H_0 .

7. A farmer is trying out a planting technique that he hopes will increase the yield on his pea plants. The average number of pods on one of his pea plants is 145 pods with a standard deviation of 100 pods. This year, after trying his new planting technique, he takes a random sample of his plants and finds the average number of pods to be 147. He wonders whether or not this is a statistically significant increase. What are his hypotheses and the test statistic?

Following are the hypotheses:

$H_0: \mu \leq 145$ (null hypothesis)

$H_a: \mu > 145$ (alternate hypothesis)

Given, Sample mean, $X_{\text{bar}} = 147$

Standard deviation, $\sigma = 100$

Let, Sample size, $n = 144$

Level of significance, $\alpha = 0.05$

Z-value at $\alpha = 0.05 = 1.64$

$$Z_{\text{critical}} = (X_{\text{bar}} - \mu) / (\sigma / \sqrt{n})$$

$$= 147 - 145 / (100 / \sqrt{144})$$

$$= 0.24$$

As, $Z_{\text{critical}} < Z_{0.5}$

So, we fail to reject the null hypothesis, H_0 .

8. Duracell manufactures batteries that the CEO claims will last an average of 300 hours under normal use. A researcher randomly selected 20 batteries from the production line and tested these batteries. The tested batteries had a mean life span of 270 hours with a standard deviation of 50 hours. Do we have enough evidence to suggest that the claim of an average lifetime of 300 hours is false?

Following are the hypotheses:

$H_0: \mu = 300$ (null hypothesis)

$H_a: \mu \neq 300$ (alternate hypothesis)

Given, Sample size, $n = 20$

Sample mean, $X_{\text{bar}} = 270$

Standard deviation, $\sigma = 50$

Let, Level of significance, $\alpha = 0.05$

Z-value at $\alpha = 0.05 = 1.64$

But, since, here the test is a two-tailed test, we have, Z-value as $Z_{\alpha/2} = Z_{0.025} = 1.96$

$$Z_{\text{critical}} = (X_{\text{bar}} - \mu) / (\sigma / \sqrt{n})$$

$$= 270 - 300 / (50 / \sqrt{20})$$

$$= -2.68$$

As, $Z_{\text{critical}} > Z_{0.025}$

So, we can reject the null hypothesis, H_0 and we can say that the claim of an average lifetime of 300 hours is false.

9. You have just taken ownership of a pizza shop. The previous owner told you that you would save money if you bought the mozzarella cheese in a 4.5 pound slab. Each time you purchase a slab of cheese, you weigh it to ensure that you are receiving 72 ounces of cheese. The results of 7 random measurements are 70, 69, 73, 68, 71, 69 and 71 ounces. Are these differences due to chance or is the distributor giving you less cheese than you deserve?
- State the hypotheses.
 - Calculate the test statistic.
 - Would the null hypothesis be rejected at the 10% level? The 5% level? The 1% level?

a. Following are the hypotheses:

$H_0: \mu = 72$ (null hypothesis)

$H_a: \mu \neq 72$ (alternate hypothesis)

b. Given, Sample size, $n = 7$

Samples are = 70, 69, 73, 68, 71, 69 and 71

Sample mean, $X_{\text{bar}} = \text{Sum} / n$

$$= (70 + 69 + 73 + 68 + 71 + 69 + 71) / 7 = 491 / 7 = 70.14$$

Standard deviation, $\sigma = \sqrt{\sum (X_i - X_{\text{bar}})^2 / (n - 1)}$ (for sample)

$$= \sqrt{[(70 - 70.14)^2 + (69 - 70.14)^2 + (73 - 70.14)^2 + (68 - 70.14)^2 + (71 - 70.14)^2 + (69 - 70.14)^2 + (71 - 70.14)^2] / 6}$$

$$= \sqrt{16.8572 / 6} = \sqrt{2.8095}$$

$$= 1.676$$

c. For level of significance, $\alpha = 10\% = 0.1$

As, the test is two-tailed test, Z-value will be $Z_{\alpha/2} = Z_{0.05} = 1.64$

$$Z_{\text{critical}} = (X_{\text{bar}} - \mu) / (\sigma / \sqrt{n})$$

$$= 70.14 - 72 / (1.676 / \sqrt{7})$$

$$= -2.9315$$

As, $Z_{\text{critical}} > Z_{0.05}$

So, we can reject the null hypothesis, H_0 for 10% level of significance.

For level of significance, $\alpha = 5\% = 0.05$

As, the test is two-tailed test, Z-value will be $Z_{\alpha/2} = Z_{0.025} = 1.96$

$$\begin{aligned} Z_{\text{critical}} &= (\bar{X} - \mu) / (\sigma / \sqrt{n}) \\ &= 70.14 - 72 / (1.676 / \sqrt{7}) \\ &= -2.9315 \end{aligned}$$

As, $Z_{\text{critical}} > Z_{0.025}$

So, we can reject the null hypothesis, H_0 for 5% level of significance.

For level of significance, $\alpha = 1\% = 0.01$

As, the test is two-tailed test, Z-value will be $Z_{\alpha/2} = Z_{0.005} = 4.32$

$$\begin{aligned} Z_{\text{critical}} &= (\bar{X} - \mu) / (\sigma / \sqrt{n}) \\ &= 70.14 - 72 / (1.676 / \sqrt{7}) \\ &= -2.9315 \end{aligned}$$

As, $Z_{\text{critical}} < Z_{0.005}$

So, we fail to reject the null hypothesis, H_0 for 1% level of significance.

10. The high school athletic director is asked if football players are doing as well academically as the other student athletes. We know from a previous study that the average GPA for the student athletes is 3.10. After an initiative to help improve the GPA of student athletes, the athletic director randomly samples 20 football players and finds that the average GPA of the sample is 3.18 with a sample standard deviation of 0.54. Is there a significant improvement? Use a 0.05 significance level.

Following are the hypotheses:

$H_0: \mu = 3.10$ (null hypothesis)

$H_a: \mu \neq 3.10$ (alternate hypothesis)

Given, Sample size, $n = 20$

Sample mean, $\bar{X} = 3.18$

Standard deviation, $\sigma = 0.54$

Level of significance, $\alpha = 0.05$

Z-value at $\alpha = 0.05 = 1.64$

But, since, here the test is a two-tailed test, we have, Z-value as $Z_{\alpha/2} = Z_{0.025} = 1.96$

$$\begin{aligned} Z_{\text{critical}} &= (\bar{X} - \mu) / (\sigma / \sqrt{n}) \\ &= 3.18 - 3.1 / (0.54 / \sqrt{20}) \\ &= 0.663 \end{aligned}$$

As, $Z_{\text{critical}} < Z_{0.025}$

So, we fail to reject the null hypothesis, H_0 .