**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: We have *μ* = 45 and *σ* = 8 as data is normally distributed.

X be the amount of time taking to repair a customer’s car.

As per the given condition the work will start after 10 min so the mean will be 45+10=55

The car will be ready after 1 hour(X)= 60

Therefore,

Z= (60-55)/8 = 0.625

According to Z table, the Z value will be 0.73237

Therefore, the probability that service manager cannot meet his commitment = 1-0.73237

= 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: a) *μ* = 38 and *σ* =6

P(X>44) = 1- P(X<=44)

Z score = (X-*μ*)/ *σ* = (44-38)/6 = 1

Therefore, from Z table, P(X<=44) = P(Z<=1) = 0.84134

Probability that employees will be greater than 44

P(X>44) = 1- P(X<=44) = 1-0.84134 = 0.1587 = 15.87%

Probability of employees between 38 and 44:

= P(X<=44)-P(X>=38)

For P(X>=38) = P(Z>=(38-38/6))=P(Z>=0)=0.5

Here, P(X<=44) = 0.84134

= 0.84134-0.5 = 0.34134 = 34.13%

So, the given statement “More employees at the processing center are older than 44

than between 38 and 44” is False.

b) P(X<30)

Z = (X-*μ*)/ *σ* = (30-38)/6 = -1.3333

P(X<30) = P(Z<30)= P(Z<-1.3333)= 0.09176 = 9.17%

Total number of employees with probability 0.0917 being under 30:

= 400\*0.09176 = 36.68

The statement, “training program for employees under the age of 30 at the center

would be expected to attract about 36 employees” is True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: X1 ~ N(μ, σ2)

2X1 ~ N(2μ,4 σ2)

If X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) then

X1+X2~N(2μ,2 σ2)

Hence, 2X1-(X1+X2)~ (2μ-2μ ,4 σ2 +2 σ2 )

The distribution remains the same for every sample subset therefore it will fall under normal distribution.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: The probability of getting value between a and b is 0.99

So, the probability of getting value outside a and b = 1-0.99 = +/- 0.01

The probability towards left a = -0.01/2 =-0.005

The probability towards right b = 0.01/2 = 0.005

To calculate probability of X, we have to find X from the Z distribution :

Z= (X-*μ*)/ *σ*

For the probability of 0.005, value of z is -2.57

Therefore,

Z\**σ*  + *μ* = X

-(-2.57)\*20+100=151.4

(-2.57)\*20+100= 48.6

The correct ans is D.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Total profit = profit1+profit2

Mean = profit1(mean)+profit2(mean)

=5+7=12

Std= sqrt(9+6)

= sqrt(25)

=5

Mean in rs = 12\*45= 540

Std in rs = 5\*45=225

1. Range for 95%

Stats.norm.interval (0.95,540,225)

Range is (99.008,980.991) i.e., 9.9 to 98.1 crore rs

1. 5th percentile

Z\_score=stats.norm.ppf(0.05)

= -1.6448

Z\_value=x\_bar-mean/std

X\_bar (5th per) = (z\_value\*std)+mean

=(-1.6448)\*225+540

= 169.92= 170 in million = 17.0 crore rs

1. Probability of 1st division making loss = stats.norm.cdf(0,5,3)

= 0.0479

Probability of 2nd division making loss =stats.norm.cdf(0,7,4)

= 0.04005

1st division can make more loss than 2nd division